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GREG HUTKO: Welcome back to the 14.01 problem-solving videos. Today we're going to work on Fall 2010 Problem Set 4, Problem Number 3. And this problem is really going to take us through two scenarios. We're dealing with producer decisions. So now instead of dealing with utilities, we're going to be working with cost functions. And we're going to first go through the short-run scenario. And then we're going to talk about the long-run scenario and the implications of both of these cases.

Problem Number 3 says, suppose the process of producing corn on a farm is described by the function q equals $8k$ to the $1/3$ times quantity L minus 40 raised to the $2/3$, where q is the number of units of corn produced, k is the number of machine hours used, and L is the number of person-hours of labor. In addition to capital and labor, the farmer needs to pay a \$15 transportation fee to deliver corn to downtown. So the cost can be written as total cost equals 15 times the quantity produced plus the rental cost of capital plus the wage rate times the quantity of labor.

Part A says, suppose in the short-run the machine hours rented are fixed at k equals 8 , and its rental rate equals 64 , and the wage rate equals 16 . Derive the short-run total cost and the average costs as a function of output level q .

So to start off this problem, we're going to start by working with the short-run scenario. And typically, the only difference between the short-run scenario and the long-run scenario in economics problems is that in the short-run, the amount of capital that a firm can use is going to be fixed. It means that because machines are a fixed cost in the short-run, you can't actually change how many machines or how often you use the machines. So we're going to set that equal to 8 for this scenario.

And we also know that each hour that we use this machine is going to cost us 64 . And we know that for each hour that we're using labor, it's going to cost us 16 .

We also know that in addition to the cost of the capital and the cost of the labor, which is

represented in our total cost function here, for each unit q that we produce, we have to transport it to market. So we also have this $15q$ added into our total cost function, which is something that we might not always see in all of our cost functions. So let's start off by solving for the total cost function.

And to do this, the first thing that we're going to do is we're going to plug in to our production function here what we know the capital is fixed at. And we're going to solve for labor, or L , in terms of q . So plugging in for k we're going to be left with this equation. And from here, we can solve for L in terms of q . And this is going to be useful for us because what we're going to do is we're going to take this L and we're going to plug it into the total cost function, so that our cost function is no longer in terms of k and L . But it's only going to be in terms of q . So isolating L in this equation, we're going to have that L equals 40 plus $q/16$ raised to the $3/2$. So now let's go to our total cost function.

We're going to plug in for k and r . So we know that r is 64 and k is 8 . We're going to plug in for w 16 . And now for L we're going to plug in what we solved for using our production function.

So from this equation when we do the algebraic manipulation, we're going to get the total cost function in terms of only q . And solving out for this, we find that the total cost-- and I'm going to denote that this is in the short-run with the capital fixed by a little sr as a subscript. The total cost is going to be equal to $1,152$ plus $15q$ plus 16 times quantity $q/16$ raised to the $3/2$.

Now to find the average cost, all the average cost is it's the total cost divided by q . So it's per unit, how much on average does the producer have to spend to actually produce one unit? So to find the average cost, we're going to divide this whole thing through by q . And we're going to find the average cost in the short-run is going to be equal to $1,152$ divided by q plus 15 divided by $q/16$ raised to the $1/2$.

In part B, what we're going to do is we're going to take the total cost function, we're going to plug in a fixed quantity, and we're going to find, what is the actual amount of money that a producer would have to pay to produce a fixed quantity?

Part B says, suppose the farm wants to produce 64 units of corn. Based on the answer to part A, what is the total short-run cost?

So using our solution from part A, all you have to do now for part B is you're going to plug in for q the number 64 . So for part B, plugging in for q , you're going to find that the total cost in

the short-run is going to be equal to 2,240.

Now the more interesting part of this problem is what actually happens when instead of having to fix the capital at 8, what happens when the producer can change the amount of capital that they're producing?

Part D says, in the long-run, the farm can change its capital level by minimizing the cost subject to the production function, derive the cost-minimizing demands for k and L as a function of output q , the wage rates w , and the rental rates of machine r .

So now we're going to go back to a similar problem like we saw with consumer theory where we saw the marginal rate of substitution had to be equal to the price ratio. In this case, we're going to use something called the marginal rate of technical substitution.

Simply put, the marginal rate of technical substitution asks us, how many machines or how many people would we be willing to basically lay off for one additional machine? And we call that the marginal product of capital, or how much we're actually getting from each unit of capital, divided by the marginal product of labor. We're going to set that equal to the price of the capital and the price of the labor.

To find these marginal products of capital and the marginal product of labor, what we're going to do is we're going to take the derivative of our production function with respect to q , or with respect to k , and with respect to L . And when we do that, we find that the marginal product of capital is going to be equal to r . And we can also solve for the marginal product of labor. And we're simply going to divide here and we can find that L minus 40 over $2k$ is going to equal r/w .

And finally, the last thing we're going to do here is we're going to solve for the amount of labor and the amount of capital rented in terms of the other variables. We're going to plug this back into our production function that we have over here in the middle of the board. And then we can solve for how much capital and how much labor is demanded in terms of w and in terms of r . So I'll go through this process first for solving for the demand function for capital.

To start off, we're going to get this equation in terms of L . So we have that L is equal to $2rk$ over w plus 40. And then we're going to take this L and we're going to plug it back into our production function.

And when we do this, we're going to have that $8k$ to the $1/3$ times quantity $2rk$ over w raised to

the $2/3$. And now we're just going to solve through for k .

And when we solve through for k , we're going to find that k equals $q/8$ w over $2r$ raised to the $2/3$. This is our conditional demand for capital. And we call it the conditional demand because it's dependent on the price that we're going to pay for labor, the price that we're going to pay for labor, the price we're going to pay for capital, and the quantity that we're going to produce.

Now to do this same process only solving for the conditional demand for labor, you're going to go back to this equation right here. And instead of solving through for L , you're going to solve through for k . And when you solve through for k , you're going to find that k equals quantity L minus 40 times w over $2r$.

You're going to take this k , just like we did for labor you're going to plug it back into the production function. And you can solve through for the conditional demand for labor. And when you do that, you're going to find that labor is just going to equal $q/8$ times quantity $2rw$ raised to the $1/3$ plus 40 .

So again, to summarize this problem, we started off with the short-run scenario. And what we found with the short-run scenario, we were able to plug-in for the capital that was fixed at 8 . We were able to find the total cost and the average cost functions. Given a fixed quantity that they wanted to produce, we solved for the total cost in the short-run. And then finally we said, let's let the producer change how much capital they are actually using. And let's figure out based on letting the wage rate and the rental rate of capital change, let's get a conditional demand that lets those variables change that would let us then solve for the amount of capital and the amount of labor that would be needed to produce a certain amount of quantity.