Internal Conflict

14.123 Microeconomic Theory III
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Motivation

- DM is assumed to be a unitary agent, trying to improve his well being –so far…
- But internal conflict may be the rule for homopsychologicus
  - Procrastination
  - Temptation and self-control
  - Self-image
  - Self-deception…
- We may better model a DM a collection of agents…

Main Models of Multi-self agents

- Hyperbolic Discounting; conflicting time preferences of selves
  - Strotz; Phelps & Pollak; Laibson
  - Procrastination
  - Commitment
- Temptation and Control; Gul&Pesendorfer
- Planner & Doer models; Thaler; Fudenberg & Levine
- Models of Self-deception— "Egonomics"; Benabou&Tirole

Time Preferences

- Which do you prefer:

a) \$1 today (Tuesday)

b) \$1.5 on next Thursday

- Which do you prefer:

a) \$1 today on April 1st (Tuesday)

b) \$1.5 on April 3rd (Thursday)

- Standard Exponential Discounting: stationary impatience.
- Hyperbolic Discounting: decreasing impatience

Time preferences, formally

- (x,t) = getting $x at time t
- Utility from (x,t) for the DM at time s:

  $$\delta(t,s)u(x)$$

- Stationary impatience:
  - $\delta(t+1,s)/\delta(t,s)$ is independent of s
  - Exponential discounting
- Decreasing impatience:
  - $\delta(t+1,s)/\delta(t,s)$ is decreasing in s
  - Hyperbolic/Quasi-hyperbolic discounting
- Time invariance: $\delta(t,s) = f(t-s)$
- A condition for decreasing impatience: log(f) is convex

Functional forms

- Exponential Discounting:

  $$f(t) = e^{-rt} = \delta^t$$

- Hyperbolic Discounting:

  $$f(t) = (1 + \alpha t)^{-\beta/\alpha}$$

- Quasi-hyperbolic Discounting:

  $$f(t) = \beta\delta^t \text{ and } f(0) = 1$$

- Consumption sequences:

  $$x = (x_0, x_1, \ldots)$$

- Separable payoffs at time s:

  $$U(x|s) = \sum_{t=s}^{\infty} f(t-s)u(x_t)$$

Optimal consumption under exponential discounting

- DM has
  - w_0 units of initial wealth, perfectly storable,
  - Utility function u(x) = ln(x),
  - Exponential discounting
- DM at s wants to maximize

$$U(x|s) = \sum_{t=s}^{\infty} \delta^{t-s} \ln(x_t) \text{ s.t. } \sum_{t=s}^{\infty} x_t \leq w_s$$

- Solution:

$$x_t = \delta^{t-s} x_s = \delta^{t-s}(1-\delta)\, w_s$$

- Dynamic Consistency: At any time s, DM chooses

$$x_t = \delta^{t}(1-\delta)\, w_0$$

Dynamic Consistency and lack of internal conflict under exponential discounting

- DM in previous slide will retire at time s > 0 with wealth w_s.
- The consumption plan of time 0 self contingent on w_s:

$$x_t = \delta^{t-s} x_s = \delta^{t-s}(1-\delta)\, w_s$$

- The consumption plan of time s self contingent on w_s:

$$x_t = \delta^{t-s} x_s = \delta^{t-s}(1-\delta)\, w_s$$

- Dynamic Consistency: Time 0 self and time s self have the same contingent plan.
- Lack of internal conflict: Time 0 self and time s self have the same preferences on consumption plans (under the same information).

Optimal consumption under quasi-hyperbolic discounting & commitment

- DM at time s has initial wealth w_s and can commit to a consumption plan.
- He wants to maximize

$$U(x|s) = \ln(x_s) + \sum_{t=s+1}^{\infty} \beta\delta^{t-s}\ln(x_t)$$
$$\text{s.t. } \sum_{t=s}^{\infty} x_t \le w_s$$

- Solution:

$$x_s = (1-\delta)\, w_s/(1-\delta+\beta\delta)$$
$$x_t = \beta\delta^{t-s}x_s = \frac{1-\delta}{1-\delta+\beta\delta}\beta\delta^{t-s}\, w_s$$

for t > s.

Dynamic Consistency and internal conflict under quasi-hyperbolic discounting

- DM in previous slide will retire at time s > 0 with wealth w_s.
- The consumption plan of time 0 self contingent on w_s:

$$x_t = \delta^{t-s}x_s = \delta^{t-s}(1-\delta)\, w_s$$

- The consumption plan of time s self contingent on w_s:

$$x_s = (1-\delta)\, w_s/(1-\delta+\beta\delta)$$
$$x_t = \beta\delta^{t-s}x_s = \frac{1-\delta}{1-\delta+\beta\delta}\beta\delta^{t-s}\, w_s \text{ for } t > s.$$

- Dynamic Inconsistency: Time s self want to revise the contingent plan of time 0 self.
- Internal conflict: Time 0 self and time s self have different preferences on consumption plans (under the same information).

Naively-Optimal consumption under quasi-hyperbolic discounting

- At each time s, DM thinks that he can commit to a consumption path moving forward—but the future selves can revise the plan.
- At each time s, DM chooses:

$$x_s = (1-\delta)\, w_s/(1-\delta+\beta\delta)$$
$$x_t = \beta\delta^{t-s} x_s = \frac{1-\delta}{1-\delta+\beta\delta}\beta\delta^{t-s}\, w_s$$

- The consumption path chosen at time 0:

$$x_t = \frac{1-\delta}{1-\delta+\beta\delta}\beta\delta^t\, w_0$$

- Actual consumption path

$$x_t = \left(\frac{\delta\beta}{1-\delta+\beta\delta}\right)^t \frac{1-\delta}{1-\delta+\beta\delta} \quad w_0$$

Sophisticated-Optimal consumption under quasi-hyperbolic discounting

- DM recognizes that the future selves deviate from his plan.
- We have a game in which each self chooses his own consumption, leaving the rest to the next self.
- Sophisticated Solution: a subgame-perfect Nash equilibrium of this game.
- In a stationary SPNE, for some α, the self at each s chooses

$$x_s = \alpha w_s.$$

- The payoff of the self at t is

$$\ln(x_t) + \frac{\beta\delta}{1-\delta}\ln(w_t - x_t) + \mathrm{K}$$

where $K = \beta\delta \sum_{s\geq 0} \delta^s \ln(\alpha(1-\alpha)^s)$.

- Best response: $x_t = \frac{1-\delta}{1-\delta+\beta\delta} w_t$.
- SPNE condition: $\alpha = \frac{1-\delta}{1-\delta+\beta\delta}$.

A "more sophisticated" solution

- Consider the following strategy profile:
  - At time t, consume $x_t = (1-\delta)\, w_t$ if all previous selves followed this plan; otherwise consume $x_t = \frac{1-\delta}{1-\delta+\beta\delta}\, w_t$.
- This is a SPNE ⇔ the former (exponential) plan is better than the latter (quasi-hyperbolic) for all selves.