Fall 2018 14.01 Problem Set 6

Problem 1: True/False/Uncertain (20 points)

Please fully explain your answer. Points are awarded based on explanations.

1. (4 points) In a two-player game, a Nash equilibrium is the outcome that maximizes the sum of the players' payoffs.
2. (4 points) In a Nash equilibrium in a two-player game, both players must have selected a dominant strategy.
3. (4 points) Repeatedly playing the Prisoner's Dilemma may or may not result in a cooperative solution.
4. (4 points) In the models of oligopoly considered in class, the equilibrium price will be strictly lower if there are $n + 1$ firms than if there are n firms.
5. (4 points) In the models of oligopoly considered in class, consumers are no better off than in a perfectly competitive market.

Problem 2: Game Theory (15 points)

For each of the following games:

(i) (2.5 points) Find the dominant strategies (if any)

(ii) (2.5 points) Find the Nash equilibria (if any).

1. (5 points total)

		Player 2	
		Left	Right
Player 1	Up	$(0, 0)$	$(2, 2)$
	Down	$(1, 1)$	$(0, 0)$

2. (5 points total)

		Player 2	
		Left	Right
Player 1	Up	$(1,1)$	$(2,0)$
	Down	$(0,1)$	$(1,5)$

3. (5 points total)

		Player 2	
		Left	Right
Player 1	Up	$(1,2)$	$(2,1)$
	Down	$(2,1)$	$(1,2)$

Problem 3: Rock Paper, Scissors (10 points)

Consider the game Rock, Paper, Scissors. Both players simultaneously choose one of the three options Rock, Paper or Scissors. A player who plays R will beat another player who has chosen S ("rock crushes scissors") but will lose to one who has played P ("paper covers rock"); a play of P will lose to a play of S ("scissors cut paper"). Assume that the payoffs are the following: when a player wins, the payoff is 1 and when she loses the payoff is -1. If both players choose the same item and there is a tie, each get 0 points.

		Player 2		
		Rock	Paper	Scissors
	Rock	$(0,0)$	$(-1,1)$	(1,-1)
Player 1	Paper	$(1,-1)$	$(0,0)$	(-1,1)
	Scissors	$(\delta,1)$	$(1,-1)$	(0,0)

1. (5 points) Let $\delta = -1$. Find the dominant strategies (if any) and pure strategy Nash equilibria (if any).

2. (5 points) For which values of δ (Scissors, Rock) becomes a pure strategy Nash equilibrium?

Problem 4: Comparing Models (27 points)

Suppose that the food truck Cheesy Burgers has the monopoly of burgers in the MIT campus and its cost function is given by $C(Q) = 2Q$. The food truck studied the demand for burgers at MIT and found out that the inverse demand function can be written as $P(Q) = 38 - 2Q$.

1. (5 points) What is the socially efficient output level Q^e and the optimal price P^e? What would the firm's profits be?

2. (5 points) Solve for the equilibrium quantity Q^m produced by the monopolist and the equilibrium price P^m. What are the firm's profits?

3. (5 points) Suppose that Greasy Burgers, with the same cost function above, sees that Cheesy Burgers is making a lot of money and enters the market. Both firms compete as in the Cournot model. Solve for each firm's equilibrium quantity (q_C^d, q_G^d), the equilibrium price P_d^*, each firms' profits and total profits.

4. (5 points) Suppose further that Cheesy Burgers and Greasy Burgers make an agreement to avoid competition and form a cartel. What are the equilibrium quantities (q_C^c, q_G^c), the equilibrium price P^c and firms' profits?

5. (7 points) Compute the consumer surplus for each equilibrium, as well as the aggregate surplus (calculated as the sum of consumer surplus and total profits). How do the 4 different equilibria compare in terms of CS, total profits and aggregate surplus? Explain the intuition. Calculate the deadweight loss associated to each non-competitive equilibrium.

Problem 5: Variations in Cournot Competition (28 points)

In class we saw the Cournot competition model for two firms with the same cost function. Now, we are going to consider assymetric cost functions. Assume that demand for a good is given by $p = a - bQ^d$, and that there are 2 firms competing in quantities. Both have no fixed costs and a constant marginal cost. Firm 1 has a marginal cost c_1, and firm 2 has a marginal cost c_2. We have that $a > c_1 > c_2$.

1. (7 points) Find the reaction functions of firms 1 and 2 in this market: how does the optimal quantity produced depend on the quantity produced by the other firm?

2. (7 points) Solve for the quantity produced by each firm and the equilibrium price. Which firm produces a higher quantity? Give an intuitive reason for this.

3. (7 points) What will be the equilibrium price and the quantity produced by each firm if they compete in prices (Bertrand competition)?

4. (7 points) What is the competitive equilibrium? How does it compare to the Bertrand case and why do they differ (or not)?

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Fall 2018