14.384: Time Series Analysis.

Final Exam.

due Wednesday, December 19 (1:30PM - 4:30PM)

You may turn it in earlier by sending me an e-mail or stopping by my office. The exam should take you approximately 3 hours. Late works will not be accepted. Good luck!

1. *Posterior for variance of normals.* The Inverse-chi-square distribution $IG(\nu, \sigma^2)$ has density:

$$p(x|\nu, \sigma^2) = \frac{(\sigma^2\nu/2)^{\nu/2}}{\Gamma(\nu/2)} x^{-(1+\nu/2)} \exp\left\{-\frac{\sigma^2\nu}{2x}\right\}, \quad x \geq 0, \nu > 0,$$

with ν being the degrees of freedom and σ^2 being the scale parameter. It has mean $\frac{\sigma^2\nu}{\nu-2}$ for $\nu > 2$ and variance $\frac{2\nu^2\sigma^4}{(\nu-2)^2(\nu-4)}$ for $\nu > 4$.

Consider an iid sample $Y^T = \{y_1, ..., y_T\}$ from $N(\mu, \sigma^2)$.

(i) Assume that the prior has the following convenient parametrization:

$$\mu|\sigma^2 \sim N(\mu_0, \frac{\sigma^2}{k_0}), \quad \text{and} \quad \sigma^2 \sim IG(\nu_0, \sigma_0^2).$$

Note, that μ and σ^2 are dependent. The goal is to find the joint posterior distribution of (μ, σ). More specifically, find the posterior densities $p(\mu|\sigma^2, Y^T)$ and $p(\sigma^2|Y_T)$. Is the described prior conjugate? How would you simulate a draw from the joint posterior for μ and σ^2? (All statistical packages have simulators from known distributions).

(ii) Compare the prior moments $E\sigma^2$ and $Var(\sigma^2)$ with their posterior counterparts for the prior defined in (i) . What happens as $T \to \infty$?

2. *MCMC for probit.* Assume that you have a sample from probit model:

$$z_i = x_i'\beta + u_i, \quad u_i \sim i.i.d.N(0, 1), \quad y_i = I\{z_i > 0\}.$$

You observe $\{x_i, y_i\}_{i=1}^n$. The variables z_i's are not observable. Prior for β is normal with mean β_0 and variance matrix B_0. You want: 1) to get the Bayesian estimate of β minimizing the quadratic loss function; 2) to test $H_0 : \beta > a$ vs $H_0 : \beta < a$ if your losses from a type 1 error are two times bigger than the losses from a type 2 error.

(i) Suggest a Gibbs sampling procedure to get draws from the posterior.

(ii) Describe the simulation procedure and calculation of results in detail. Imagine you have an RA who knows nothing about econometrics, but knows mathematics (with matrix algebra) and programs well. Write step-by-step instructions for him.

(iii) Describe how you would check the convergence of your procedure.

(iv) Now assume that instead of a normal prior, you decide to use some other continuous prior $p(\beta)$ with bounded pdf (which you can calculate). Could you suggest a modification of the procedure described above? (*Hint:* think about M-H inside Gibbs sampling.)

14.384 Time Series Analysis
Fall 2013