14.123 Microeconomics III—Problem Set 2

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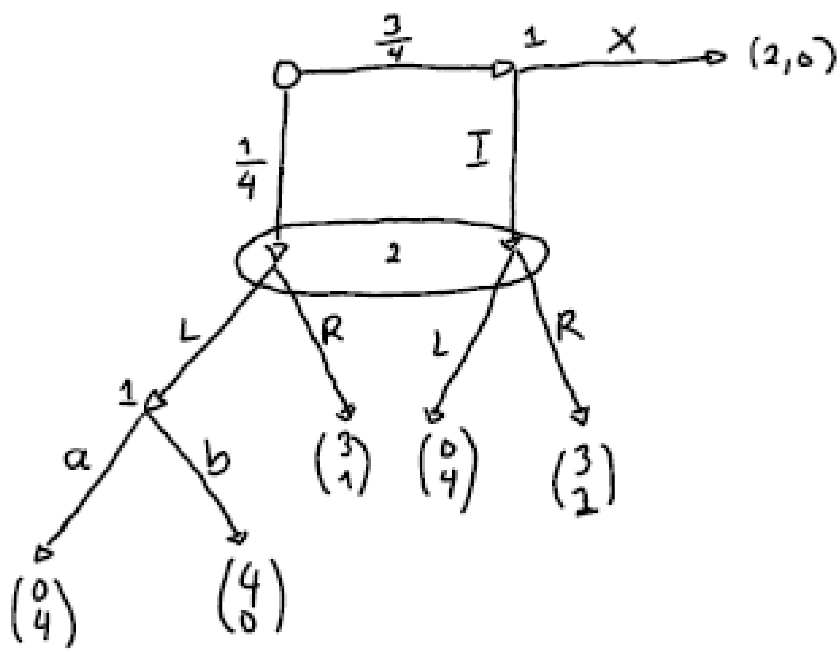
Instructions. Each question is 33 points. Good Luck!

1. Compute a sequential equilibrium of the following game.



2. Consider the following centipede game. There are $2T$ dates $t = 1, 2, \ldots, 2T$. At each odd date $t = 1, 3, \ldots$, player 1 gets to choose between exit, which ends the game, and stay, after which the game proceeds to $t+1$. At each even date $t = 2, 4, \ldots, 2T$, player 2 chooses between exit and stay. At $2T$, the game ends even after stay. Player 1 has two types, namely, rational and irrational, with probabilities $1 - \varepsilon$ and ε, respectively for some $\varepsilon \in (0, 1/2)$, and player 2 has only one type. The irrational type gets -1 if he exits and 0 otherwise. For all the other types, if player i exits at t, player i gets $t+1$ and the other player gets $t-1$. At $t = 2T$, after stay, rational player 1 gets $2T+2$ and player 2 gets $2T$.

   (a) Compute the sequential equilibrium. (You do not need to show that it is unique.)

   (b) For every $T > 2$, find the smallest ε under which the rational type of player 1 stays with probability 1 at $t = 1$. Briefly discuss your finding.

   (c) (Bonus) Prove or disprove the following statement. There exists an $\bar{\varepsilon} > 0$ such that for every $\varepsilon \in (0, \bar{\varepsilon})$, the unique Nash equilibrium outcome is that either (the rational) player 1 exits at $t = 1$ or player 2 exits at $t = 2$ (if player 1 happens to be irrational).

3. Fix a finite extensive-form game G^* and consider a family of extensive-form games G^m in which everything is as in G^* except for the probabilities assigned by the nature at the histories the nature moves. Assume that for any history h at which nature moves and for any available action $a \in A(h)$, the probability $\pi^m(a|h)$ nature assigns to a at h in game G^m converges to the probability $\pi^*(a|h)$ nature assigns to a at h in game G^*. Show that for any sequence of assessments (σ^m, μ^m), if (σ^m, μ^m) is a sequential equilibrium of G^m for each m and $(\sigma^m, \mu^m) \to (\sigma^*, \mu^*)$, then (σ^*, μ^*) is a sequential equilibrium of G^*.