14.23 Spring 2003
Problem Set 2
Due in class #10

1. A natural monopolist has total costs $C(Q) = 400 + 25Q$ and faces market demand $Q = 200 - 2P$.

   Solve for monopolist's profits, output, and consumer surplus when:

   (a) Price is set equal to marginal cost.

   (b) Price is set equal to average cost.

   (c) There is *two-part pricing*. Now the monopolist sets price equal to marginal cost and charges a fixed participation fee per consumer. Assume that the demand comes from 10 identical consumers. What participation fee would a profit-maximizing monopolist set?

2. Suppose a company providing cable TV and broadband is a natural monopolist in some city. It has a fixed operation cost of \$1 million per day regardless of the number of households that subscribe to either service. The number of households subscribing to cable TV is $Q_1 = 2 - P_1$ and the number of households subscribing to broadband is $Q_2 = 1 - \frac{1}{4}P_2$. Here Q is measured in millions and P in dollars per day.

   (a) Find the inverse demand curves and the total consumer surplus as a function of the prices.

   (b) Find the Ramsey prices and the associated consumer surplus.

   (c) What are the prices and consumer surplus if a regulator forces the company to charge prices such that the fixed cost is divided equally between the two products and that the company makes zero profits?

3. The demand for electricity is $Q^p = 10 - P^p$ in peak periods and $Q^o = 4 - P^o$ in off-peak periods. Both periods take up half of each day. Variable cost is 0.2 per unit of output per period and capital costs capacity are 0.3 per unit of capacity per day. Capacity costs are sunk and can not be adjusted between periods.

   (a) Find the optimal capacity, peak price, and off-peak price.

   (b) With price constrained to be the same all through the day, what price and capacity would maximize social surplus?

   (c) Compare consumer surplus in the two cases.

4. The cost of producing two types of products is described by the cost function $C(x, y)$. Is there a natural monopoly (why or why not) when

   (a) $C(x, y) = x + y$

   (b) $C(x, y) = \sqrt{xy}$

   (c) $C(x, y) = \sqrt{xy} + 1$