

Fall 2018 14.01 Problem Set 6 - *Solutions*

Problem 1: True/False/Uncertain (20 points)

Please fully explain your answer. Points are awarded based on explanations.

1. (4 points) In a two-player game, a Nash equilibrium is the outcome that maximizes the sum of the players' payoffs.

Solution: False. Consider the prisoner's dilemma. The Nash equilibrium does not maximize utility.

2. (4 points) In a Nash equilibrium in a two-player game, both players must have selected a dominant strategy.

Solution: False. A dominant strategy is one that is a best response no matter what the other player does. A Nash equilibrium is one in which both players play the optimal strategy, given the other player's strategy. If both players are playing a dominant strategy, this must be a Nash Equilibrium, but not vice-versa.

3. (4 points) Repeatedly playing the Prisoner's Dilemma may or may not result in a cooperative solution.

Solution: True. Whether or not cooperation is sustainable will depend on the time horizon, the relative benefit to cooperation, and how much the players value the future.

4. (4 points) In the models of oligopoly considered in class, the equilibrium price will be strictly lower if there are $n + 1$ firms than if there are n firms.

Solution: False, in the Bertrand model, with two firms the price will be the marginal cost. If we keep increasing the number of firms, the price will remain at marginal cost.

5. (4 points) In the models of oligopoly considered in class, consumers are no better off than in a perfectly competitive market.

Solution: True, both in the Cournot and Bertrand models the equilibrium price will be either marginal cost (in Bertrand) or above marginal cost (in Cournot), so the consumers can't be better off than in the competitive market.

Problem 2: Game Theory (15 points)

For each of the following games:

- (i) (2.5 points) Find the dominant strategies (if any)
 - (ii) (2.5 points) Find the Nash equilibria (if any).
1. (5 points total)

		Player 2	
		Left	Right
Player 1	Up	(0, 0)	(2, 2)
	Down	(1, 1)	(0, 0)

Solution: There are no dominant strategies. If player 2 plays Left, player 1 prefers Down, while if player 2 plays Right, player 1 prefers Up. A symmetric argument holds for player 2. There are two Nash equilibria – (Up,Right) and (Down,Left).

2. (5 points total)

		Player 2	
		Left	Right
Player 1	Up	(1, 1)	(2, 0)
	Down	(0, 1)	(1, 5)

Solution: Playing Up is a dominant strategy for player 1. There is no dominant strategy for player 2. Player 2 chooses Left when player 1 plays Up, and chooses Right when player 1 plays Down. There is one Nash equilibrium - (Up, Left).

3. (5 points total)

		Player 2	
		Left	Right
Player 1	Up	(1, 2)	(2, 1)
	Down	(2, 1)	(1, 2)

Solution: There are no dominant strategies. Player 1 chooses Up when player 2 plays Right, and chooses Down when player 2 chooses Left. Player 2 chooses Right if Player 1 plays Down and chooses Left when Player 1 plays Up. There are no Nash equilibria.

Problem 3: Rock Paper, Scissors (10 points)

Consider the game Rock, Paper, Scissors. Both players simultaneously choose one of the three options Rock, Paper or Scissors. A player who plays R will beat another player who has chosen S ("rock crushes scissors") but will lose to one who has played P ("paper covers rock"); a play of P will lose to a play of S ("scissors cut paper"). Assume that the payoffs are the following: when a player wins, the payoff is 1 and when she loses the payoff is -1. If both players choose the same item and there is a tie, each get 0 points.

		Player 2		
		Rock	Paper	Scissors
Player 1	Rock	(0, 0)	(-1, 1)	(1, -1)
	Paper	(1, -1)	(0, 0)	(-1, 1)
	Scissors	(δ , 1)	(1, -1)	(0, 0)

- (5 points) Let $\delta = -1$. Find the dominant strategies (if any) and pure strategy Nash equilibria (if any).

Solution: There is no Nash equilibrium with pure strategy, and no dominant strategy.

- (5 points) For which values of δ (Scissors, Rock) becomes a pure strategy Nash equilibrium?

Solution: the payoff associated to (Scissors, Rock) would have to change from (-1, 1) to (δ , 1) with $\delta > 1$.

Problem 4: Comparing Models (27 points)

Suppose that the food truck Cheesy Burgers has the monopoly of burgers in the MIT campus and its cost function is given by $C(Q) = 2Q$. The food truck studied the demand for burgers at MIT and found out that the inverse demand function can be written as $P(Q) = 38 - 2Q$.

- (5 points) What is the socially efficient output level Q^e and the optimal price P^e ? What would the firm's profits be?

Solution: The socially efficient level of output is where the willingness to pay of consumers equals the marginal cost of producing one more unit:

$$P(Q) = 38 - 2Q = 2 \quad \Rightarrow \quad Q^e = 18$$

Consequently, $P^e = 2$. The firms profits would be zero, $\pi^e = 0$.

2. (5 points) Solve for the equilibrium quantity Q^m produced by the monopolist and the equilibrium price P^m . What are the firm's profits?

Solution: The problem of the monopolist is:

$$\max_Q P(Q)Q - c(Q)$$

The monopolist will produce where $MR(Q) = MC(Q)$. Since $MR(Q) = 38 - 4Q$ and $MC(Q) = 2$, $Q^m = 9$ and $P^m = 20$. Profits are given by $\pi^m = (P^m - c)Q^m = (20 - 2) \times 9 = 162$.

3. (5 points) Suppose that Greasy Burgers, with the same cost function above, sees that Cheesy Burgers is making a lot of money and enters the market. Both firms compete as in the Cournot model. Solve for each firm's equilibrium quantity (q_C^d, q_G^d), the equilibrium price P_d^* , each firm's profits and total profits.

Solution: The inverse demand function can be expressed as $P(q_C, q_G) = 38 - 2(q_C + q_G)$. Take the maximization problem that Cheesy Burgers is solving:

$$\max_{q_C} P(q_C, q_G)q_C - C(q_C) = \max_{q_C} [38 - 2(q_C + q_G)]q_C - 2q_C$$

Taking the FOC with respect to q_C (note that Cheesy Burgers takes the quantity sold by Greasy Burgers as given), we obtain Cheesy Burgers' best response for a given strategy of its competitor, $q_C(q_G)$:

$$38 - 2(q_C + q_G) - 2q_C = 2 \quad \Rightarrow \quad q_C(q_G) = 9 - \frac{q_G}{2}$$

By symmetry (both firms have the same cost function and enter equally the demand function), Greasy Burgers' best response function is given by $q_G(q_C) = 9 - \frac{q_C}{2}$. Following the same reasoning, we know that in equilibrium both firms will produce the same quantity, so we can set $q_C^d = q_G^d$ and substitute in any of the response functions:

$$q_C^d = 9 - \frac{q_C^d}{2} \quad \Rightarrow \quad q_C^d = q_G^d = 6$$

Hence, $Q^d = q_C^d + q_G^d = 12$ and $P^d = 14$. Each firm's profits are given by $\pi_C^d = \pi_G^d = (P^d - c)q^d = (14 - 2) \times 6 = 72$ and total profits by $\pi^d = \pi_C^d + \pi_G^d = 144$.

4. (5 points) Suppose further that Cheesy Burgers and Greasy Burgers make an agreement to avoid competition and form a cartel. What are the equilibrium quantities (q_C^c, q_G^c), the equilibrium price P^c and firms' profits?

Solution: Their agreement would be such that they maximize their joint profits, $\pi^c = \pi_C^c + \pi_G^c$ —which is equivalent to the monopolist problem but splitting the profits. Therefore, $Q^c = 9$ and $P^c = 20$. Each firm will produce $q_C^c = q_G^c = 4.5$. Each firm's profits are $\pi_C^c = \pi_G^c = 81$ and total profits are $\pi^c = 162$.

5. (7 points) Compute the consumer surplus for each equilibrium, as well as the aggregate surplus (calculated as the sum of consumer surplus and total profits). How do the 4 different equilibria compare in terms of CS, total profits and aggregate surplus? Explain the intuition. Calculate the deadweight loss associated to each non-competitive equilibrium.

Solution: First, compute the consumer surplus for each equilibrium. Denote the corresponding equilibrium quantity and prices by Q^ and P^* , respectively. Then, consumer surplus is can be calculated as follows (graphically, it is the usual triangle, where $P(0)$ is the y-axis intercept):*

$$CS(Q^*, P^*) = \frac{1}{2}(P(0) - P^*)Q^*$$

The deadweight loss (DWL) can be computed by comparing the aggregate surplus of every non-competitive equilibrium to the competitive case (which is socially efficient). Intuitively, the higher the degree of market power in a market, the higher the price and profits of the firm, but the lower the quantity traded in the market, CS and AS —so the higher the DWL. Table 1 compares the 4 equilibria:

Table 1: Equilibria comparison

Equilibrium	P^*	Q^*	CS	Profits	AS	DWL
Competitive	2	18	324	0	324	0
Cournot	14	12	144	144	288	36
Cartel	20	9	81	162	243	81
Monopoly	20	9	81	162	243	81

Problem 5: Variations in Cournot Competition (28 points)

In class we saw the Cournot competition model for two firms with the same cost function. Now, we are going to consider assymmetric cost functions. Assume that demand for a good is given by $p = a - bQ^d$, and that there are 2 firms competing in quantities. Both have no fixed costs and a constant marginal cost. Firm 1 has a marginal cost c_1 , and firm 2 has a marginal cost c_2 . We have that $a > c_1 > c_2$.

1. (7 points) Find the reaction functions of firms 1 and 2 in this market: how does the optimal quantity produced depend on the quantity produced by the other firm?

Solution: The first order condition of this problem is given by: $MR = MC$, we have that $MR = a - 2bq_i - bq_j$. Thus we should have that:

$$a - 2bq_i - bq_j = c_i$$

Solving for q_i we have that:

$$q_i = \frac{a - c_i - bq_j}{2b}$$

Thus, we have that $q_1 = \frac{a - c_1 - bq_2}{2b}$, $q_2 = \frac{a - c_2 - bq_1}{2b}$.

2. (7 points) Solve for the quantity produced by each firm and the equilibrium price. Which firm produces a higher quantity? Give an intuitive reason for this.

Solution: We know that the equilibrium is where both reaction functions intersect. Thus, we need to solve a system of two equations and two unknowns. By plugging in the reaction function of firm 2 in firm 1 reaction function, we have that:

$$q_1 = \frac{a + c_2 - 2c_1}{3b}$$

$$q_2 = \frac{a + c_1 - 2c_2}{3b}$$

$$p = a - b \left(\frac{a + c_2 - 2c_1}{3b} + \frac{a + c_1 - 2c_2}{3b} \right) = \frac{a - c_1 - c_2}{3}$$

Intuitively, the firm with the lowest cost is producing a higher quantity, because it is more efficient.

3. (7 points) What will be the equilibrium price and the quantity produced by each firm if they compete in prices (Bertrand competition)?

Solution: If firms compete in prices then we know that $p_2^* = c_1 - \varepsilon$ for a small ε . Since firms have no fixed costs, we know that firm 1 will be having losses if it charges a price lower than p_2^* , thus we have that the market price will be given by $p = p_2^* = c_1$. Total quantity produced will be given by $Q = \frac{a - c_1}{b}$, and it will only be produced by firm 2.

4. (7 points) What is the competitive equilibrium? How does it compare to the Bertrand case and why do they differ (or not)?

Solution: In the competitive equilibrium, we know that the market equilibrium would be given by $p = c_2$, $Q = \frac{a - c_2}{b}$, and everything is produced by firm 2. The difference with the Bertrand case is that, even if only firm 2 is producing in both equilibria, firm 2 is selling at $c_1 > c_2$ in the Bertrand case, so there is still an efficiency loss due to the market structure.

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