

Practice Problems for Consumer Surplus, Edgeworth Boxes

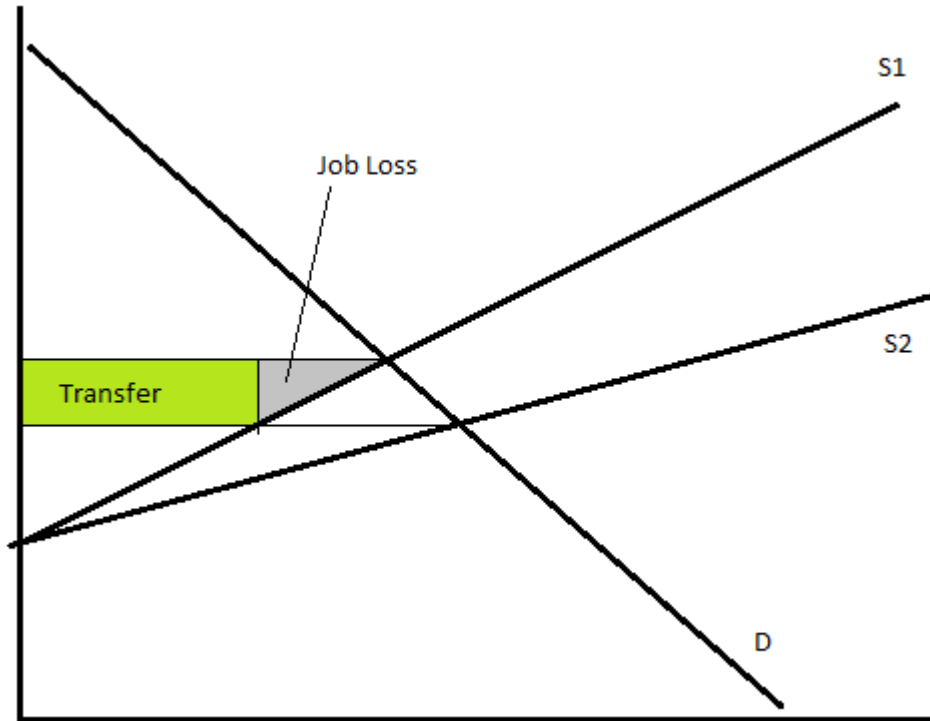
1 Subsidies and Incidence (2014 PS3)

1. Consider the labor market for high skill workers characterized by the following supply and demand:

$$S(w) = -50 + 5w$$

$$D(w) = 1000 - 10w$$

- (a) Calculate the equilibrium wage in this market. Draw the supply and demand curves on a graph, and calculate worker and employer surplus. **[4 points]**
In equilibrium, $S(w) = D(w)$. Solving the equation gives $w = 70$. At these wages, quantity is $S(70) = -50 + 5(70) = 300$. Worker surplus is all of the area above the supply curve below $w=70$. This is a triangle with height 60 and width 300, area 9000. $WS = 9000$. Employer surplus is the area between the demand curve and the $w=70$ line. This is a triangle with height 30 and width 300, area 4500. $ES = 4500$



- (b) There is another group of workers who may be high skilled, but have not been tested. The government institutes a free testing service which verifies workers as being skilled. The result is that a second, identical group of high skilled workers join the workforce, doubling the supply of high skilled labor. What is the new supply equation that the employer faces? Draw the new supply curve on your previous graph (and verify that at each level of wage the supply of labor doubled). What is the new equilibrium wage? [4 points]

The new supply equation is $S(w) = -100 + 10w$. When $S(w) = D(w)$, $w = 55$ and quantity is 450. As before, employer surplus is equal to the area below demand and above w . $ES = 10125$, an increase. This is expected because supply increased, and as a result quantity increased and wage decreased. Worker surplus for each population is derived from their individual supply functions ($S'(w) = -50 + 5w$). Using these supply functions, we can find the area above supply curve and below $w = 55$. For each group, the area is a triangle with height and width of 45×225 and area 5062.5. Total worker surplus increased but the original population is worse off.

- (c) At the new equilibrium, calculate the employer's surplus, the surplus for the original group of high skilled workers and the surplus for the new group of high skilled

workers. How would each group be willing to individually pay to implement this government testing service? **[4 points]**

Each group is willing to pay an amount equal to their change in surplus. The employer would pay $10125 - 4500 = 5625$ for this. The original employees are worse off and would not pay. The new workers would pay up to 5062.5.

- (d) The original group of workers rightfully views this new development as a wage decrease for themselves. Associated with this wage decrease is a transfer of wealth to the employer (since each working employee is now getting paid less) as well as a loss of welfare from workers who were productive but no longer have a job. On your graph label the areas associated with the transfer and job loss. What is the sum of the two areas? **[3 points]**

The transfer is a loss of 15 for each worker still employed, an area of 3375. The job loss resulted in 75 workers losing their jobs, with the loss in welfare ranging from 0 to 15, with an area of 562.5. The sum of the areas is equal to 3937.5, their loss of utility.

- (e) If the program costs K to implement, for what values of K does this service increase total welfare? If $K = 0$, what are the deadweight losses associated with this policy change? **[3 points]**

Total change in welfare for this program is $6749 - K$. For K less than 6749, the service increases welfare. For $K=0$, there is no deadweight loss because total welfare increases.

2. Return to our original market for labor

$$S(w) = -50 + 5w$$

$$D(w) = 1000 - 10w$$

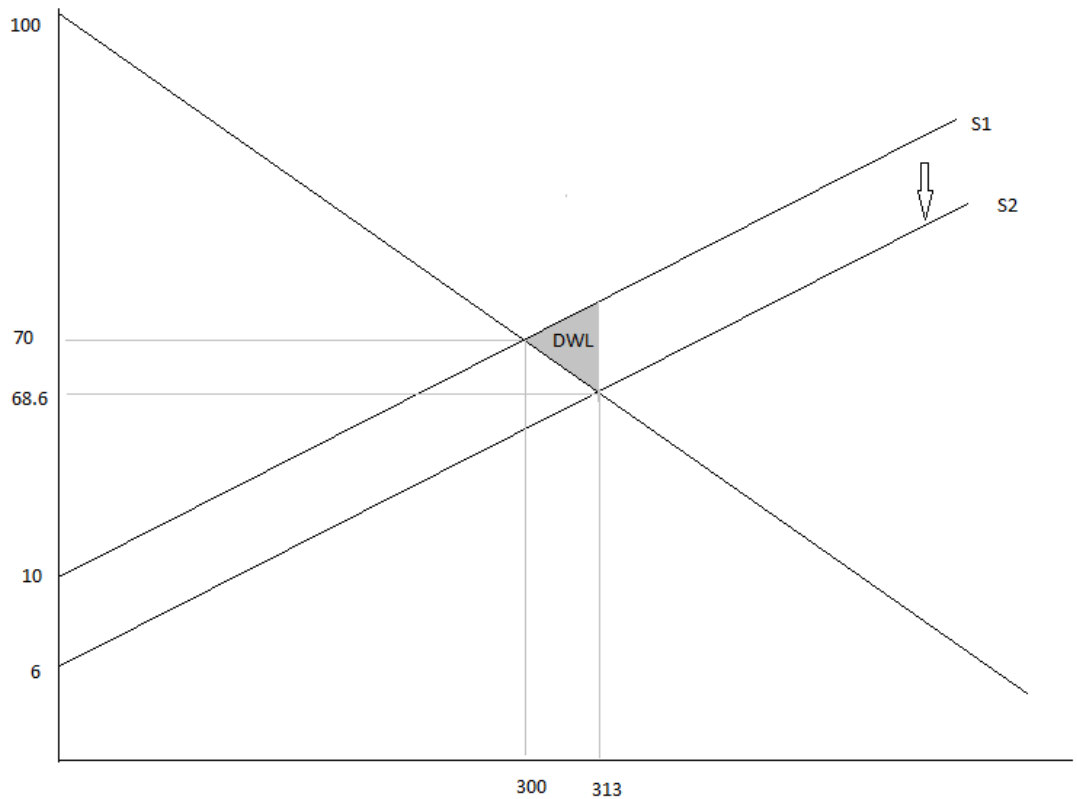
Suppose that the government implements an earned income tax credit. Any worker in the market that is employed gets a government subsidy of 4, so our new labor supply is $S(w) = -50 + 5(w + 4)$.

- (a) For subsidies, we are often interested in the incidence, which describes how much each party gains from the subsidy. To calculate this, find the new equilibrium wage. What is the difference in the amount the worker gets paid? What is the difference in the amount the employer pays? The ratio of the difference for the worker and the employer is the incidence. This is also the ratio of the elasticities of supply and demand. What is this value? **[4 points]**

The equilibrium is where the new $S(w) = D(w)$. At that point, $w = 68.6$, and quantity is equal to 313.4 employees. At these wages, employers pay 1.33 dollars less per employee. Employee gets paid $68.6 + 4$, or 2.66 more per person. This tells us that the incidence is 2. Supply elasticities are equal to $\frac{dQ/Q}{dP/P} = 5 \frac{68.8}{313.4} = 1.09$. The demand elasticity is $-10 \frac{68.8}{313.4} = 2.18$. The ratio of elasticities is 2, equal to the incidence.

- i. Is there deadweight loss associated with this program? On a new graph draw the shift in supply and demand and label on the graph the deadweight loss associated with the subsidy, and calculate the change in total welfare and the cost of the subsidy to determine the deadweight loss. [4 points]

13.4 more employees are hired than efficient. The loss for these employees are getting paid more than they are producing, with the first extra employee costing 0 and the last one costing 4, with an average of 2. The total deadweight loss is 26.8



- (b) Imagine instead that the labor supply curve was $S(w) = -400 + 10w$. Recalculate the incidence as in part (a) and compare to your answer above. Explain intuitively why the incidence of the subsidy on equilibrium wages is higher or lower in this case. [4 points]

The equilibrium without the subsidy is $w = 70$. With the subsidy, $S(w) = -400 + 10(w + 4)$, the new equilibrium at $S(w) = D(w)$ is $w = 68$. Employers pay 68 and employees get 72, so each group benefits by 2, exactly splitting the subsidy. Incidence is 1. Employees are much more elastic in this scenario, and as a result they benefit less from the subsidy.

2 General Equilibrium in an Exchange Economy (2014 PS3)

Romeo is an MIT undergraduate who lives in East Campus. Juliet is a Wellesley student. Suppose Romeo has utility

$$u_r(R, C) = R^\alpha C^{1-\alpha}$$

where R are roses and C are chocolates. Juliet has utility

$$u_j(R, C) = R^\beta C^{1-\beta}$$

with $\alpha \in (0, 1)$ and $\beta \in (0, 1)$. Initially Romeo and Juliet do not know each other, and so are unable to trade the roses and chocolates they own, only consuming their own endowment. Romeo begins with 1 rose and 9 chocolates ($\bar{R}_r = 1, \bar{C}_r = 9$), Juliet begins with 1 chocolate and 9 roses ($\bar{C}_j = 1, \bar{R}_j = 9$).

1. What are the initial utility levels of Romeo and Juliet? **[2 points]**

[Solution: Romeo has $u_r = 1^\alpha 9^{1-\alpha} = 9^{1-\alpha}$ and Juliet has $u_j = 9^\beta 1^{1-\beta} = 9^\beta$]

2. Suppose that Romeo bumps into Juliet at Toscanini's and after talking about life over ice cream, they decide to enter into a trading agreement. Draw an Edgeworth box and show clearly all feasible allocations of chocolates and roses for Romeo and Juliet. Draw Romeo and Juliet's indifference curves within the box [you can assume $\alpha = \beta = \frac{1}{2}$ for this part if you would like, but the indifference curves need not be completely accurate] and mark the endowment allocation. Shade in the allocations which would be Pareto improvements over the endowment allocation. **[5 points]**

[Solution: See Figure:

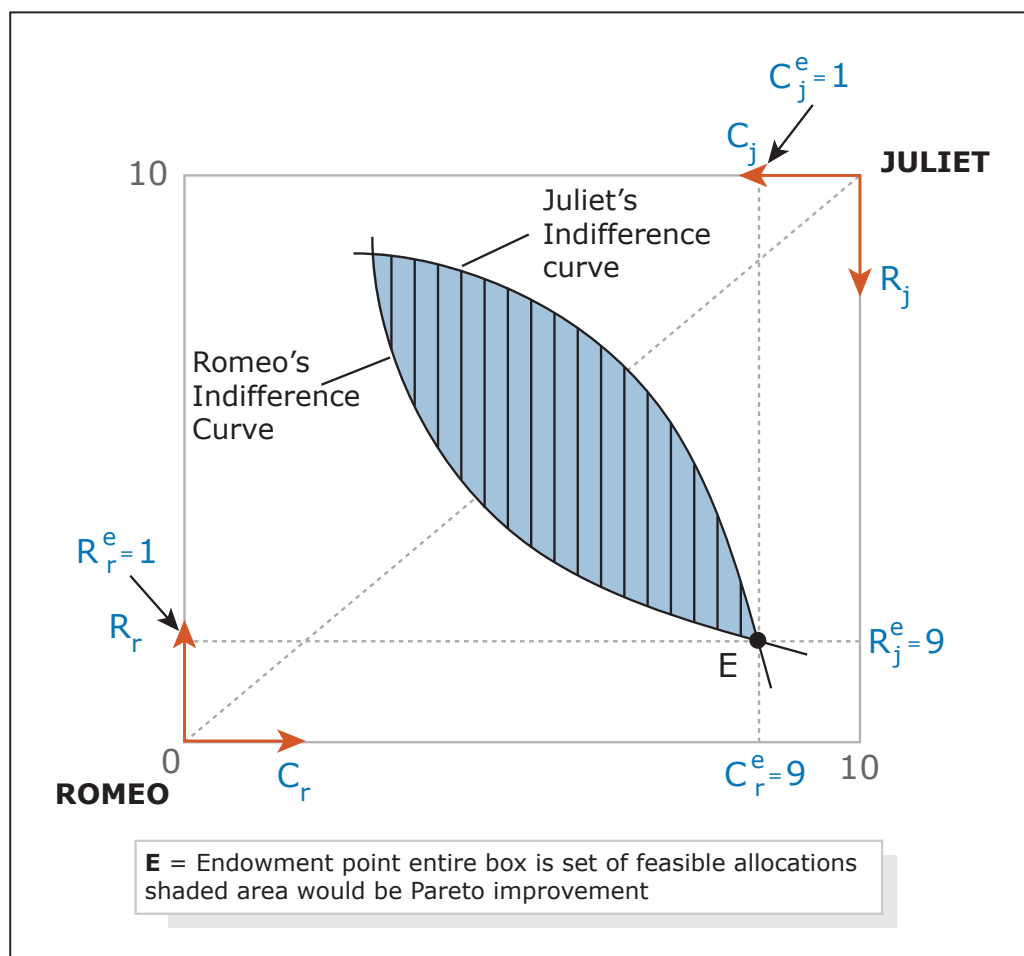


Image by MIT OpenCourseWare.

3. Suppose that Romeo and Juliet sell/buy roses at price p_R and sell/buy chocolates at price p_C . We can normalise and set $p_R = 1$. What relative price $\frac{p_C}{p_R} = p_C$ clears these mini-markets for roses and chocolates? [Note that chocolates and roses are divisible for the purposes of this question, that is, you can buy a fraction of a rose or take a small bite of a chocolate.] **[6 points]**

[Solution: For this question we want to find the relative price p_C such that total demand for roses equals total supply (it will follow automatically that this price also clears the market for chocolates – i.e. we only need to check that one market clears. This is due to something called ‘Walras’ Law’). We know total supply is just equal to the sum of Romeo and Juliet’s endowments of roses. For total demand we find Marshallian

demand of both people. Romeo maximises u_r subject to constraint

$$p_R R + p_C C = p_R \bar{R}_r + p_C \bar{C}_r = p_R + 9p_C$$

where \bar{R}_r is Romeo's endowment of roses and \bar{C}_r is his endowment of chocolates. Since utility is of the Cobb-Douglas form, Romeo has demands (taking prices as given) such that fraction α of income is spent on roses and the remaining fraction $1 - \alpha$ on chocolates. I.e. we have

$$p_R R_r^* = \alpha (p_R + 9p_C) \Rightarrow R_r^* = \alpha \frac{p_R + 9p_C}{p_R}$$

$$p_C C_r^* = (1 - \alpha) (p_R + 9p_C) \Rightarrow C_r^* = (1 - \alpha) \frac{p_R + 9p_C}{p_C}$$

and taking into account Juliet's different endowment, Juliet has demands

$$p_R R_j^* = \beta (9p_R + p_C) \Rightarrow R_j^* = \beta \frac{9p_R + p_C}{p_R}$$

$$p_C C_j^* = (1 - \beta) (9p_R + p_C) \Rightarrow C_j^* = (1 - \beta) \frac{9p_R + p_C}{p_C}$$

and we now have enough to find the relative price that clears the market. First note that total demand for roses is $R_r^* + R_j^*$ and total supply is 10 (again, we could instead look at the market for chocolates, but we would find the same result). Setting supply equal demand we have

$$\begin{aligned} \alpha \frac{p_R + 9p_C}{p_R} + \beta \frac{9p_R + p_C}{p_R} &= 10 \\ \Rightarrow \alpha (p_R + 9p_C) + \beta (9p_R + p_C) &= 10p_R \\ \Rightarrow (9\alpha + \beta) p_C + (9\beta + \alpha) p_R &= 10p_R \\ \Rightarrow \frac{p_C}{p_R} = p_C &= \left(\frac{10 - 9\beta - \alpha}{9\alpha + \beta} \right) \end{aligned}$$

which pins down the relative price of chocolates to roses.]

4. Assume now that $\alpha = \beta$. Suppose that Romeo and Juliet trade at the relative price p_C that you just found. Can you show that the utility of each person is higher than it was in part (1) (i.e. that we have a Pareto improvement, a.k.a. gains from trade)? **[7 points]**

[**Solution:** Romeo's indirect utility is

$$v_r = (R_r^*)^\alpha (C_r^*)^{1-\alpha} = \left(\alpha \frac{p_R + 9p_C}{p_R} \right)^\alpha \left((1-\alpha) \frac{p_R + 9p_C}{p_C} \right)^{1-\alpha}$$

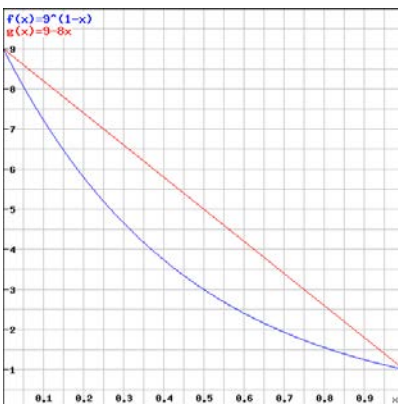
and using that $p_R = 1$ we have

$$v_r = \alpha^\alpha (1-\alpha)^{1-\alpha} (1+9p_C) (p_C)^{\alpha-1}$$

and substituting in for p_C and using $\alpha = \beta$ we get

$$\begin{aligned} v_r &= \alpha^\alpha (1-\alpha)^{1-\alpha} \left(1 + 9 \left(\frac{1-\alpha}{\alpha} \right) \right) \left(\frac{1-\alpha}{\alpha} \right)^{\alpha-1} \\ &= \alpha \left(1 + 9 \left(\frac{1-\alpha}{\alpha} \right) \right) = 9 - 8\alpha \end{aligned}$$

which is always greater than Romeo's pre-trade utility $9^{1-\alpha}$ for $\alpha \in (0, 1)$. See this graph to convince yourself of this:



Now we check Juliet's indirect utility:

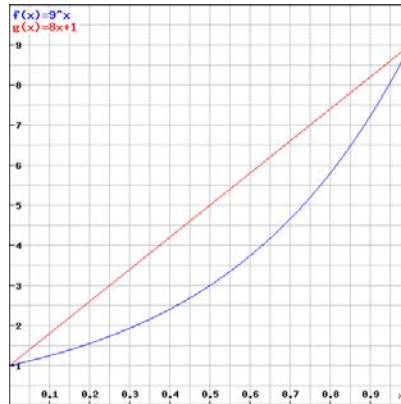
$$v_j = (R_j^*)^\beta (C_j^*)^{1-\beta} = \left(\beta \frac{9p_R + p_C}{p_R} \right)^\beta \left((1-\beta) \frac{9p_R + p_C}{p_C} \right)^{1-\beta}$$

and simplifying as before, including that $\beta = \alpha$, we get

$$v_j = \alpha^\alpha (1-\alpha)^{1-\alpha} \left(9 + \frac{1-\alpha}{\alpha} \right) \left(\frac{1-\alpha}{\alpha} \right)^{\alpha-1}$$

$$= \alpha \left(9 + \frac{1 - \alpha}{\alpha} \right) = 8\alpha + 1$$

which is always greater than Juliet's pre-trade utility 9^α for $\alpha \in (0, 1)$. Again, the graph makes this clear:



This shows that trade yields a Pareto improvement. Beyond this, it shows that Toscanini's may serve a useful market matching role.]

5. Still assuming that $\alpha = \beta$, what happens to the relative price of chocolates p_C and equilibrium consumption levels as α increases (i.e. as preferences of both shift toward roses)? Can you give any intuition for these effects? **[5 points]**

[Solution: we have that $p_C = \frac{1-\alpha}{\alpha}$ so it is straightforward to see that as α increases the relative price of chocolate falls (equivalently the relative price of roses increases). This is because at the current relative price there would be excess demand for roses – the demand from both Romeo and Juliet increases as α increases, but the total supply stays fixed at the initial endowment. To clear the market, the relative price of roses must rise.

Now let's consider equilibrium consumption levels. Recall that

$$R_r^* = \alpha \frac{p_R + 9p_C}{p_R} = \alpha \left(1 + 9 \left(\frac{1 - \alpha}{\alpha} \right) \right) = 9 - 8\alpha$$

$$C_r^* = (1 - \alpha) \frac{p_R + 9p_C}{p_C} = (1 - \alpha) \frac{1 + 9 \left(\frac{1 - \alpha}{\alpha} \right)}{\frac{1 - \alpha}{\alpha}} = 9 - 8\alpha$$

So Romeo has $R_r^* = C_r^*$ and both are falling in α – as preferences shift towards roses, his equilibrium demand of both roses and chocolates falls. Intuitively this is because roses are in greater demand, but his endowment is mostly chocolates. In some sense the economic power has shifted toward Juliet. Now consider Juliet's consumption levels:

$$R_j^* = \beta \frac{9p_R + p_C}{p_R} = \alpha \left(9 + \frac{1 - \alpha}{\alpha} \right) = 8\alpha + 1$$

$$C_j^* = (1 - \beta) \frac{9p_R + p_C}{p_C} = (1 - \alpha) \frac{9 + \left(\frac{1-\alpha}{\alpha}\right)}{\frac{1-\alpha}{\alpha}} = 8\alpha + 1$$

which shows as suspected that Juliet's equilibrium consumption levels are both increasing in α .]

6. Now suppose we start with a scenario where $\alpha \neq \beta$ and Romeo has 9 roses and 9 chocolates ($\bar{R}_r = 9, \bar{C}_r = 9$), Juliet has 1 rose and 1 chocolate ($\bar{R}_j = 1, \bar{C}_j = 1$). The endowments are very unequal. Are there still gains from trade? What does this say about the possibility of true love between the rich and poor? **[5 points]**

[Solution: even though the endowments are very unequal, there will still be gains from trade provided that the initial endowment is not on the contract curve (the set of Pareto efficient allocations). The contract curve is the set of allocations where the indifference curves of Romeo and Juliet are tangent (same MRS), i.e. where

$$\frac{MU_R^r}{MU_C^r} = \frac{MU_R^j}{MU_C^j}$$

so we can just check that this condition does not hold at the initial endowment. For Romeo we have

$$\frac{MU_R^r}{MU_C^r} = \frac{\alpha \left(\frac{C}{R}\right)^{1-\alpha}}{(1-\alpha) \left(\frac{R}{C}\right)^\alpha} = \frac{\alpha}{1-\alpha} \frac{C}{R} = \frac{\alpha}{1-\alpha}$$

substituting in that $C = R = 9$ at the initial endowment. For Juliet we have

$$\frac{MU_R^j}{MU_C^j} = \frac{\beta \left(\frac{C}{R}\right)^{1-\beta}}{(1-\beta) \left(\frac{R}{C}\right)^\beta} = \frac{\beta}{1-\beta} \frac{C}{R} = \frac{\beta}{1-\beta}$$

substituting in $C = R = 1$. Since $\alpha \neq \beta$ we have $\frac{MU_R^r}{MU_C^r} \neq \frac{MU_R^j}{MU_C^j}$ – so the indifference curves are not tangent at the initial endowment. It follows that there are still gains

from trade (Pareto-improving allocations) despite the very unequal endowment. This says that Romeo and Juliet would still want to trade goods even though one is very 'rich' and one is very 'poor'. But probably true love is not affected that much by being able to trade goods, so this result tells us little about that.]

3 Sugar cases (2012 PS3)

1. **(10 points)** The 2004 *New York Times* article, "In Mexico, Sugar vs. U.S. Corn Syrup" (on the class web site) reports: "What is nominally at issue is a 20 percent Mexican tax on soft drinks made with high-fructose corn syrup imposed in January 2002. The tax was a response, legislators and government officials say, to the United States' unwillingness to accept imports of the Mexican sugar that were displaced in Mexico by cheaper American corn syrup." Mexican sugar is generally cheaper than U.S. sugar, while the per pound production cost of high fructose corn syrup (HFCS) generally exceeds the world price of sugar. Explain how (1) U.S. imports of Mexican sugar could be displaced by HFCS and (2) why Mexico might be importing soft drinks made with HFCS from the U.S.

On (1), it's clear that the the U.S. sugar program creates a price window for HFCS to enter. It would not be competitive in the U.S. at world prices, but it's cheaper than raw sugar in the U.S. at the quota price. As HFCS production expands, the U.S. cuts back on import quotas, and this has apparently displaced the Mexican quota. On (2), it sounds from the NYT article like the price of sugar in the Mexican market is also manipulated by the government to keep prices high as a subsidy to poor farmers (from NYT: "In Mexico, sugar's political power grows out of its social importance. Some 155,000 cane growers, who each farm an average of 9 or 10 acres, sell cane to about 60 mills. The farmers are organized in two associations linked to the P.R.I., the Institutional Revolutionary Party, which ruled Mexico for 71 years until 2000. The government sets the price the mills must pay growers"). This price distortion means that HFCS is also cheaper than sugar in the Mexican domestic market, so Mexican soda bottlers U.S. Coca Cola with HFCS rather than producing Coke using domestic sugar.

2. **(10 points)** Imagine that the U.S. sugar program were ended today. Putting aside the issue of the surplus lost by producers and gained by consumers, this policy change would reduce employment in sugar cane and sugar beet farming, remove close to a million acres of U.S. land from sugar production, and reduce demand for HFCS. Reasoning from the principle that "all costs are opportunity costs," how should we think about the *economic*

costs of this policy change for workers in the sugar industry, owners of land used for cane and beet production, and firms that have built HFCS plants? [Note: this question does not have a trivial answer. Think carefully about the opportunity cost of each of these factors.]

This question doesn't have a "right" answer. We were looking for an intelligent discussion that raised such issues as: capital and labor resources (workers, land, plants) that are currently working in the sugar industry have optimized their production decisions under the market conditions given by the federal US sugar policy. If this policy is abandoned, all these players will have to re-optimize their decisions under new prices in the new equilibrium without the sugar program. Some of these players will choose to stay in the industry and decrease their capacity. Thus, they will encounter extra restructuring costs (for producers) or loose jobs (for workers). Some of them will choose to exit the industry altogether. However, the capital and labor resources will then most probably be used for alternative production. Thus, the economic cost calculation will have to take into account the alternative productive activities and the economic surplus they generate. For example, if all the resources were now moved to biotechnological products with higher economic value-added, it is possible that in net ending the program result in higher total economic surplus.

4 Tariffs and Quotas (2012 PS3)

Let us consider the worldwide market for cars. Suppose for a moment that consumers value all makes and types of cars equally. (In that case, we would call cars a homogeneous good). This problem considers how, hypothetically, U.S. domestic car producers might compete with newly emerging Indian car producers. Suppose that Indian producers can supply cars perfectly elastically at a price of $P = 150$. The domestic supply function of U.S. auto producers is given by:

$$P = 100 + 5Q$$

where prices and quantities are stated in thousands of dollars and millions of cars respectively. The U.S. domestic demand for cars is given by:

$$Q = -\frac{1}{10}P + 100$$

1. **(4 points)** Find the equilibrium of the U.S. car market in autarky (that is, no imports or exports).

In autarky, the equilibrium of the US car market will be at the intersection of the US demand and US domestic supply. In other words,

$$Q^* = -\frac{1}{10}(100 + 5Q^*) + 100$$

$$\frac{3}{2}Q^* = 90$$

$$Q_A^* = \frac{2}{3}90 = 60$$

$$P_A^* = 100 + 5 * 60 = 400$$

2. **(4 points)** Now suppose the US opened its market to trade and allowed Indian manufacturers to import their cars to the US without any additional tariffs or quotas. Find the new equilibrium. How many cars would U.S. manufacturers produce and at what prices would they sell them? How many cars would be imported?

Indian producers would be willing to supply as many cars as they can sell at $P = 150$. At this price, consumers want to buy: $Q_D^{P=150} = -\frac{150}{10} + 100 = 85$. At the same time at this price, the US producers would be willing to supply:

$$150 = 100 + 5Q_S^{P=150}$$

$$Q_S^{P=150} = 10$$

Therefore, the US producers will sell 10 cars at price $P = 150$, while 75 cars will be imported.

3. **(4 points)** Suppose that the US car manufacturers are concerned that they are not competitive with the Indian manufacturers and lobby the government to establish a tariff on Indian car exports of $t = 100$. How would this tariff change the equilibrium prices and quantities of cars sold by US manufacturers?

With the tariff in place, the price at which Indian producers can sell the cars in the US market is now $P^{tariff} = 250$. At this price, consumers demand $Q_D^{P=250} = -\frac{250}{10} + 100 = 75$. At the same time at this price, the US producers would be willing to supply:

$$250 = 100 + 5Q_S^{P=250}$$

$$Q_S^{P=250} = 30$$

Therefore, the US producers will sell 30 cars at price $P = 250$, while 45 cars will be

imported. Note that in addition the government will collect $T = 100 * imports = 4500$

4. **(4 points)** Now suppose instead of the tariff, the government used a quota instead. Calculate the quota that would result in the same equilibrium quantity of U.S. production as the $t = 100$ tariff.

We need to find a quota that will result in $Q_{domestic} = 30$. For the US producers to supply this quantity, the effective market equilibrium price has to be $P = 250$. As we saw in the previous part, the consumers are willing to buy 75 cars at this price. That is, we would implement the same equilibrium quantity of US production if we set the quota at $q = 45$. Note that the imports will be sold at the market price of $P = 250$. Note also that the problem is exactly equivalent in terms of prices and quantities to the tariff case, but as will see below, they are not equivalent in terms of surplus.

5. **(4 points)** After the US introduced the tariff in part (3), the Indian government decided to provide a subsidy of $s = 50$ per car to its producers, so that they could sell at an even lower price. How would that change the equilibrium? How much does Indian government spend on the subsidy? How much does US government collect in tariff?

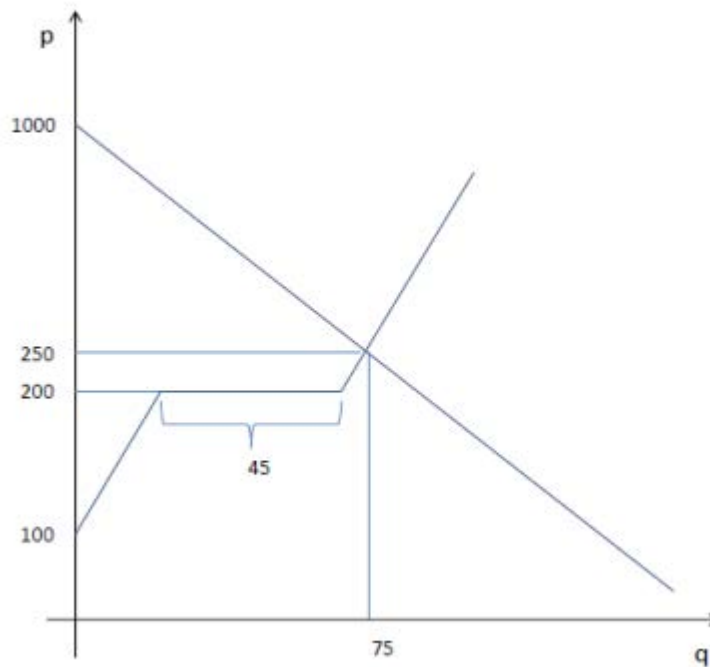
With the subsidy and the tariff, Indian producers can sell at $P = 150 - 50 + 100 = 200$. At this price, consumers are willing to purchase: $Q_D^{P=200} = -\frac{200}{10} + 100 = 80$. US producers are willing to supply:

$$200 = 100 + 5Q_S^{P=200}$$

$$Q_S^{P=200} = 20$$

In the new equilibrium, US producers supply 10 cars fewer, while Indian producers supply 60 cars, which is 15 cars more. At the same time, the US government collects $T = 100 * imports = 6000$ and Indian government spends $S = 50 * imports = 3000$.

6. **(4 points)** Draw a graph that represents the free trade equilibrium and the equilibria with the quota and tariff policies you found above in parts (3) through (5).



7. (6 points) Find consumer and producer surplus (for domestic producers) and government revenues under the free trade, tariff and the quota [you do not need to analyze the Indian subsidy scenario]. Would Indian producers prefer the quota or the tariff policy? What about the US producers?

Under free trade:

- Consumer surplus (we calculate the area of a triangle in between the equilibrium price and the demand function): $CS^A = \frac{1}{2} * 85(1000 - 250) = 36,125$
- Domestic producer surplus (we calculate the area of a triangle in between the equilibrium price and the supply function) : $PS^A = \frac{1}{2} * 75 * (250 - 100) = 2,250$
- Government revenue: 0

Under tariff:

- Consumer surplus (we calculate the area of a triangle in between the equilibrium price and the demand function): $CS^A = \frac{1}{2} * 75(1000 - 250) = 28,125$
- Domestic producer surplus (we calculate the area of a triangle in between the equilibrium price and the supply function) : $PS^A = \frac{1}{2} * 30 * (250 - 100) = 2,250$
- Government revenue: 4500

Under quota:

- Consumer surplus (we calculate the area of a triangle in between the equilibrium price and the demand function): $CS^A = \frac{1}{2} * 75(1000 - 250) = 28,125$
- Domestic producer surplus (we calculate the area of a triangle in between the equilibrium price and the supply function) : $PS^A = \frac{1}{2} * 30 * (250 - 100) = 2,250$
- Government revenue: 0

US producers are indifferent between quota and tariff, because either policy has the same effect on them. Indian producers sell 45 cars at 150 in case of tariff. They sell 45 cars at 250 in case of quota, so they certainly prefer the latter.

5 General equilibrium (2012 PS3)

Ann and Bob are two MIT students endowed with Mac computers (M) and Coffee (C). Suppose Mac computers are divisible for the purposes of this question (that is, you buy any fraction of a computer). Ann and Bob wish to trade with one another in order to maximize their individual utilities. We will consider how their trading decisions depend on the initial endowments of Macs and Coffee and on their utility functions.

Suppose both Ann and Bob are each endowed with one unit of Macs and one unit of Coffee, i.e. $e_A = (1, 1)$ and $e_B = (1, 1)$. Suppose their utility functions are given by:

$$U_A(M, C) = MC$$

$$U_B(M, C) = M + 2C$$

1. **(5 points)** Draw an Edgeworth box and indicate clearly all feasible allocations for this two-individual economy. Draw Ann and Bob's indifference curves in your Edgeworth box. Mark clearly the endowment allocation.

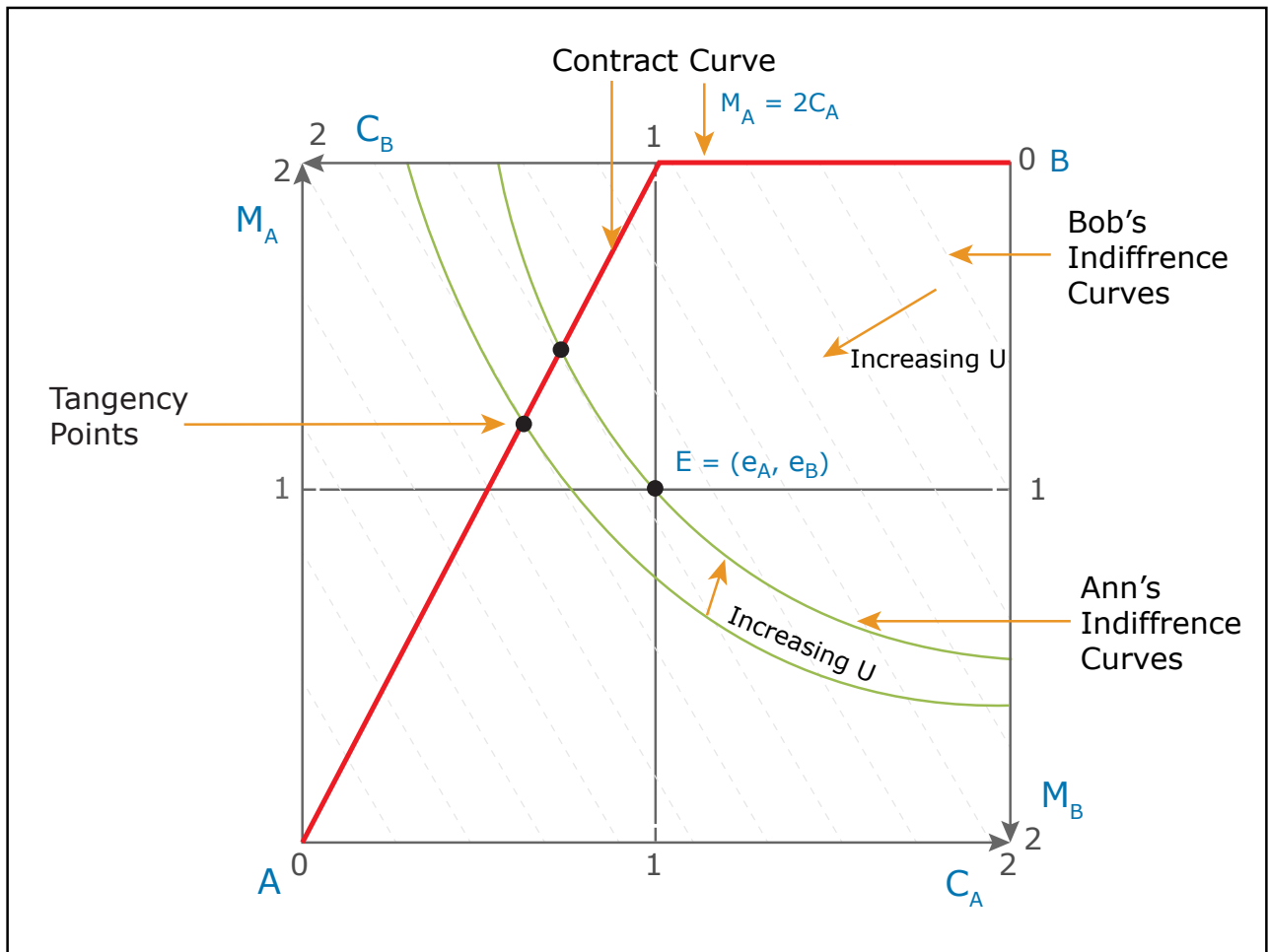


Image by MIT OpenCourseWare.

2. (5 points) Find the set of Pareto Optimal Allocations in this economy (this is also known as the contract curve). Draw the contract curve in an Edgeworth box and explain its meaning.

The Pareto Optimal Allocations (or the Contract Curve) are a set of allocations, such that by moving away from these allocations you cannot make one of the consumer's better off without making another worse off. This set of allocations is given by the set of tangency points of the consumers' utility functions. In other words, POA are those where consumers have the same marginal rate of substitution. Therefore, to find these points we equate the MRS for Ann and Bob:

$$MRS_A = MRS_B$$

$$\frac{U_M^A}{U_C^A} = \frac{U_M^B}{U_C^B}$$

$$\frac{C_A}{M_A} = \frac{1}{2}$$

Therefore, the contract curve is given by: $M_A = 2C_A$ when $C_A < 1$ and then, it is on the edge of the Edgeworth box, since no more than 2 Macs are available in the economy.

3. **(7 points)** Find the equilibrium consumption of Macs and Coffee by Ann and Bob in this economy and determine the price ratio that supports this equilibrium [You will find it helpful to normalize $P_C = 1$]

The equilibrium price and allocation have to satisfy three conditions:

- (a) Each person must be choosing a bundle that maximizes their utility subject to their budget constraint, i.e. $\frac{U_M^A}{U_C^A} = \frac{P_M}{P_C}$ and $\frac{U_M^B}{U_C^B} = \frac{P_M}{P_C}$. These imply: $\frac{C_A}{M_A} = \frac{P_M}{P_C}$ and $\frac{1}{2} = \frac{P_M}{P_C}$
- (b) Satisfy the resource constraints (we cannot consume more than is available): for Macs: $M_A + M_B = e_A^M + e_B^M = 2$ and for Coffee: $C_A + C_B = e_A^C + e_B^C = 2$
- (c) Satisfy the budget constraints (individuals cannot spend more than the value of their endowment): for Ann: $p_M M_A + p_C C_A = p_M e_A^M + p_C e_A^C$ and for Bob: $p_M M_B + p_C C_B = p_M e_B^M + p_C e_B^C$

Since only the price ratio rather than price levels matter in the equilibrium, we can normalize $p_C = 1$. Then, we get our system of equilibrium equations:

$$\begin{cases} \frac{C_A}{M_A} = P_M \\ \frac{1}{2} = P_M \\ M_A + M_B = 2 \\ C_A + C_B = 2 \\ p_M M_A + C_A = p_M + 1 \\ p_M M_B + C_B = p_M + 1 \end{cases}$$

Since the utility functions are quite simple in this case, we immediately see that the equilibrium price is $p_M^* = \frac{1}{2}$ and the equilibrium allocations are: $(M_A^*, C_A^*) = (\frac{3}{2}, \frac{3}{4})$ and $(M_B^*, C_B^*) = (\frac{1}{2}, \frac{5}{4})$

4. **(3 points)** Will this equilibrium allocation be Pareto Efficient? Please explain without reference to any algebra.

According to the First Welfare Theorem under certain conditions (check whether these are satisfied here!), the competitive equilibrium allocation is pareto efficient.

6 Import quotas and tariffs (2011 PS3)

Under the North American Free Trade Agreement (NAFTA), preferential treatment was offered to Mexican chili peppers in the American market. The current exercise explores stylized consequences of such treatment. Suppose that American demand for chili peppers is given by

$$D(p) = 100 - \frac{1}{10}p,$$

as measured in daily tons of chili pepper while the American domestic supply is given by

$$S^{US}(p) = -20 + \frac{p}{5}.$$

Suppose further that Mexican suppliers (S^{MX}) are capable of producing a perfectly elastic supply of chili peppers at a cost of U\$200/ton.

1. Find the equilibrium quantity and price if the American market is in autarky (no imports are allowed). Represent the solution graphically.
2. Find the equilibrium consumption, domestic production, imports and price if free trade with Mexico were allowed. Provide a graphic description of the equilibrium.
3. Suppose chili pepper producers from New Mexico and California constitute the most efficient firms in the US, with marginal costs up to U\$250/ton. These firms lobby Congress to set a quota that allows them to continue operating. (Assume that less efficient non-New Mexican Non-Californian producers, those with costs above \$250/ton, are not part of this producer lobby.) Find the largest quota that would allow the New Mexican and Californian firms to continue to operate. Again, provide a graphic description of this situation.
4. Find a unit tariff (a fixed additional dollar value per-ton) on Mexican imports that would achieve the same quantity of the U.S. production and, again, represent this situation with a graph.
5. Assume that other international producers can be represented by a perfectly elastic supply at a price of U\$220 and are subject to a tariff of \$10/ton. [Almost no math required for the next two items.]
 - (a) Would you expect Mexican chili pepper producers to argue in favor of free trade or of an untaxed quota if they are capable of selling their output in the US market at the ongoing price?

- (b) Without varying that tariff rate, what size of quota would maximize US import-tariff revenue? What about Mexican producer surplus?
- (c) What combination of tariffs on other international producers and quotas on Mexican producers would maximize total surplus?

Solution

1. The equilibrium price can be obtained from the condition which sets equal the demand and supply quantities

$$S(p) = D(p),$$

$$\begin{aligned} -20 + \frac{2p}{10} &= 100 - \frac{p}{10} \\ 3p &= 1200 \end{aligned}$$

which leads to $p^* = 400$. Quantities are therefore $S(400) = D(400) = 60$.

2. If imports from Mexico were allowed, aggregate supply (including domestic and Mexican producers) can be summarized by

$$S^{US+MX}(p) = \begin{cases} -20 + \frac{p}{5} & , \text{ for } p \in [0, 200] \\ [20, +\infty) & , \text{ for } p = 200, \end{cases}$$

representing that some American producers can produce below $p = 200$ but supply is perfectly elastic once that price is reached and imports from Mexico enter the market. The equilibrium can be found by the intersection of demand and this supply, which is graphically represented below,

The intersection occurs at $p = 200$ and $D(200) = 80$. Of these 80 tons, 20 are supplied domestically and the remaining 60 are imported.

3. To ensure that these producers remain in business, domestic prices have to be at least U\$250/ton. No price above that value would can lead to the largest possible quota for imports, as an increase in the quota would still allow these producers to survive and bring prices down, closer to U\$250. At $p = 250$, we have

$$D(250) = 100 - \frac{250}{10} = 75$$

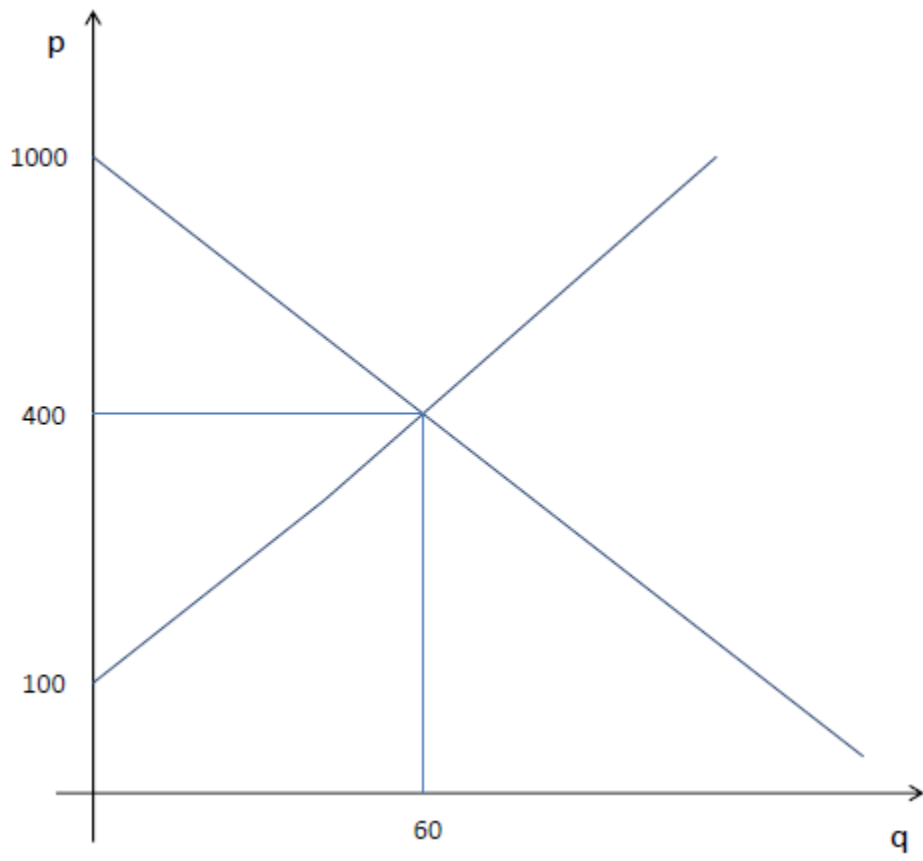


Figure 1: 1.a - Autarky Equilibrium, with US supply only.

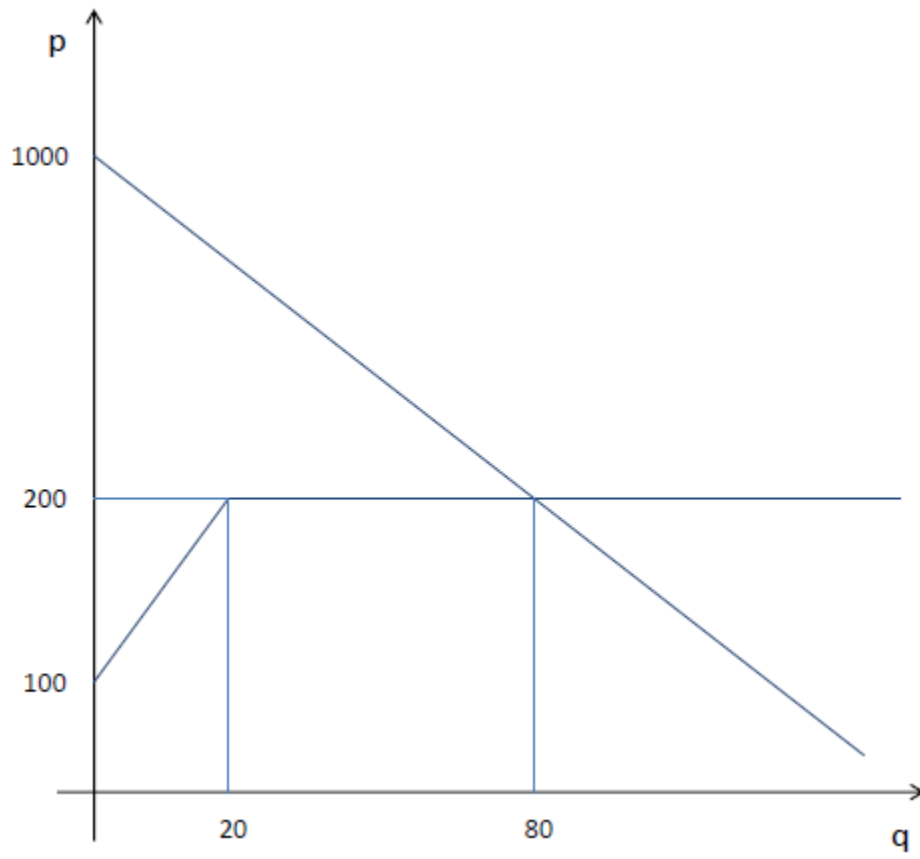


Figure 2: 1.b - US supply and MX quota.

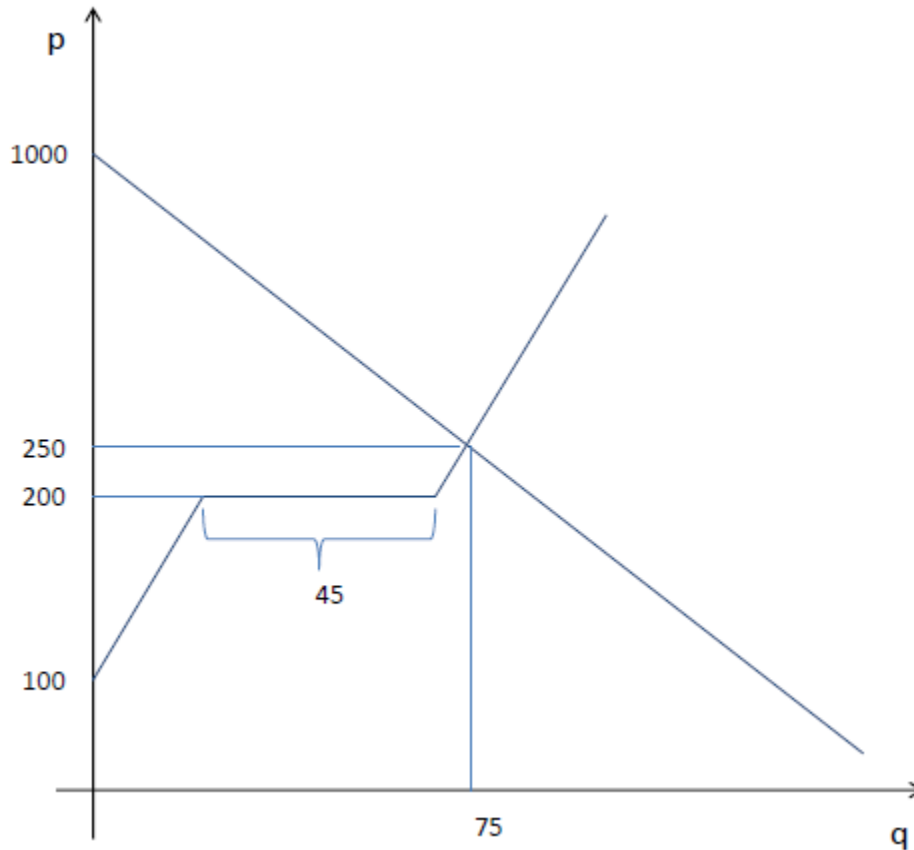


Figure 3: 1.c - Equilibrium with 45 ton quota.

and

$$S^{US}(250) = 30.$$

Therefore, the maximum quota possible is given by $\bar{S}^{MX} = 45$. A graphic representation is provided below.

4. A tariff that brings the domestic price to U\$250 will have the same effect as the quota. That tariff is the one necessary to shift the supply of Mexican chilli peppers as perceived by American consumers to be perfectly elastic at U\$250. Therefore, a tax of \$50/ton is required. A graphic representation (not requested) is provided below.

(i) If they argue in favor of free trade, the outcome is the one obtained in part (b), where prices are brought to U\$200 in the US market and no surplus is obtained by Mexican producers. If instead they request a quota, these producers can sell some units at U\$230 and capture

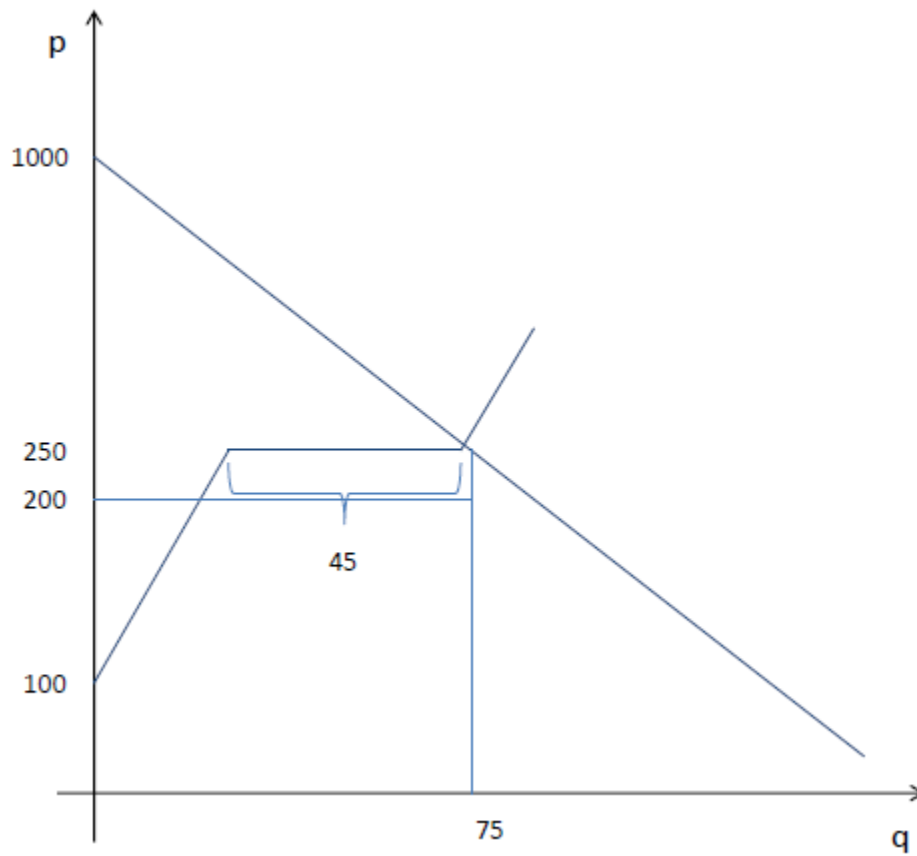


Figure 4: 1.d- Tax instead of quota, which raises the price of imported sugar to consumer to U\$250.

the U\$30/ton surplus. Therefore, we are likely to see Mexican producers arguing in favor a special treatment in the US market, with the right to a untaxed quota.

(ii) Naturally, a tariff exemption on any number of imported tons reduces revenues. Therefore, US revenues are maximized when no quota is offered to Mexican producers. On the other hand, Mexican producer welfare is maximized when they are responsible for selling all the imports in the US market at the price of U\$230, totally driving out other international producers. That means a quota of $D(230) - S^{US}(230) = 120 - \frac{3 \cdot 230}{10} = 120 - 69 = 51$ tons.

(iii) Surplus is maximized under the free-trade situation described on item (b), with domestic prices being \$200/ton. At those prices, at least 60 tons of Mexican chili peppers have to be allowed in the US (so, any quota above 60 tons is equivalent). No imports from other countries would be efficient, given that Mexican producers can supply it at U\$200, which is lower than U\$230. As a consequence, tariffs on these other international producers are irrelevant in this situation and any non-negative tariff would work¹

7 General Equilibrium (2011 PS3)

Ann and Bertha are two roommates who derive utility from the consumption of two goods with which they are initially endowed: cookies (c) and milk (m). Utility functions are given by

$$U_a(c_a, m_a) = \frac{1}{3} \ln c_a + \frac{2}{3} \ln m_a$$

and

$$U_b(c_b, m_b) = \frac{1}{2} \ln c_b + \frac{1}{2} \ln m_b.$$

Ann is initially endowed with 8 cookies and 4 gallons of milk, while Bertha owns 4 cookies and 8 gallons of milk. Answer the following:

1. What are Ann and Bertha's uncompensated (Marshallian) demands for cookies and milk?
2. Find the equilibrium allocation and price ratio ($\frac{p_c}{p_m}$). [hint: it is a good idea to normalize $p_m = 1$]
3. Derive an equation for the contract curve (that is, the set of Pareto efficient allocations).
4. Sketch an Edgeworth box showing endowments, competitive equilibrium prices and consumption choices, and indifference curves through the endowments and through the

¹Subsidies can be treated as negative tariffs and would also be neutral as long as they do not undo the \$30/ton difference.

equilibrium consumption choices (these do not have to be to scale). Indicate the area of mutually beneficial alternative allocations.

Solution

1. We can write the Lagrangians associated to their utility maximization problems as

$$\mathcal{L}_a = \frac{1}{3} \ln c_a + \frac{2}{3} \ln m_a + \lambda [p_c (8 - c_a) + p_m (4 - m_a)]$$

and

$$\mathcal{L}_b = \frac{1}{2} \ln c_a + \frac{1}{2} \ln m_a + \lambda [p_c (4 - c_a) + p_m (8 - m_a)].$$

The FOCs for Anna are

$$\begin{aligned} \frac{1}{3c_a} &= \lambda p_c \\ \frac{2}{3m_a} &= \lambda p_m \end{aligned}$$

and the budget constraint is

$$p_c (8 - c_a) + p_m (4 - m_a) = 0.$$

That leads to

$$c_a(p_c, p_m, 8, 4) = \frac{p_c 8 + p_m 4}{3p_c}$$

and

$$m_a(p_c, p_m, 8, 4) = \frac{2(p_c 8 + p_m 4)}{3p_c}.$$

Analogously, for Bertha, we get

$$\begin{aligned} \frac{1}{2c_b} &= \lambda p_c \\ \frac{1}{2p_m} &= \lambda p_m \end{aligned}$$

and

$$p_c (4 - c_a) + p_m (8 - m_a) = 0.$$

We obtain the following Marshallian demands

$$\begin{aligned}c_b(p_c, p_m, 4, 8) &= \frac{p_c 4 + p_m 8}{2p_c} \\m_b(p_c, p_m, 4, 8) &= \frac{p_c 4 + p_m 8}{2p_m}.\end{aligned}$$

2. Market clearing for cookies imposes

$$\frac{p_c 8 + 4}{3p_c} + \frac{p_c 4 + 8}{2p_c} = 12.$$

We can manipulate that to obtain $p_c = \frac{8}{11}$. That leads to the following allocation

$$\begin{aligned}c_a\left(\frac{8}{11}, 1, 8, 4\right) &= \frac{9}{2} \\m_a\left(\frac{8}{11}, 1, 8, 4\right) &= \frac{72}{11} \\c_b\left(\frac{8}{11}, 1, 4, 8\right) &= \frac{15}{2} \\m_b\left(\frac{8}{11}, 1, 4, 8\right) &= \frac{60}{11}\end{aligned}$$

3. First, we use the feasibility conditions, which ensure that consumption of the two goods by the two consumers is defined inside the Edgeworth box, and write

$$c_b = 12 - c_a$$

and

$$m_b = 12 - m_a.$$

The contract curve is given by the following tangency condition

$$\frac{m_a}{2c_a} = \frac{12 - m_b}{12 - c_a},$$

for $c_a \in [0, 12]$ and $m_a \in [0, 12]$.

4. The description is offered in the graph below. The area of mutually beneficial trades lies within the lens defined by the indifference curves that go through the endowment, which are represented in blue.

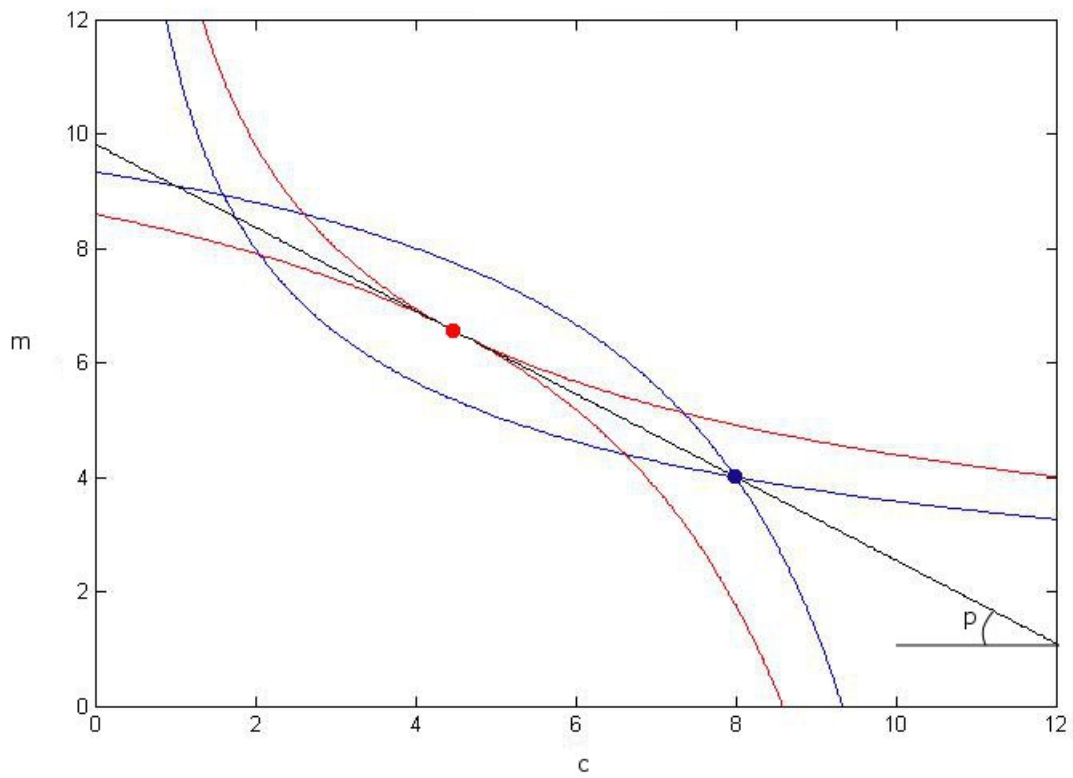


Figure 5: Edgeworth Box and key indifference curves. Initial endowment in blue, final allocation in red.

This checks out at E .

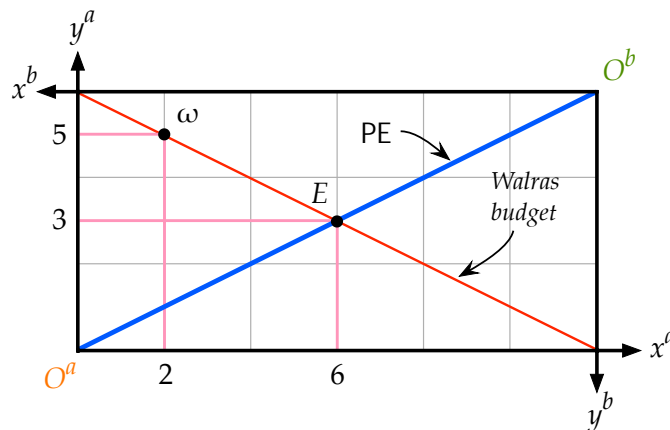
- 6.3. Two individuals, a and b , consume goods x and y . Their endowments are $\omega^a = (2, 5)$ and $\omega^b = (10, 1)$. Both have identical Cobb-Douglas utility functions $u^i(x^i, y^i) = x^i y^i$ where $i = a, b$. The price p_y is normalized to 1; for simplicity we write p_x as just p . Then consumer i 's demand for each good is

$$x^i = \frac{m^i}{2p} \quad \text{and} \quad y^i = \frac{m^i}{2},$$

where m^i refers to the value of consumer i 's endowment.

- (a) Draw the set of interior Pareto efficient allocations in an Edgeworth box for this economy.

For interior Pareto efficient allocations, the consumers' indifference curves must be tangent. Therefore, set $MRS^a = MRS^b$ to obtain $y^a/x^a = y^b/x^b$. Since $x^a + x^b = 12$ and $y^a + y^b = 6$, substitute for $y^b = 6 - y^a$ and $x^b = 12 - x^a$ and solve to obtain $y^a = x^a/2$. Therefore, the set of interior Pareto efficient allocations is the diagonal from O^a to O^b as shown.



- (b) Calculate the Walras equilibrium price \hat{p} and Walras allocation $((\hat{x}^a, \hat{y}^a), (\hat{x}^b, \hat{y}^b))$. Check that the Walras allocation is Pareto efficient graphically and algebraically.

Pick the market for good y . Since p_y is normalized to 1, note that $m^a = 2p_x + 5$ and $m^b = 10p_x + 1$. Therefore, the demand

of y for consumer a is $p_x + 2.5$, while the demand for consumer b is $5p_x + 0.5$. Adding them, the total demand is $6p_x + 3$ which has to equal the total supply of good y of 6. Solving, we obtain $\hat{p}_x = \$0.50$. Therefore, the Walras prices are $(\hat{p}_x, \hat{p}_y) = (0.50, 1)$. Then $m^a = m^b = 6$. Calculating the demands for each consumer, we obtain the Walras allocation $E = ((6, 3), (6, 3))$.

Graphically, since E is on the blue diagonal, it is Pareto efficient. Algebraically, we need to verify that $MRS^a = MRS^b$, i.e., $\hat{y}_a / \hat{x}^a = \hat{y}_b / \hat{x}^b$. This checks out at E .

- 6.4. For the following three-person two-commodity pure exchange economy, the price of good y is normalized to \$1 and p_x is written as p . The table below gives the utility functions, endowments, and demands for goods x and y , where m^i denotes the value of consumer i 's endowment. Calculate the Walras equilibrium price of good x , \hat{p} , and the Walras allocation, $((\hat{x}^a, \hat{y}^a), (\hat{x}^b, \hat{y}^b), (\hat{x}^c, \hat{y}^c))$. Check that the Walras allocation is Pareto efficient.

Person i	u^i	ω^i	x^i	y^i
a	$x^a y^a$	$(2, 0)$	$m^a / (2p)$	$m^a / 2$
b	$(x^b)^3 y^b$	$(0, 12)$	$3m^b / (4p)$	$m^b / 4$
c	$x^c (y^c)^2$	$(12, 0)$	$m^c / (3p)$	$2m^c / 3$

Pick the market for good y . Since price of good y is normalized to 1, $m^a = 2p$, $m^b = 12$, and $m^c = 12p$. Substituting in the demand of y for each consumer and adding, the total demand is $9p + 3$ which has to equal the total supply of good y of 12. Solving, we obtain $\hat{p} = \$1$. Therefore, the Walras prices are $(\hat{p}_x, \hat{p}_y) = (1, 1)$. Then $m^a = 2$, $m^b = 12$ and $m^c = 12$. Hence, the Walras allocation $E = ((1, 1), (9, 3), (4, 8))$.

To check for Pareto efficiency, check the marginal rates of substitution at the Walras allocation: $MRS^a = \hat{y}^a / \hat{x}^a = 1/1 = 1$, $MRS^b = 3\hat{y}^b / \hat{x}^b = 9/9 = 1$, and $MRS^c = \hat{y}^c / (2\hat{x}^c) = 8/8 = 1$. Since they are the same, the Walras allocation is Pareto efficient.

- 6.5. For the following two-person three-commodity pure exchange economy, the price of good z is normalized to \$1. Calculate the Walras equilibrium prices, (\hat{p}_x, \hat{p}_y) , as well as the Walras allocation for this

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