

# 14.451: Introduction to Economic Growth

## Problem Set 5

Due date: Never.

**Question 1:** Here we examine the policy implications of the Romer model (expanding product varieties), examined in section 7.1 of the class notes.

1. Show that the government can ensure a first-best equilibrium if it uses a lump-sum tax to finance the appropriate subsidy of the intermediate goods. What rate of subsidy is required? In a richer model, why would it be difficult to carry out the required form of policy?
2. Can the government ensure a first-best solution if it relies solely on a subsidy to R&D (financed again by a lump-sum tax)? Explain your answer. What modifications to the model would make it important for the government to subsidize research?

**Question 2:** Consider the following endogenous growth model due to Uzawa and Lucas. The economy admits a representative consumer and preferences are given by

$$\int_0^{\infty} \exp(-\rho t) \frac{C^{1-\theta} - 1}{1-\theta} dt, \quad (1)$$

where  $C$  is consumption defined over the final good of the economy. The final good is produced as

$$Y(t) = AK(t)^\alpha H_P^{1-\alpha}(t)$$

where  $K(t)$  is capital and  $H(t)$  is human capital, and  $H_P(t)$  denotes human capital used in production. The accumulation equations are as follows:

$$\dot{K}(t) = I(t) - \delta K(t)$$

for capital and

$$\dot{H}(t) = BH_E(t) - \delta H(t)$$

where  $H_E(t)$  is human capital devoted to education (further human capital accumulation), and the depreciation of human capital is assumed to be at the same rate as physical capital for simplicity ( $\delta$ ). The resource constraints of the economy are

$$I(t) + C(t) \leq Y(t)$$

and

$$H_E(t) + H_P(t) \leq H(t).$$

1. Interpret the second resource constraint.
2. Denote the fraction of human capital allocated to production by  $\phi(t)$ , and calculate the growth rate of final output as a function of  $\phi(t)$  and the growth rates of accumulable factors.
3. Assume that  $\phi(t)$  is constant, and characterize the balanced growth path of the economy (with constant interest rate and constant rate of growth for capital and output). Show that in this balanced growth path, we have  $r^* \equiv B - \delta$  and the growth rate of consumption, capital, human capital and output are given by  $g^* \equiv (B - \delta - \rho) / \theta$ . Show also that there exists a unique value of  $k^* \equiv K/H$  consistent with balanced growth path.
4. Determine the parameter restrictions to make sure that the transversality condition is satisfied.
5. (More difficult): Now analyze the transitional dynamics of the economy starting with  $K/H$  different from  $k^*$  (Hint: to do this, look at dynamics in three variables,  $k \equiv K/H$ ,  $\chi \equiv C/K$  and  $\phi$ ). Consider both the cases  $\alpha < \theta$  and  $\alpha \geq \theta$ .