Lecture 10
Subgame-perfect Equilibrium

14.12 Game Theory

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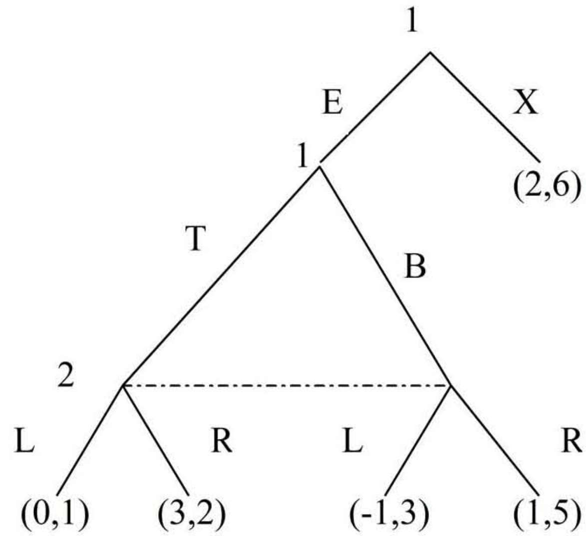
Road Map

1. Subgame-perfect Equilibrium
    1. Motivation
    2. What is a subgame?
    3. Definition
    4. Example
2. Applications
    1. Bank Run
    2. Infinite-horizon Bargaining

A game



Backward induction

- Can be applied only in perfect information games of finite horizon.

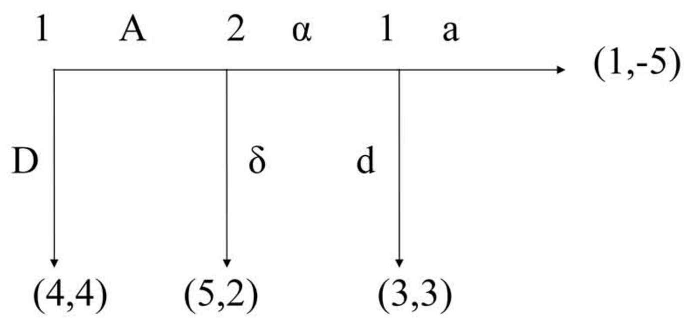
How can we extend this notion to infinite horizon games, or to games with imperfect information?

A subgame

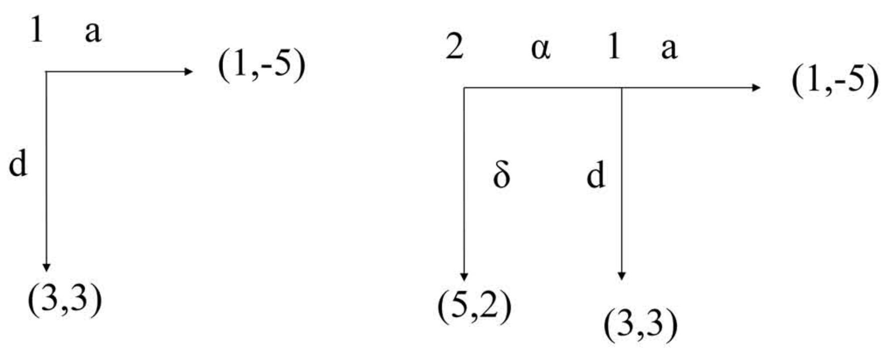
A *subgame* is part of a game that can be considered as a game itself.

- It must have a unique starting point;
- It must contain all the nodes that follow the starting node;
- If a node is in a subgame, the entire information set that contains the node must be in the subgame.

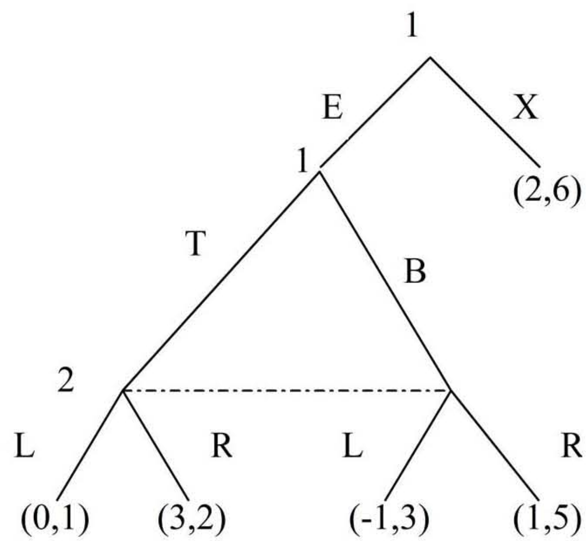
A game



And its subgames



A game

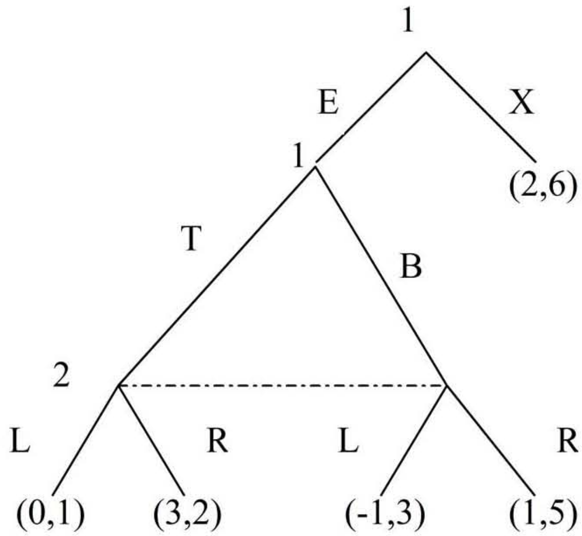


Definitions

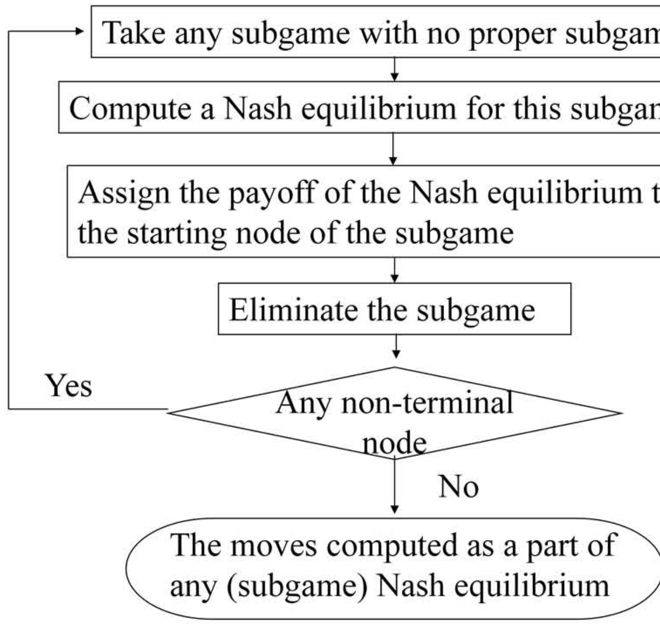
A *substrategy* is the restriction of a strategy to a subgame.

A subgame-perfect Nash equilibrium is a Nash equilibrium whose substrategy profile is a Nash equilibrium at each subgame.

Example



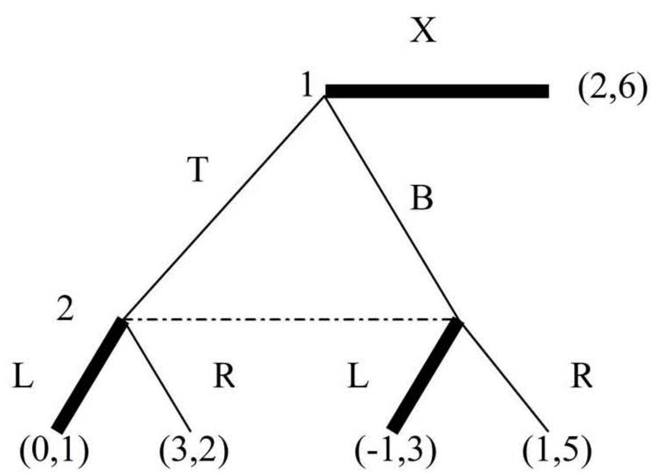
A "Backward-Induction-like" method



In a finite, perfect-information game, …

… the set of subgame-perfect equilibria is the set of strategy profiles that are computed via backward induction.

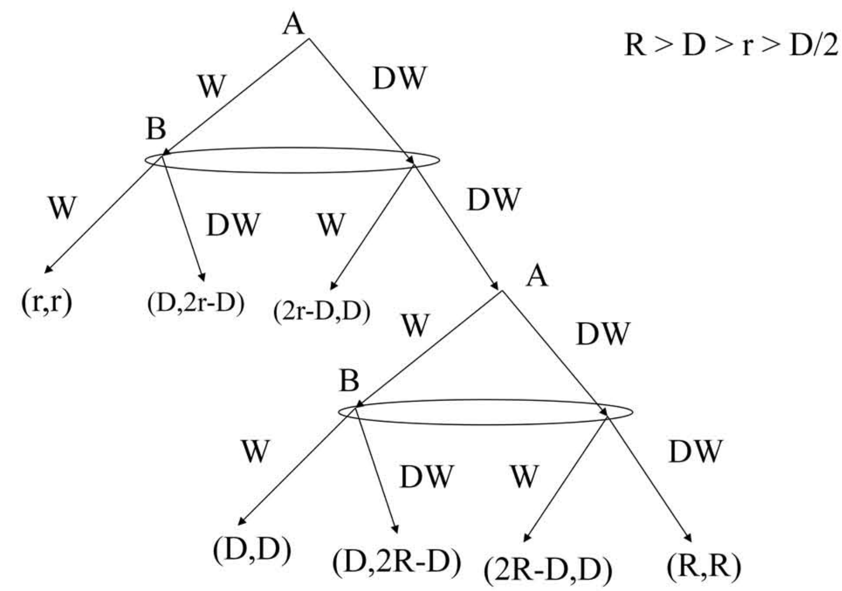
A subgame-perfect equilibrium?



Bank Run

- Alice and Bob each deposit D = \$1M in a bank
- Bank invests the money in a project, which pays 2r if liquidated at t=1, 2R if waited to t=2, where $R > D > r > D/2$
- Either player has the option of withdrawing at either date, getting D if bank has the money
- If they do not withdraw, bank pays R to each

Bank Run



Infinite-horizon Bargaining

T = {1,2,…, n-1,n,…}

If t is odd,

- Player 1 offers some (x_t,y_t),
- Player 2 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding $\delta^t(x_t,y_t)$,
- Otherwise, we proceed to date t+1.

If t is even

- Player 2 offers some (x_t,y_t),
- Player 1 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding payoff (x_t,y_t),
- Otherwise, we proceed to date t+1.

$n \longrightarrow \infty$

t = 2n – 2k – 1

$$x_t = \frac{1-\delta^{2k+1}}{1+\delta} = \frac{1-\delta^{2n-t}}{1+\delta} \xrightarrow{n\to\infty} \frac{1}{1+\delta}$$

A SPE: At each t,

- proposer offers δ/(1+δ) to the other
- and keeps 1/(1+δ) for himself;
- responder accepts an offer iff
- she gets at least δ/(1+δ) .

14.12 Economic Applications of Game Theory
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