14.123 Microeconomics III—Problem Set 1

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Instructions. You are encouraged to work in groups, but everybody must write their own solution to the problem that is for grade. Good Luck!

(i) (For Grade) There are n individuals. Each individual i has constant absolute risk aversion $\alpha_i > 0$ and an asset that pays X_i where $(X_1, \ldots, X_n) \sim N((\mu_1, \ldots, \mu_n), \Sigma)$.

(a) What are the optimal risk sharing contracts? What is the vector of payoffs from an optimal risk-sharing contract? Characterize the set of the vectors of certainty equivalents from optimal risk sharing contracts.

(b) Answer (a) for $\alpha_1 = \cdots = \alpha_n$, $\mu_1 = \cdots = \mu_n$ and

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \ldots & \rho & \rho \\ \rho & 1 & \ldots & \rho & \rho \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \rho & \rho & \ldots & 1 & \rho \\ \rho & \rho & \ldots & \rho & 1 \end{pmatrix}.$$

How much the society as a whole are willing to pay all of these assets? Assuming that they write a symmetric contract, what is the preference relation of an individual on (σ^2, ρ) pairs? Briefly discuss.

(ii) Exercise 2.1 in lecture notes.

(iii) Consider the set of lotteries (p_x, p_y, p_z) on the set of outcomes $\{x, y, z\}$ where p_x, p_y, and p_z are the probabilities of x, y, and z, respectively.

(a) For each (partial) preference below, determine whether it is consistent with expected utility maximization. (If yes, find a utility function; if no, show that it cannot come from an expected utility maximizer.)

i. $(0,1,0) \succ (1/8, 6/8, 1/8)$ and $(7/8, 0, 1/8) \succ (6/8, 1/8, 1/8)$

ii. $(1/4, 1/4, 1/2) \succ (3/4, 0, 1/4) \succ (5/6, 1/6, 0) \succ (1/2, 1/3, 1/6)$

(b) For each family of indifference curves below, determine whether it is consistent with expected utility maximization. (If yes, find a utility function; if no, show that it cannot come from an expected utility maximizer.)

i. $p_y = c - 2p_x$ (where c varies)

ii. $p_y = c(p_x + 1)$ (where c varies)

iii. $p_y = c - 2\sqrt{p_x}$ (where c varies)

(c) Find a complete and transitive preference relation on the above lotteries that satisfies the independence axiom but cannot have an expected utility representation.

(iv) Alice has M dollars and has a constant absolute risk aversion α (i.e. $u(x) = -e^{-\alpha x}$) for some $\alpha > 0$. With some probability $\pi \in (0, 1)$ she may get sick, in which case she would need to spend L dollars on her health. There is a health-insurance policy that fully covers her health care expenses in case of sickness and costs P to her. (If she buys the policy, she needs to pay P regardless of whether she gets sick.)

(a) Find the set of prices P that she is willing to pay for the policy. How does the maximum price $\bar{P}$ she is willing to pay varies with the parameters M, L, α, and π?

(b) Suppose now that there is a test $t \in \{-1, +1\}$ that she can take before she makes her decision on buying the insurance policy. If she takes the test and the test t is positive, her posterior probability of getting sick jumps to $\pi^+ > \pi$ and if the test is negative, then her posterior probability of getting sick becomes 0. What is the maximum price c she is willing to pay in order to take the test? (Take $P \leqslant \bar{P}$.)