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Problem Set #8

No Due Date

1. In a two player model where the player's valuations are independent uniform on $[0, 1]$, find the equilibrium of a variant of a first price sealed bid auction in which the seller announces a reserve price of R and sells the good to the highest bidder only if the winning bid is at least R. What value of R maximizes the seller's expected revenue from the auction?

(Note: If the calculations are difficult just try to describe what you'd think an equilibrium would have to look like.)

2. Consider a population of high school students who use the internet to exchange photos and "instant messages." Assume that students find it easier to do this if they and their friends all use AOL so that the amount a student is willing to pay to get AOL internet access is increasing in the fraction of people in his or her school who use AOL. In particular, assume that a student of type θ (which might be interpreted as a reflection of his parents' income) is willing to pay $20\theta q$ dollars per month for AOL if the fraction of students in the school using AOL is q. Suppose θ is uniform on $[0, 1]$.

(a) By considering the valuation of the marginal consumer show that if some but not all students in the school purchase AOL then AOL usage and price must satisfy $(1 - q)20q = p$. Is q uniquely determined by p in this model? What is the profit maximizing price for AOL assuming that AOL has no marginal costs and market demand ends up being the q that AOL prefers for each p?

(b) Suppose now that there is a second competing ISP. Assume that its service is identical to AOLs, but that two people can only exchange instant messages and photos if they are using the same ISP. Also, modify the model slightly by assuming that there are a finite number N of students with each students' valuation being drawn from a uniform $[0, 1]$ distribution. Show that regardless of what assumes about how students coordinate the model does not have an equilibrium in which both firms make positive sales.

3. (a) Consider the following simple model of the adoption of technologies with network externalities. Two potential buyers each have unit demands for one of two goods. Consumer i's utility when he buys good $j \in \{A, B\}$ at price p_j is

$$u_{ij} = \begin{cases} a_j + e - p_j & \text{if the other player also buys good } j \\ a_j - p_j & \text{otherwise.} \end{cases}$$

Show that if $|(a_A - a_B) - (p_A - p_B)| < e$ then this model has an equilibrium in which technology A is adopted and an equilibrium in which technology B is adopted.

(b) Consider a somewhat more complicated two stage model (motivated by the idea that technology A is an old standard which both players initially are using, and that players can preannounce and commit to changing to B by purchasing technology B and publicizing the fact that they are doing so) in which in the first stage firms are able to take a costless action which has the effect of

committing them to choose B in the second stage. Suppose also that $a_A = a_B$ and $p_A = p_B$ so that the technologies are in expectation identical but that player i also receives an idiosyncratic payoff θ_i from consuming product B, with θ_i being known only to player i and player $-i$'s belief being that θ_i is uniformly distributed on $[-1, 1]$.

Show that is is a perfect Bayesian equilibrium for player i to commit to and then adopt B if $\theta_i \in [\sqrt{e(1-e)}, 1]$, to not commit but to "jump on the bandwagon" and adopt B only if the other player is observed to commit if $\theta_i \in [-e, \sqrt{e(1-e)})$, and to not commit and to not adopt if $\theta_i \in [-1, -e]$.

In what sense does this model exhibit the underadoption of new technologies with network externalities. Are there other equilibria?

4. Tirole exercise 10.2, p. 392

5. Tirole exercise 10.4, p. 392