14.01 Fall 2010: Final Exam Solution Set

1. True/False/Uncertain Questions (48 points)

In this section, write whether each statement is True, False or Uncertain. You should fully explain your answer, including diagrams where appropriate. Points will be given based on your explanation.

(a) (4 points) A uniform pricing monopolist has an upward sloping MC curve. Claim: a price ceiling set below the monopoly price will increase welfare relative to the monopoly outcome (assume the monopolist does not shut down).

Uncertain. For a certain range of prices, a binding price ceiling will increase welfare because it forces the monopolist to be a price taker and output increases (q moves toward the perfectly competitive outcome). However, if the price ceiling is low enough then the monopolist could produce even less output than the unregulated monopolist and therefore welfare is lower.

(b) (4 points) A large box of paper towels which will last you 2 years is on deep discount at Wal-Mart, saving you $1 per roll. Claim: a rational consumer who has perfectly inelastic demand for paper towels should always purchase this large box instead of paying a higher price for individual paper towel rolls.

False. For this purchase, you pay a large upfront cost to save dollars in the future. If interest rates are sufficiently high, it is better for you to not buy this box, since you can earn interest on the money you would have used to buy the big box of paper towels.

(c) (4 points) In an oligopolistic market with identical firms, the market price is higher when there are two firms in the market than when there are 100 firms in the market.

Uncertain / False - This may be true in a market with Cournot competition, but under Bertrand competition any market with more than one firm will have price equal to marginal cost. Even in the Cournot case, if the demand curve is perfectly elastic price will not depend on the number of firms.

(d) (4 points) The government of a developing country is worried about the adverse effect that the high interest rate in the economy has on investment. Claim: Since a low interest rate makes more projects have positive NPV, investment in this economy will increase if the government imposes an interest rate cap.

False. An interest rate cap below the equilibrium interest rate will lower the supply of funds and increase the demand, leading to excess demand for funds in the capital market. The level of investment in the economy will fall, since the supply of funds is lower.

(e) (8 points) There are three individuals in society: Bob, Milton and Paul. There are three possible social states which result in different utility levels for the three individuals:

	Bob	Milton	Paul
A	12	50	10
B	20	20	20
C	15	15	15

Are the following True/False/Uncertain? Explain.

(i) No government would choose social state C.

True. Social state B always dominates state C since, everybody is better off under B compared to C.

(ii) There is a government election in this society and there are two candidates: a Rawlsian and an Utilitarian government candidates. Claim: In a democratic election (majority win election) a Utilitarian candidate will be elected since more individuals in this society prefer the Utilitarian candidate.

False. A Rawlsian government would choose social state B, whereas an Utilitarian government would choose state A. Since Bob and Paul are better off under B than under A they would vote for a Rawlsian government.

(f) (8 points) Jon spends his entire budget on espresso and gasoline. You have the following data on his choices:

Table 1: Jon's budget

	Price/ gallon gasoline	Price/ shot espresso	Gallons purchased	Shots purchased	Total income
February	2	1	9	4	22
March	5/2	3/4	10	8	31
April	3	1/2	8	14	31

Assume Jon's preferences are monotone, the same over the three months, and that he has no way to save or borrow across periods. Are Jon's choices consistent with utility maximization?

The key to these problems is to see if two bundles are affordable in a period and if bundle A is chosen over bundle B, then A is preferred to B. Neither the March nor April bundles are affordable in February. The April bundle is affordable in March but it is not chosen, so March is preferred to April. Checking to see if March is affordable in April, we find that it is not, so this is consistent with utility maximization.

(g) (4 points) Marco's monthly income is \$1000. He spends 40% of his income on food and the rest on buying designer clothes. The City Council thinks it is unfair that people spend more than 35% of their income for food. In order to lower the proportion of income going to food, the City Council gives Marco \$200. Claim: given that Marco's income elasticity of food is 2, the City Council accomplishes its goal.

Originally Marco spends $\$400(\$1000 * 40\%)$ *on food. After Marco receives* \$200 *from the City Council, his income increases by* 20%. *Since his income elasticity of food is* 2, *his spending on food will increase by* $40\%(20\% * 2)$, *which is equal to* $\$560(\$400 * (1 + 40\%))$. *Then the proportion of Marco's income going to food is* $47\%(560/1200)$, *which is higher than* 40%. *Thus, the City Council will not accomplish its goal.*

(h) (4 points) Venus Williams likes both tennis rackets and tennis shoes. She has many of both. Her marginal rate of substitution (MRS) of rackets for shoes is 3, meaning

that given the opportunity, she is willing to trade 3 tennis rackets for 1 pair of shoes, or vice versa. Unused rackets and shoes may be returned to the local sporting goods store for a refund. The current price for a racket is \$200 and the price for a pair of shoes is \$100. Claim: Venus can make herself better off by trading in some tennis rackets in return for some shoes at the market rate.

True. Since Venus has many tennis rackets (Y) and tennis shoes (X) she hasn't worn, she can return or exchange them. Since her $MRS_{Y\,for\,X} = 3$*, she would be willing to give up 3 rackets for one pair of shoes. However, since the price ratio is* $P_X/P_Y = \$100/\$200 = 1/2$*, she's only required to give up 1/2 racket for each extra pair of shoes (if you don't like the way that sounds, an equivalent statement is that she's required to give up 1 racket for every 2 extra pairs of shoes). Venus would make herself happier by trading in some tennis rackets in return for some shoes at the market rate.*

(i) (4 points) Suppose that there are two types of drivers: speeders and slowpokes. Speeders are more likely to have accidents, and have expected costs of \$5,000 a year in car repair bills. Slowpokes on the other hand have expected costs of only \$100 a year. Suppose further that speeders are risk-loving and that slowpokes are risk-averse. Claim: a (risk-neutral) private car insurance company will insure only slowpokes even if there are no asymmetric information or moral hazard problems in this market.

True - speeders will never want to buy fair insurance, so the insurance market will lose money on average by insuring speeders and make money by insuring slowpokes. Insurers can offer a policy aimed at slowpokes and speeders will never try to buy the policy.

(j) (4 points) Please outline the pros and cons of the U.S. adopting a tax system with a heavier reliance on a consumption tax.

A consumption tax encourages savings, which may be positive for growth, especially given that savings rates in the U.S. are currently extremely low. On the other hand, a consumption tax is generally regressive since the poor consume a higher fraction of their income than the rich, and will thus pay a higher proportion of their income in tax than the rich (unlike the income tax, which is progressive).

2. Uncertainty (20 points)

An economy has two agents, Bill and Bob. Bill has \$110, and Bob has \$200. Utility of agents in this economy is characterized by the following function of income:

$$U = u(y) = \begin{cases} \log(y - 60) & \text{if } y < 160 \\ \frac{1}{80}y & \text{if } y \geq 160 \end{cases}$$

The minimum level of income possible in this economy is 60.

Each agent is about to choose a new business venture, and has a choice between project A and project B. Neither project requires any investment up front. Project A yields revenues of 20 with probability $\frac{1}{2}$ and revenues of -10 with probability $\frac{1}{2}$. Project B yields revenues of 4 with probability one-half and revenues of 5 with probability one-half. Throughout this problem, assume that fractional income is possible.

(a) (5 points) Which project would each agent choose? Provide intuition for your answer.

Bill is in the risk-averse portion of the utility function, and so he will choose project B where there is no possibility of a loss. Bob is risk-neutral, and so he chooses the project with higher expected returns, which is project A. We can verify that Bill receives higher utility from project B.

$$\begin{aligned} E[U_{Bill}, B] &= \frac{1}{2}\log 54 + \frac{1}{2}\log 55 = \frac{1}{2}\log 2970 \\ E[U_{Bill}, A] &= \frac{1}{2}\log 40 + \frac{1}{2}\log 70 = \frac{1}{2}\log 2800 \end{aligned}$$

Clearly, the first expression is higher.

(b) (5 points) If Bill and Bob each choose an investment project each year and receive the associated income for 20 years, will the expected gap in their incomes be larger or smaller at the end of this period than it was initially? How does this relate to attitudes toward risk? You do not need to calculate income over 20 years, just provide intuition.

The income gap will be larger. Eventually Bill will get to a point on his utility function where it is optimal for him to choose the more lucrative project, but because he starts at a lower income level and initially chooses a less lucrative project, the income gap will grow. Because there is risk aversion at lower levels of income, relatively poor agents will not choose more profitable but riskier sources of income because the disutility of a decline of income from an already low level is high.

(c) (10 points) Now, assume that there is a job available that provides fixed wage income. What salary would the job have to provide in order to induce Bill to take the job rather than entering a new business venture? What salary would the job have to provide in order to induce Bob to take the job? Which is higher, and why? Algebraic expressions are acceptable as answers.

We need to provide each person with his certainty-equivalent level of income as a wage. For Bill, this is equal to $\sqrt{2970} - 50$*, or around \$4.50. Since Bob is risk neutral, we need to provide him with the same level of income he could earn in project A, which is \$5. The salary is higher for Bob—Bill has higher utility from the job by virtue of it being a steady stream of income as opposed to a gamble, and so a lower salary will make him satisfied. Bob is indifferent to risk, so he needs a higher salary.*

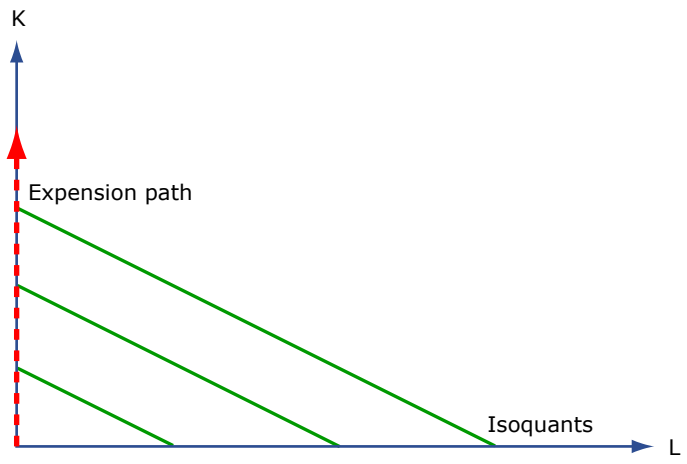
3. Costs and oligopoly (45 points)

A firm produces output q using capital and labor inputs according to the production function

$$q = f(K, L) = 4K + 2L$$

Capital and labor are both supplied in perfectly elastic input markets at prices of $r = 4$ and $w = 4$.

(a) (5 points) Draw a representative set of isoquants for this firm. On the same graph, draw and label the firm's expansion path at these prices of capital and labor.



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In this production function, capital and labor are perfect substitutes. The isoquants have a slope of $-1/2$:

$$\begin{aligned} & \bar{q} = 4K + 2L \\ \Rightarrow \quad & K = \frac{\bar{q}}{4} - \frac{L}{2} \end{aligned}$$

At these prices, the firm will only use capital in production. Therefore the expansion path is simply increasing along the y-axis.

(b) (5 points) Find the factor demands for capital and labor as functions of output.
We know that the firm will only use capital at these prices, so the demand for labor as a function of q is simply

$$L(q) = 0$$

The production function can therefore be written

$$q = 4K \quad \Rightarrow K(q) = \frac{q}{4}$$

(c) (6 points) Derive an expression for the firm's total cost as a function of q.
As usual, we can calculate the total cost function

$$TC(q) = rK(q) + wL(q) = 4\left(\frac{q}{4}\right) + 4 \cdot 0 = q$$

Marginal cost is constant at $MC(q) = 1$.

(d) (8 points) Suppose that in this market, our original firm competes with one other identical firm, and that both firms set their quantities at the same time. Furthermore, inverse demand for q is given by $p = 7 - q$. Regardless of what you found in part (c), you should now assume that both firms produce at a constant marginal cost of 1. Find the equilibrium price as well as the quantities and profits for each of these duopolists.

We solve for the Cournot quantities and prices in the usual way. For firm A, the problem is

$$\max_{q_A} q_A(p - MC(q_A))$$
$$\max_{q_A} q_A(7 - q_A - q_B - 1)$$

The first order condition with respect to q_A yields optimality condition

$$6 - 2q_A - q_B = 0 \quad \Rightarrow \quad q_A = \frac{6 - q_B}{2}$$

Since the firms are identical, we know that

$$q_B = \frac{6 - q_A}{2}$$

and we can substitute in to solve for q_A:

$$\begin{aligned} & q_A = 3 - \frac{1}{2}q_B = 3 - \frac{3}{2} + \frac{q_A}{4} \\ \Rightarrow \quad & \frac{3}{4}q_A = \frac{3}{2} \\ \Rightarrow \quad & q_A = 2 \end{aligned}$$

So we have:

$$\begin{aligned} q_A &= 2 \\ q_B &= 2 \\ Q &= 4 \\ p &= 7 - 4 = 3 \\ \pi_A &= 2(3 - 1) = 4 \\ \pi_B &= 4 \end{aligned}$$

(e) (6 points) What is the maximum amount that an outside investor would be willing to pay to purchase one of these firms? Explain.

An outside investor knows that the firm will generate 4 in profits when it produces. The investor would therefore be willing to buy the firm at any price such that her net profits were nonnegative. The highest such price is 4.

(f) (8 points) What is the maximum amount that duopolist A would pay to acquire firm B before quantities are set? Explain intuitively why firm B is worth more to its competitor than to an outside investor.

If firm A acquires firm B, firm A will be a monopolist in its market. If firm A were a monopolist, it would set its quantity to solve the usual monopolist's problem:

$$\begin{aligned} & \max_q q(7 - q - 1) \\ \Rightarrow \quad & 6 - 2q = 0 \\ \Rightarrow \quad & q^* = 3 \end{aligned}$$

Price would then go up to 4, so firm A's profits would be $\pi_M = 3(4-1) = 9$ in this scenario. Firm A would therefore be willing to pay up to 5 to purchase firm B, as doing so would increase profits from 4 to 9.
Firm B is worth more to firm A than to an outside investor because firm A not only gains the profits B would have earned in the Cournot duopoly, but also increases its own market power, allowing it to maximize total industry profits.

(g) (7 points) Suppose instead that the two firms were competing on price, a la Bertrand. How much would an outside investor be willing to pay for firm B? How much would firm A have paid?

Profits in the case with identical firms in a Bertrand duopoly are zero, so an outside investor would not have been willing to pay any positive price for firm B. Firm A on the other hand would be able to raise its profits from 0 to 9 if it bought out firm B. Therefore firm A would be willing to pay up to 9 for firm B.

4. Labor supply and income and substitution effects (40 points)

There are three periods, $t = 0, 1, 2$. In $t = 1$ Mary maximizes her utility over leisure and consumption given the following function:

$$U_1(N_1, C_1) = N_1^{\frac{1}{2}} C_1^{\frac{1}{2}}$$

subject to the following budget constraint:

$$C_1 + w_1 N_1 = 24 w_1$$

where $w_1 = 10$. Note the price of the consumption good is assumed to be one in all periods. After she has made this decision, in $t = 2$ she maximizes this utility function:

$$U_2(N_2, C_2) = N_2^{\frac{1}{3}} C_2^{\frac{2}{3}}$$

subject to the following budget constraint:

$$C_2 + w_2 N_2 = 24 w_2$$

where $w_2 = 20$.

(a) (6 points) For $t = 1, 2$ calculate Mary's choice of leisure and consumption in each period.

For $t = 1$, $MRS = w_1$ implies $C_1 = w_1 N_1$. Plugging back into the budget constraint yields $N_1 = 12$, $H_1 = 12$, $C_1 = 120$. For $t = 2$, $w_2 = w_1 + 2.5(H_1 - 8) = 20$. $MRS = w_2$ implies $C_2 = 2 w_2 N_2$. Plugging back into the budget constraint yields $N_2 = 8$, $H_2 = 16$, $C_2 = 320$. Note that the optimal amount of labor/leisure does not depend on the wage rate.

(b) (6 points) For $t = 1$, provide economic intuition for the income and substitution effects of a wage increase on leisure. Can you say anything about the relative magnitudes of these income and substitution effects?

Substitution effect: when the wage increases, it is relatively more expensive to take leisure so you will work more.
Income effect: when the wage increases, you are richer and since leisure is a normal good you take more leisure.
These effects are equal in magnitude because leisure does not depend on the wage rate.

(c) (7 points) Go back to your solution in part (a). If the interest rate is 10% per period, what is the present value of her consumption in $t = 0$? Please use 0.9 and 0.8 as approximations for $1/(1.1)$ and $1/(1.1)^2$ respectively.

$$PV = 120/(1+i) + 320/(1+i)^2 = 120 * 0.9 + 320 * 0.8 = 108 + 256 = 364$$

(d) (7 points) Mary now has the option of obtaining additional job training in $t = 0$ at an investment cost of \$200. As a result, her wage rate increases in $t = 1$ to $w_1 = 20$ and in $t = 2$ to $w_2 = 30$. Calculate the net present value of this investment on consumption. Consider only the value of consumption (and not the value of leisure).

Since the optimal amount of work/leisure does not depend on the wage rate, we can use the same answers we found in part (a), $H_1 = 12$ and $H_2 = 16$ which implies $C_1 = 240$ and $C_2 = 480$. As a result, the increase in consumption is $C_1 = 120$, $C_2 = 160$ with an upfront investment of \$200.

$$NPV = -200 + 120/(1.1) + 160/(1.1)^2 = -200 + 108 + 128 = 36$$

(e) (7 points) For more general utility functions, when will the net present value of the investment on consumption from part (d) likely be negative? Use income and substitution effects in your explanation.

For more general utility functions, it's possible that an increase in wage will cause you to work less because the income effect is larger than the substitution effect. In other words, it's possible that your consumption of goods barely changes but instead you decide to take more leisure. This could result in $NPV < 0$.

(f) (7 points) Does Mary have a Laffer curve for income taxes (as opposed to consumption taxes)?

No, Mary does not have a Laffer curve because as you increase the income tax rate, it does not change her labor/leisure decision. She continues to work the same amount regardless of the income tax, so revenues will always increase monotonically with the tax rate. The only way that she could have a Laffer curve is if she worked less when you increased her tax rate.

5. Trade and price discrimination (20 points)

A U.S. pharmaceutical firm sells its patent-protected drug Levemir in the U.S. and E.U. markets. The domestic demand function is $Q_{US} = 120 - 2p_{US}$, and the E.U. demand function is $Q_{EU} = 60 - p_{EU}$, where all prices are measured in U.S. dollars and quantity is measured in vials. The firm's marginal cost is $MC = 10$ in both countries.

(a) (6 points) Initially, the U.S. and E.U. governments prevent resale of Levemir. What are the firm's optimal p_{US} and p_{EU}? (The same price has to be charged to all consumers in the U.S. market and all consumers in the E.U. market.) How many vials does it sell in the U.S. and E.U. markets?

If U.S. and E.U. government can prevent resale, then this firm can implement the multi-market price discrimination. That is, this monopolistic firm solves its profit maximization problem separately in two markets, U.S. and E.U. market.

U.S. Market:

- *demand curve:* $Q_{US} = 120 - 2P_{US}$;

- *inverse demand curve:* $P_{US} = 60 - 0.5Q_{US}$;
- *marginal revenue curve:* $MR_{US} = 60 - Q_{US}$.

Profit maximization condition:

$$MR_{US} = MC \Longrightarrow 60 - Q_{US} = 10 \Longrightarrow Q_{US} = 50$$

Substituting Q_{US} *into the inverse demand function:*

$$P_{US} = 60 - 0.5Q_{US} = 60 - 0.5(5) = 35$$

E.U. Market:

- *demand curve:* $Q_{EU} = 60 - P_{EU}$;
- *inverse demand curve:* $P_{EU} = 60 - Q_{EU}$;
- *marginal revenue curve:* $MR_{EU} = 60 - 2Q_{EU}$.

Profit maximization condition:

$$MR_{EU} = MC \Longrightarrow 60 - 2Q_{EU} = 10 \Longrightarrow Q_{EU} = 25$$

Substituting Q_{EU} *into inverse demand function:*

$$P_{EU} = 60 - Q_{EU} = 60 - 25 = 35$$

Thus, this firm sells 50 and 25 vials at the same price, 35 per vials, in the U.S. and E.U. markets, respectively.

(b) (6 points) Now assume that the U.S. and E.U. governments permit resales and per unit transportation and other transaction costs are negligible, so that the pharmaceutical monopoly can no longer price discriminate. What price will the firm charge and how many vials will it sell in the U.S. and in the E.U. markets?

Assume that this monopoly charges a single price such that $P_{US} = P_{EU} = P$.

- *aggregate demand curve:* $Q = Q_{US} + Q_{EU} = (120 - 2P_{US}) + (60 - Q_{EU}) = 180 - 3P$;
- *inverse demand curve:* $P = 60 - \frac{1}{3}Q$;
- *marginal revenue curve:* $MR = 60 - \frac{2}{3}Q$.

Profit maximization condition:

$$MR = MC \Longrightarrow 60 - \frac{2}{3}Q = 10 \Longrightarrow Q = 75$$

Substituting $Q = 75$ *into the inverse demand curve:*

$$P = 60 - \frac{1}{3}(75) = 35$$

Substituting $p = 35$ *into individual demand functions:*

$$Q_{US} = 120 - 2P = 50$$
$$Q_{EU} = 60 - P = 25$$

(c) (8 points) Use one graph for the U.S. market and one graph for the E.U. market to show the welfare impacts of the policy change in (b). What happens to consumer and producer surplus in each nation? Overall, is this a social welfare improvement or reduction? Please provide intuition for the overall welfare impact.

i. U.S. Market:

- *consumer surplus:* $A_1 + B_1$;
- *producer surplus:* $C_1 + D_1$;
- *deadweight loss:* E_1.

ii. E.U. Market:

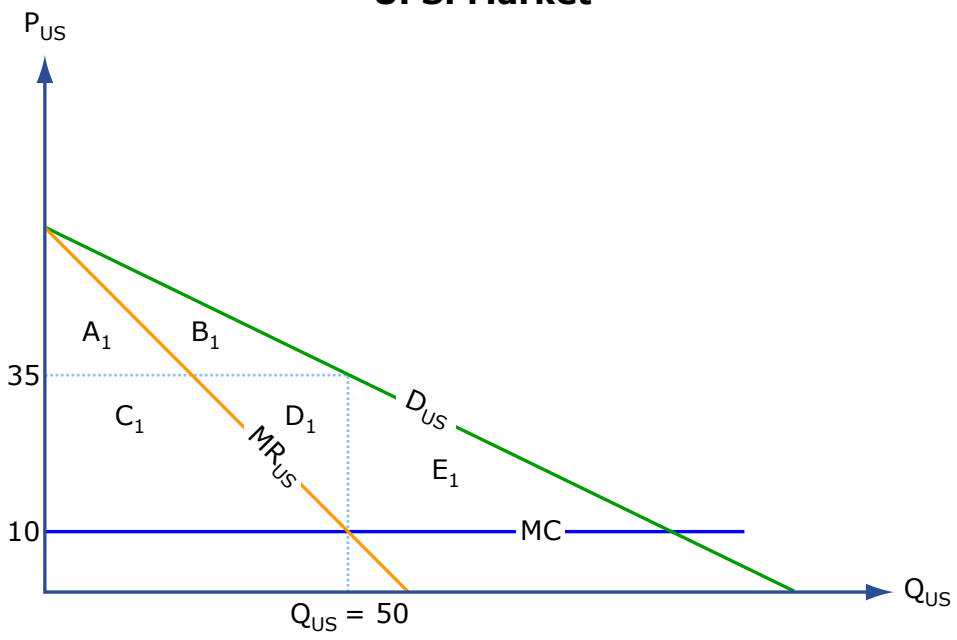
- *consumer surplus:* $A_2 + B_2$;
- *producer surplus:* $C_2 + D_2$;
- *deadweight loss:* E_2.

iii. Total

- *consumer surplus:* $(A_1 + B_1) + (A_2 + B_2)$;
- *producer surplus:* $(C_1 + D_1) + (C_2 + D_2)$;
- *deadweight loss:* $E_1 + E_2$.

Generally, the monopolist is better off under price discrimination (part a), because at worse he can always charge the unifrom price in each market. The monopolist's pricing decision depends on the elasticity of demand. In general, consumers in low-elasticity markets are adversely affected by the discrimination and would prefer a uniform price; consumers in high-elasticity markets prefer price discrimination. However, the two demand functions in this problem have the same elasticity. Hence the monopolist's pricing decisions are the same in parts (a) and (b).

U. S. Market



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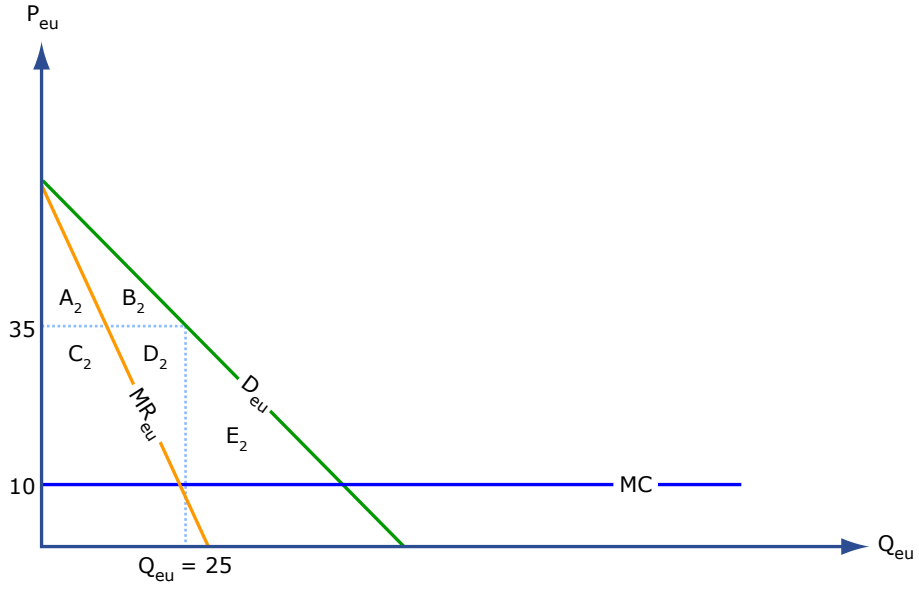
E.U. Market



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