14.126 GAME THEORY

PROBLEM SET 4

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Question 1

Consider a finitely repeated game with perfect monitoring, played by a long-run player (Player 1) against short run players (Player 2) for T rounds, where the stage game is

	B	P
H	$1,1$	$-1,0$
L	$2,-1$	$0,0$

with probability $1-\varepsilon$, and

	B	P
H	$1,1$	$-1,0$

with probability $\varepsilon \in (0,1)$. (The stage game is the same in all periods.) There is no discounting, and player 1 knows the stage game while the short run players do not. Find a sequential equilibrium of this game. For each $\varepsilon > 0$, find the minimal T under which Player 1 plays H at the beginning for sure in the equilibrium you found. (You do not have to show that the equilibrium is unique.)

Question 2

Consider an infinitely repeated game as in the Fudenberg-Levine setup discussed in the class, with the following stage game. Each player i selects $x_i \in X = \{0, 0.01, 0.02, \ldots, 0.99, 1\}$ and the payoff of player i is

$$u_i(x_1, x_2) = \begin{cases} x_i + (1 - x_1 - x_2)/2 & \text{if } x_1 + x_2 \leq 1; \\ 0 & \text{otherwise.} \end{cases}$$

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Assume that for each $x \in X$, there is a commitment type who plays x at every history. Let the probabilities of these types be fixed, and let the discount factor δ vary. Let $v = \lim_{\delta \to 1} U_1(\sigma[\delta])$ for some sequence $(\sigma[\delta])$ where $\sigma[\delta]$ is a Nash equilibrium under discount factor δ and U_1 is the expected average discounted utility of the rational type of the long-run player. What are the bounds on v given by Fudenberg and Levine?

Question 3

For each case below, show that $(X, \geq)$ is a lattice. Determine the join and meet operators and check whether it is complete.

(1) X is the set of all probability distributions on the real line; $\geq$ is the relation of first-order stochastic dominance.

Hint: You can take X as the set of CDFs $F : \mathbb{R} \to [0,1]$ and write

$$F \geq G \iff [F(x) \leq G(x) \qquad \forall x].$$

If you feel more comfortable, you can confine X to continuous CDFs and/or restrict the domain to $[0,1]$.

(2) X is the set of all partitions of a fixed set A. $\geq$ is the refinement ordering: for any $P, P' \in X$, $P \geq P'$ iff P is finer than P', i.e., for any $S \in P$, $S' \in P'$, if $S \cap S' \neq \varnothing$, then $S \subseteq S'$. (You can take A finite if you feel more comfortable.)

(3) Fix a finite type space (Θ^*, T^*, p), where $T^* = T_1^* \times \cdots \times T_n^*$ and each type $t_i \in T_i^*$ is associated a belief $p_{t_i} \in \Delta(\Theta^* \times T_{-i}^*)$. A *belief-closed subspace* is a pair (Θ, T), with T a nonempty set of the form $T_1 \times \cdots \times T_n$, where $\Theta \subseteq \Theta^*$ and $T_i \subseteq T_i^*$ for each i, and such that $p_{t_i}(\Theta \times T_{-i}) = 1$ for each i and each $t_i \in T_i$. Take X to be the set of all belief-closed subspaces, together with $(\varnothing, \varnothing)$, and the ordering to be set inclusion: $(\Theta, T) \geq (\Theta', T')$ if $\Theta \supseteq \Theta'$ and $T \supseteq T'$.

Question 4

Consider a Cournot oligopoly where players choose quantities q_i, with set of firms N, inverse-demand function P, and with cost functions $C_i(q_i)$ for players $i \in N$.

(1) For the case of duopoly, find conditions on P and C_i that guarantee there are extremal equilibria that bound all rationalizable strategies.

(2) For the case of oligopoly with three or more players and *linear* P and C_i, find the set of all rationalizable strategies and the set of all Nash equilibria in pure strategies. Assume P is decreasing and $C_1 = \cdots = C_n$ is increasing and that each firm's strategy space is restricted so that it cannot produce above the monopoly quantity.

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