

Lectures 16
Incomplete Information
Static Case

14.12 Game Theory
Muhamet Yildiz

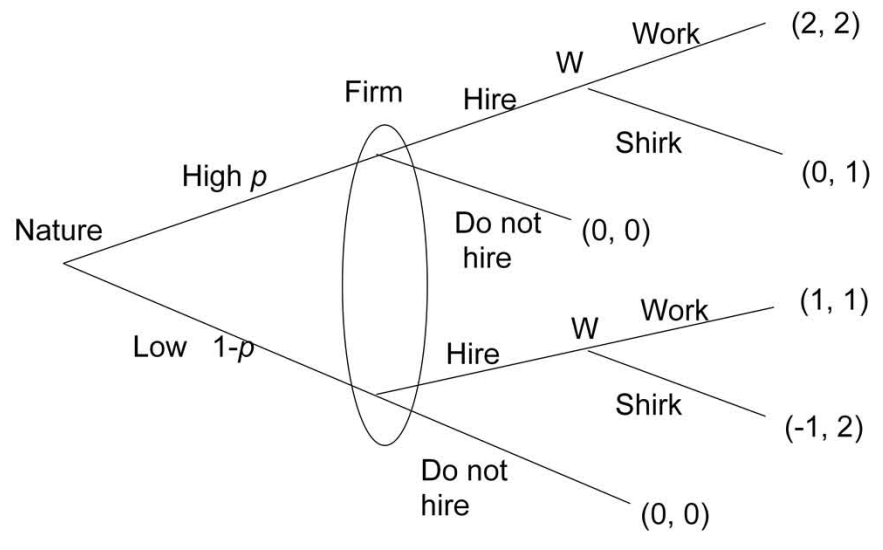
Road Map

1. Example
2. Bayesian Games
3. Bayesian Nash Equilibrium
4. More Examples
5. Bayes' Rule

Incomplete information

one player knows something (relevant)
that some other player does not know.

Example



Bayesian Game (Normal Form)

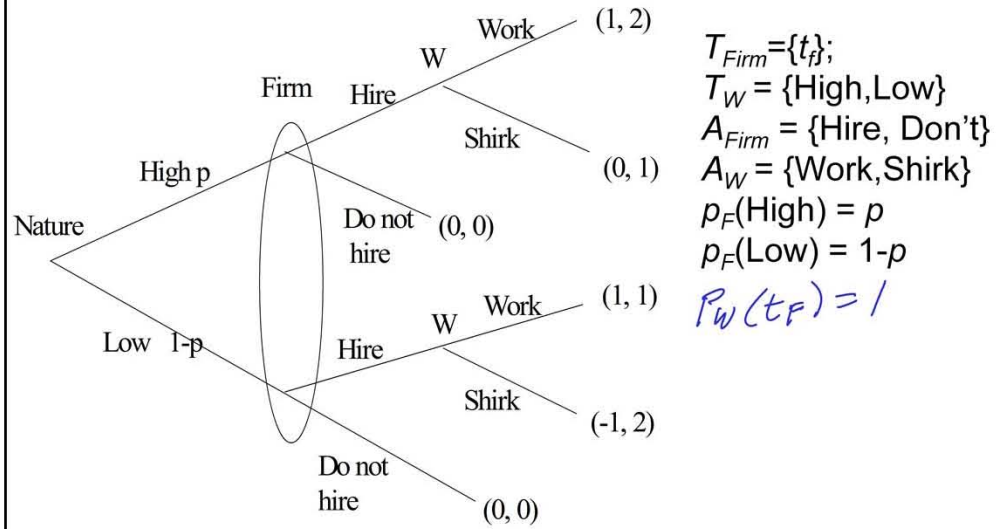
A Bayesian game is a list

$$G = \{A_1, \dots, A_n; T_1, \dots, T_n; p_1, \dots, p_n; u_1, \dots, u_n\}$$

where

- A_i is the action space of i (a_i in A_i)
- T_i is the type space of i (t_i in T_i)
- $p_i(t_{-i}|t_i)$ is i 's belief about the other players
- $u_i(a_1, \dots, a_n; t_1, \dots, t_n)$ is i 's payoff.

An Example



Bayesian Nash equilibrium

A Bayesian Nash equilibrium is a Nash equilibrium of a Bayesian game (when each type has positive prob).

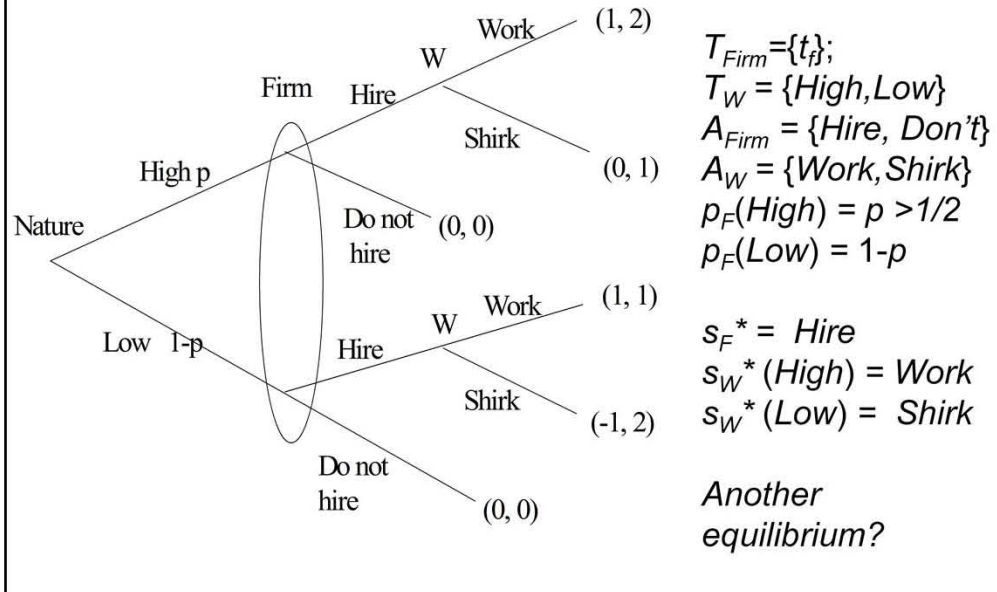
- Bayesian game

$$G = \{A_1, \dots, A_n; T_1, \dots, T_n; p_1, \dots, p_n; u_1, \dots, u_n\}$$

- a **strategy** of i is any function $s_i: T_i \rightarrow A_i$;
- A strategy profile $s^* = (s_1^*, \dots, s_n^*)$ is a **Bayesian Nash equilibrium** $\Leftrightarrow s_i^*(t_i)$ is a best response to s_{-i}^* for each t_i , i.e.,

$$\max_{a_i \in A_i} \sum_{t_{-i} \in T_{-i}} u_i(s_1^*(t_1), \dots, s_{i-1}^*(t_{i-1}), a_i, s_{i+1}^*(t_{i+1}), \dots, s_n^*(t_n); t) p_i(t_{-i} | t_i)$$

An Example



Another example

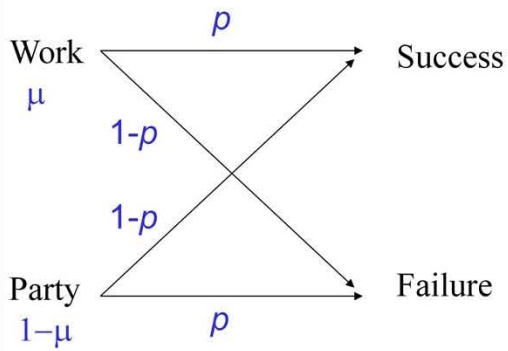
	L	R
X	θ, γ	1, 2
Y	-1, γ	$\theta, 0$

- $\theta \in \{0, 2\}$, known by Player 1
- $\gamma \in \{1, 3\}$, known by Player 1
- All values are equally likely
- $T_1 = \{0, 2\}$; $T_2 = \{1, 3\}$
- Bayesian Nash Equilibrium:
 - $s_1(0) =$
 - $s_1(2) =$
 - $s_2(1) =$
 - $s_2(3) =$

Bayes' Rule

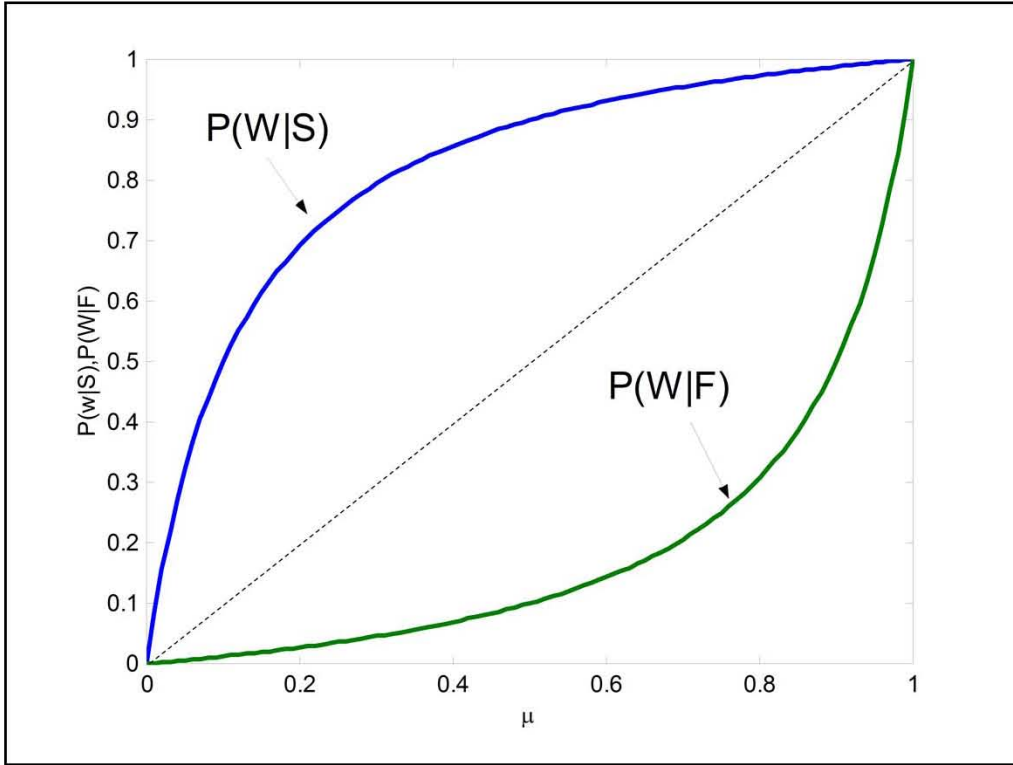
- $\text{Prob}(A|B) = \frac{\text{Prob}(A \text{ and } B)}{\text{Prob}(B)}$
- $\text{Prob}(A|B)\text{Prob}(B) = \text{Prob}(A \text{ and } B) = \text{Prob}(B|A)\text{Prob}(A)$
- $\text{Prob}(A|B) = \frac{\text{Prob}(B|A)\text{Prob}(A)}{\text{Prob}(B)}$

Example



- $\Pr(\text{Work}|\text{Success}) =$

- $\Pr(\text{Work}|\text{Failure}) =$



MIT OpenCourseWare
<http://ocw.mit.edu>

14.12 Economic Applications of Game Theory
Fall 2012

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.