

14.451 Midterm

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You have 1.5 hours. No books or notes are allowed. Please answer each question in a separate booklet.

Good Luck!

Question 1 (40 points)

Consider the following endogenous growth model. There is a continuum of entrepreneurs. The preferences of an entrepreneur are given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c) = \frac{c^{1-1/\theta}}{1-1/\theta}.$$

His budget constraint is given by

$$c_t + h_{t+1} + a_{t+1} = \pi_t + R_t a_t$$

where c denotes his consumption, h denotes his human capital, a denotes his assets, and π_t denotes the profit he makes. The latter is given by

$$\pi_t = F(k_t, h_t, H_t) - R_t k_t$$

where k denotes the capital he employs in a competitive capital market, h denotes his own human capital, H denotes the aggregate human capital, and F is a production function, given by

$$F(k, h, H) = k^\alpha h^\gamma H^\eta$$

where $\alpha, \gamma, \eta \in (0, 1)$. Note that physical capital is tradeable, whereas human capital is not.

(a) Characterize the solution to the problem of the entrepreneur for a given sequence of interest rates R_t and aggregate human capital H_t . In particular, give the three optimality conditions that characterize the optimal choice of consumption/saving (a), human capital investment (h), and capital input (k). Also, how does an increase in R affect the optimal k and h ?

(b) Now consider the general equilibrium. Note that, in equilibrium, $a_t = k_t$ and $H_t = h_t$. What is the restriction on the parameters α , γ and η that is necessary and sufficient for the economy to admit a balanced growth path (i.e., a path along which c , k , and h all grow at a constant, common, non-zero rate)? Explain this restriction. Also, when this restriction is satisfied, what is the equilibrium k/h ratio and what is the equilibrium growth rate (as functions of exogenous parameters)?

(c) What is the restriction on the parameters α , γ and η that is necessary and sufficient for the equilibrium to be efficient? What is the restriction that is necessary and sufficient for both efficiency and balanced growth? If the equilibrium is inefficient, what is the Pigou-like policy that restores efficiency?

Question 2 (30 points)

(a) Consider the following discrete time optimal growth model with full depreciation:

$$V(k_0) = \max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$k_{t+1} = f(k_t) - c_t,$$

where $u(c) = \log c$, $f(k) = k^\alpha$, and $\alpha \in (0, 1)$ (and where $k_0 > 0$ is historically given). It is easy to show that the policy rule for this problem is

$$G(k) = \alpha\beta f(k).$$

Use this to result to prove that there exists a unique (positive) steady state such that, for any initial $k_0 > 0$ the economy converges monotonically to the steady state.

(b) Now consider a variant of the above problem. There is a measure-one continuum of agents, distributed uniformly over the $[0, 1]$ interval. The preferences of individual i are given by

$$\mathcal{U}_i = \sum_{t=0}^{\infty} \beta^t u(c_{it}, C_t)$$

where c_{it} is i 's consumption and $C_t = \int_0^1 c_{it} di$ is aggregate consumption, and where

$$u(c, C) = \log c + \eta \log C,$$

for some $\eta \neq 0$. Note that we have introduced an externality from aggregate consumption. The planner maximizes average utility, $\mathcal{W} = \int_0^1 \mathcal{U}_i di$, subject to the resource constraint,

$$k_{t+1} = f(k_t) - \int_0^1 c_{it} di.$$

How does the optimal distribution of consumption across agents look like? How does the optimal saving in this economy differ (or not) from the one in part (a)?

(c) Consider the competitive equilibrium allocation of the economy in part (b). How does η , which measures the externality from consumption, affect the relation between the equilibrium and planner's allocation?

Question 3 (30 points)

True, false, or uncertain? Provide a brief explanation for your answer.

(a) The neoclassical growth model (Solow/Ramsey) can well explain the observed income and productivity differences across countries.

(b) An expected increase in TFP, and hence in the marginal product of capital, is likely to increase saving and growth in the economy.

(c) If technological progress is capital-augmenting, then it increases the return to capital relative to that of labor.