14.121 Final Exam
October 28, 2005

Answer <u>all</u> questions. You have 90 minutes in which to complete the exam. Don't spend too much time on any one question.

1. (15 Minutes – 20 Points) Answer each of the following subquestions BRIEFLY.

(a) Define equivalent variation and compensating variation. Given an example of a problem for which equivalent variation would be an appropriate concept to apply.

(b) Find the Hicksian demand function for a consumer with $u(x_1, x_2) = \sqrt{x_1} + x_2$.

(c) The traditional celebration of Thanksgiving in the United States involves families gathering together and eating a large meal that includes a whole roasted turkey. The tradition is widely followed: over 95% of Thanksgiving meals include a turkey and twenty times more turkeys are sold during the Thanksgiving week than in a normal week. An initially puzzling observation is that supermarkets typically put turkeys on sale during this week. Why might this occur?

2. (20 Minutes – 27 Points)

Suppose that Glenn Ellison is considering purchasing flood insurance for his house. If Glenn does not buy flood insurance, his wealth will be w if there is no flood and $w - L$ if there is a flood. The probability of a flood is π.

The price of a policy that pays K if a flood occurs is cK. Assume that $c < 1$. (Note that the problem is otherwise uninteresting because Glenn would never buy any insurance.)

Assume that Glenn can choose any $K \in [0, L]$, and that his choice of how much insurance to buy maximizes his expected utility. Assume that Glenn's von Neumann-Morgenstern utility function u is a differentiable, strictly increasing and concave function of his final wealth, i.e. Glenn maximizes $(1 - \pi)u(w - cK) + \pi u(w - L - cK + K)$.

(a) Find the first-order condition that characterizes Glenn's choice of K (assuming that the parameters are such that an interior optimum exists.)

(b)For what value(s) of c will Glenn purchase full insurance? Does the answer depend on the form of the utility function u? Why is this?

(c) Drop the assumptions that u is differentiable and concave – assume only that u is strictly increasing and that a utility-maximizing choice exists. Show that the K that Glenn chooses is weakly increasing in the probability π of a flood occurring.

3. (25 Minutes – 30 Points)

Consider an economy with three goods. Suppose that a consumer has a continuous utility function satisfying local nonsatiation. Suppose also that the consumer's Walrasian demands for goods 1 and 2 when $p_3 = 1$ satisfy

$$
\begin{aligned}
x_1(p_1, p_2, 1, W) &= a_1 + b_1 p_1 + c_1 p_2 + d_1 p_1 p_2 \\
x_2(p_1, p_2, 1, W) &= a_2 + b_2 p_1 + c_2 p_2 + d_2 p_1 p_2
\end{aligned}
$$

(a) State Walras' law and use it to find the Walrasian demand for good 3. (It's fine to just give the demand when $p_3 = 1$.)

(b) State a result about the homogeneity of Walrasian demands and use it to find the consumer's Walrasian demands at other values of p_3.

(c) Note that the Walrasian demands for goods 1 and 2 are independent of wealth. Show that this makes it very easy to find the Hicksian demands for goods 1 and 2. State the Compensated Law of Demand. Show that this law puts some restrictions on the possible values for $(a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2)$.

(d) Define the Slutsky substitution matrix. What properties must it have if demands are derived from maximizing a continuous, locally nonsatiated, and strictly quasiconcave utility function? Give at least one additional restriction on $(a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2)$ that this implies.

4. (25 Minutes – 23 points)

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of states. Let $\Delta(X)$ be the set of lotteries with outcomes in X. Write δ_{x_i} for the lottery in which x_i is realized with probability one.

Let $\succcurlyeq_P$ be a preference on $\Delta(X)$. Assume that $\succcurlyeq_P$ is transitive and that $\delta_{x_1} \succcurlyeq_P \delta_{x_2} \succcurlyeq_P \ldots \succcurlyeq_P \delta_{x_n}$.

(a) State the Archimedian axiom.

(b) The preference $\succcurlyeq_P$ is said to satisfy *monotonicity* if $a\delta_{x_1} + (1-a)\delta_{x_n} \succcurlyeq_P b\delta_{x_1} + (1-b)\delta_{x_n}$ if and only if $a \geq b$. Show that monotonicity implies that $\delta_{x_1} \succ_P \delta_{x_n}$.

(c) The preference $\succcurlyeq_P$ is said to satisfy *continuity* if for all $x \in \Delta(X)$ there exists an $a \in [0, 1]$ such that $a\delta_{x_1} + (1-a)\delta_{x_n} \sim_P x$. Show that if $\succcurlyeq_P$ satisfies monotonicity and continuity, then it satisfies the Archimedian axiom.

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