Lecture 7
Imperfect Competition

14.12 Game Theory

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Road Map

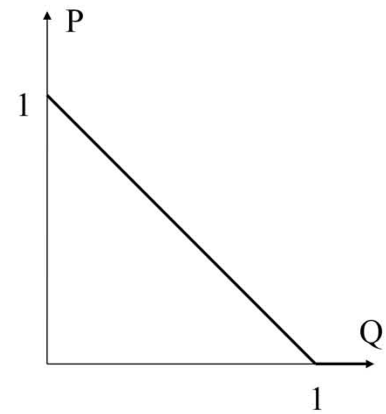
1. Cournot (quantity) competition
   1. Rationalizability
   2. Nash Equilibrium
2. Bertrand (price) competition
   1. Nash Equilibrium
   2. Rationalizability with discrete prices
   3. Search Costs

Cournot Oligopoly

- $N = \{1,2,\ldots,n\}$ firms;
- Simultaneously, each firm i produces q_i units of a good at marginal cost c,
- and sells the good at price

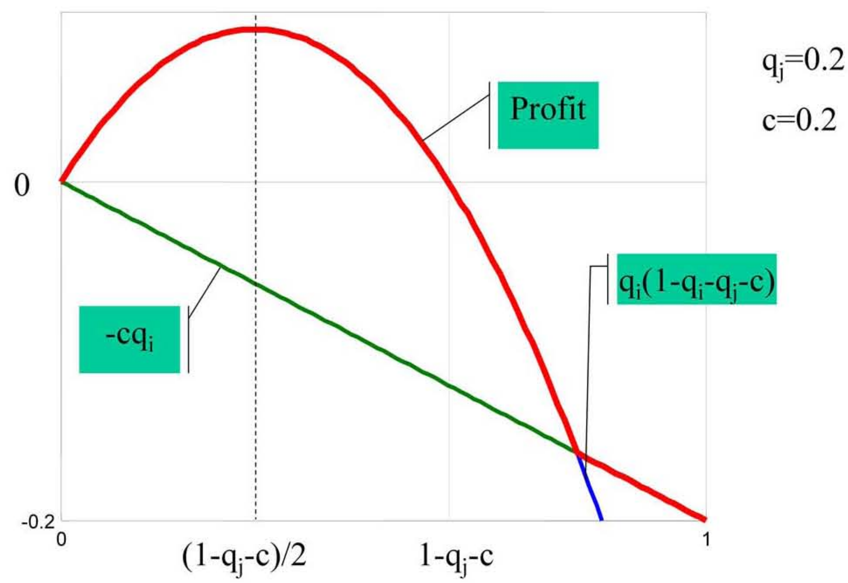
  $$P = \max\{0,1-Q\}$$

  where $Q = q_1+\ldots+q_n$.
- Game = $(S_1,\ldots,S_n; \pi_1,\ldots,\pi_n)$ where $S_i = [0,\infty)$,



$$\pi_i(q_1,\ldots,q_n) = \begin{cases} q_i[1-(q_1+\ldots+q_n)-c] & \text{if } q_1+\ldots+qn < 1, \\ -q_i c & \text{otherwise.} \end{cases}$$

Cournot Duopoly -- profit



C-D – best responses

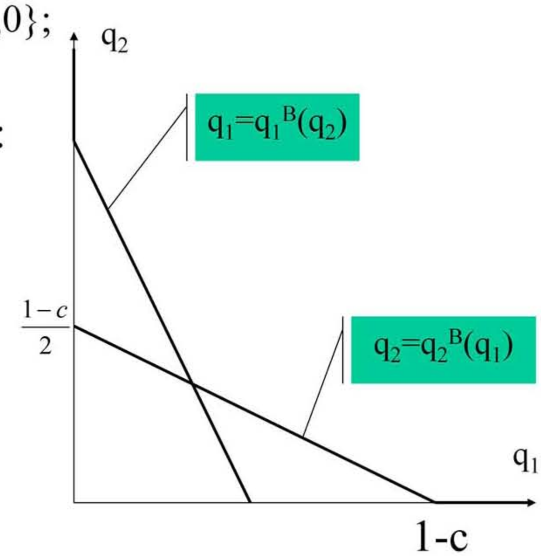
$q_i^B(q_j) = \max\{(1-q_j-c)/2, 0\}$;

- Nash Equilibrium q*:

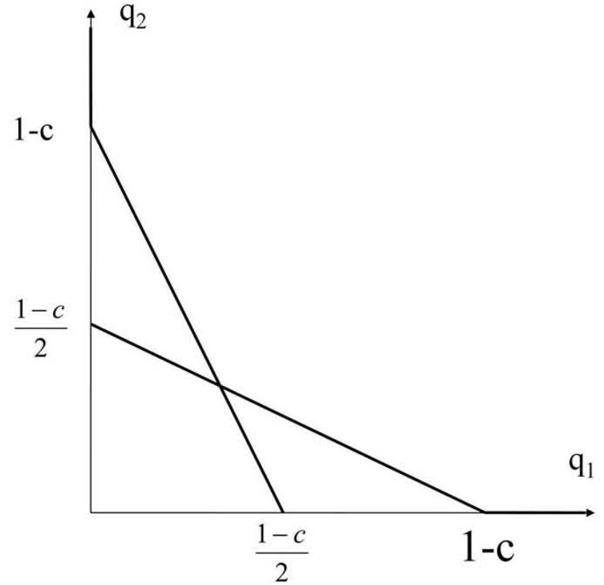
  $q_1^* = (1-q_2^*-c)/2$;

  $q_2^* = (1-q_1^*-c)/2$;

- $q_1^* = q_2^* = (1-c)/3$



Rationalizability in Cournot Duopoly



Rationalizability in Cournot duopoly

- If i knows that $q_j \leq q$, then $q_i \geq (1-c-q)/2$.
- If i knows that $q_j \geq q$, then $q_i \leq (1-c-q)/2$.
- We know that $q_j \geq q^0 = 0$.
- Then, $q_i \leq q^1 = (1-c-q^0)/2 = (1-c)/2$ for each i;
- Then, $q_i \geq q^2 = (1-c-q^1)/2 = (1-c)(1-1/2)/2$ for each i;
- …
- Then, $q^n \leq q_i \leq q^{n+1}$ or $q^{n+1} \leq q_i \leq q^n$ where
  $q^{n+1} = (1-c-q^n)/2 = (1-c)(1-1/2+1/4-\ldots+(-1/2)^n)/2$.
- As $n \to \infty$, $q^n \to (1-c)/3$.

Rationalizability in Cournot oligopoly

is not very helpful!!!

1. n = 3
2. Everybody is rational
3. => $q_i \leq (1\text{-}c)/2$;
4. Everybody is rational and knows 2
5. => $q_i \geq 0$
6. Everybody is rational and knows 4
7. => $q_i \leq (1\text{-}c)/2$;
8. Everybody is rational and knows 6
9. => $q_i \geq 0$

Cournot Oligopoly --Equilibrium

- $q > 1\text{-}c$ is strictly dominated, so $q \leq 1\text{-}c$.
- $\pi_i(q_1,\ldots,q_n) = q_i[1-(q_1+\ldots+q_n)-c]$ for each i.
- FOC:

$$\left.\frac{\partial \pi_i(q_1,\ldots,q_n)}{\partial q_i}\right|_{q=q^*} = \left.\frac{\partial[q_i(1-q_1-\cdots-q_n-c)]}{\partial q_i}\right|_{q=q^*}$$

$$= (1-q_1^*-\cdots-q_n^*-c)-q_i^* = 0.$$

- That is,

$$2q_1^* + q_2^* + \cdots + q_n^* = 1-c$$
$$q_1^* + 2q_2^* + \cdots + q_n^* = 1-c$$
$$\vdots$$
$$q_1^* + q_2^* + \cdots + 2q_n^* = 1-c$$

- Therefore, $q_1^* = \ldots = q_n^* = (1-c)/(n+1)$.

Bertrand (price) competition

- N = {1,2} firms.
- Simultaneously, each firm i sets a price p_i;
- If $p_i < p_j$, firm i sells Q = max{1 – p_i,0} unit at price p_i; the other firm gets 0.
- If $p_1 = p_2$, each firm sells Q/2 units at price p_1, where Q = max{1 – p_1,0}.
- The marginal cost is 0.

$$\pi_1(p_1, p_2) = \begin{cases} p_1(1-p_1) & \text{if } p_1 < p_2 \\ p_1(1-p_1)/2 & \text{if } p_1 = p_2 \\ 0 & \text{otherwise.} \end{cases}$$

Bertrand duopoly -- Equilibrium

Theorem: The only Nash equilibrium in the "Bertrand game" is $p^* = (0,0)$.

Proof:

1. $p^*=(0,0)$ is an equilibrium.
2. If $p = (p_1,p_2)$ is an equilibrium, then $p = p^*$.
   1. If $p = (p_1,p_2)$ is an equilibrium, then $p_1 = p_2$...
      - $p_i > p_j = 0 \Rightarrow p_j' = \varepsilon$; $p_i > p_j > 0 \Rightarrow p_i' = p_j$
   2. If $p_1 = p_2$ in equilibrium, then $p = p^*$.
      - $p_1 = p_2 > 0 \Rightarrow p_j' = p_j - \varepsilon$

Bertrand competition with discrete prices -- Rationalizability

- Allowable prices $P = \{0.01, 0.02, 0.03, \ldots\}$
- **Round 1**: Any $p_i > 0.5$ is eliminated
  - p_i is strictly dominated by σ_i with $\sigma_i(.5)=1-\varepsilon$, $\sigma_i(.01)=\varepsilon$ for small ε.
- **Round m**:
  - $P = \{0.01, 0.02, \ldots, p^m\}$ available prices at round m
  - If $p^m > .01$, it is strictly dominated by σ_i with $\sigma_i(p^m - .01) = 1-\varepsilon$, $\sigma_i(.01) = \varepsilon$ for small ε.
- **Rationalizable strategies**: $\{0.01\}$

Bertrand Competition with costly search

- $N = \{F1, F2, B\}$; F1, F2 are firms; B is buyer
- B needs 1 unit of good, worth 6;
- Firms sell the good; Marginal cost = 0.
- Possible prices $P = \{3,5\}$.
- Buyer can check the prices with a small cost $c > 0$.

Game:

1. Each firm i chooses price p_i;
2. B decides whether to check the prices;
3. (Given) If he checks the prices, and $p_1 \neq p_2$, he buys the cheaper one; otherwise, he buys from any of the firm with probability ½.

Bertrand Competition with costly search

F1 \ F2	High	Low
High		
Low		

Check

F1 \ F2	High	Low
High		
Low		

Don't Check

Mixed-strategy equilibrium

- Symmetric equilibrium: Each firm charges "High" with probability q;
- Buyer Checks with probability r.
- $U(check;q) = q^2 1 + (1-q^2)3 - c = 3 - 2q^2 - c$;
- $U(Don't;q) = q1 + (1-q)3 = 3 - 2q$;
- Indifference: $2q(1-q) = c$; i.e.,
- $U(high;q,r) = (1-r(1-q))5/2$;
- $U(low;q,r) = qr3 + (1-qr)3/2$
- Indifference: $r = 2/(5-2q)$.

14.12 Economic Applications of Game Theory
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