14.06 Macroeconomics
Spring 2003

Problem Set 7

(due on the day of Lecture # 11)

Problem 1 Romer Problem 4.3 (First-order autoregressive shocks)
Let $\ln A_0$ denote the values of A in period 0, and let the behavior of $\ln A$ be given by

$$\ln A_t = \bar{A} + gt + \tilde{A}_t,$$
$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_{At}, \text{ s.t. } -1 < \rho_A < 1,$$

where A is an input in the production function, known as technology, and $\tilde{A}$ reflects the fact that technology evolves according to a random process, specifically, a first-order autoregressive process.

1. Express $\ln A_1, \ln A_2$,and $\ln A_3$ in terms of $\ln A_0, \epsilon_{A1}, \epsilon_{A2}, \epsilon_{A3}, \bar{A}$,$g$, and ρ_A.

2. Given that expectations of the ϵ_A 's are zero, what are the expectations of $\ln A_1, \ln A_2$,and $\ln A_3$ given $\ln A_0$, $\bar{A}$, $\rho_A$and g?

Problem 2 Augmented Romer Problems 4.4 and 4.5
Suppose the period-t utility function, u_t, is $u_t = \ln c_t + b(1-l_t)^{1-\gamma}/(1-\gamma)$, $b > 0, \gamma > 0$. (Note that with $\gamma = 1$, utility reduces to the form seen in class: $u_t = \ln c_t + b\ln(1-l_t)$.)

1. Consider the household's problem of maximizing utility subject to a budget constraint $c = wl$ where is c consumption, l is hours worked, and w is the wage. Find the first order conditions and solve for the labor supply. How, if at all, does labor supply depend on the wage?

2. Consider an extension to the previous problem. Instead of a static problem, the consumer/worker lives two periods (and discounts second period utility by $\frac{1}{1+\rho}$.) There is no uncertainty.

   (a) Write the lifetime budget constraint.

   (b) Write down the first order conditions, and solve for the relative demand for leisure in the two periods.

   (c) How does the relative demand for leisure depend on the relative wage? Show that an increase in both w_1 and w_2 that leaves w_1/w_2 unchanged does not affect l_1 or l_2.

(d) Suppose output is given by $Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$, $0 < \alpha < 1$. Solve for w_t, the wage rate (assume labor is paid its marginal product). Now suppose there is a positive technology shock at time 1, $A_1 = A$, $A > 1$ (assume that at time 2, technology returns to $A_2 = 1$). What is the effect, if any, on the relative wage and on the relative demand for leisure? Does it make sense?

(e) How does the relative demand for leisure depend on the interest rate?, on the time preference rate?

(f) Explain intuitively why γ affects the responsiveness of labor supply to wages and the interest rate.

(g) Solve for the Euler equation, that is express the relationship between c_1 and c_2. What if $\rho = r$?

(h) Now assume that the household has initial wealth of amount $Z > 0$. Does the Euler equation derived in part g continue to hold?

Problem 3 Romer Problem 4.8 (A simplified RBC model with additive technology shocks). Consider an economy consisting of a constant population of infinitely-lived individuals. The representative individual maximizes the expected value of

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t), \quad \rho > 0$$
$$\text{where } u(C_t) = C_t - \theta C_t^2, \ \theta > 0$$

Assume that C is always in the range where $u\prime(C)$ is positive.
Output is linear in capital, plus an additive disturbance: $Y_t = AK_t + e_t$. There is no depreciation; thus $K_{t+1} = K_t + Y_t - C_t$, and the interest rate is A. Assume $A \equiv r = \rho$. Finally, the disturbance follows a first-order autoregressive process: $e_t = \phi e_{t-1} + \varepsilon_t$, where $-1 < \phi < 1$ and where the ε's are mean zero, i.i.d shocks.

1. Find the first-order condition (Euler equation) relating C_t and expectations of C_{t+1}.(Hint: set up the Bellman equation and maximize w.r.t K_{t+1} after substituting for C_t as functions of K_t, K_{t+1} etc.)

2. Guess that consumption takes the form $C_t = \alpha + \beta K_t + \gamma e_t$. Given this guess, what is K_{t+1} as a function of K_t and e_t?

3. What values must the parameters α, β, and γ have for the first-order condition in part 1 to be satisifed for all values of K_t and e_t?

4. What are the effects of a one-time shock to ε (suppose $\Delta\varepsilon_t = 1$) on the paths of Y, K, and C ?