14.452 Recitation #3

Ludwig Straub

MIT

November 2016

Logistics

- Problem Set 3 due tomorrow
- Piazza

Plan for today

1. Pecuniary externalities
2. Dynamic inefficiency
   - Simple model
   - Canonical OLG model
3. Why does the FWT fail?
   - Production efficiency?
   - Pecuniary externalities?
4. Market incompleteness (very preliminary)
5. Bubbles

Section 1

Pecuniary externalities

Definition of pecuniary externality

- An externality that acts on others via **prices**
- Example:
  - I decide to buy more coffee from Starbucks.
  - raises the price of coffee
  - $+$ externality on Starbucks
  - $-$ externality on all buyers (including myself)

Netting out

- Is my action to buy more coffee welfare improving? or not?
- Depends on whether externalities are positive or negative "on average"
  - include "externalities" on *my own utility*
- Determine average using **compensating transfers**
  - compensate all agents using transfers dT^h
  - $\sum_h dT^h > 0 \Rightarrow$ negative externalities
  - $\sum_h dT^h < 0 \Rightarrow$ positive externalities
  - $\sum_h dT^h = 0 \Rightarrow$ externalities "net out"

Formal setup

- $\mathcal{H}$ set of households, maximizing $U^h(x^h)$; U^h is locally non-satiated
- For simplicity: finitely many goods + endowments ω^h
- Fix equilibrium $\{p, x^h\}$
- **Experiment:** Agent h_0 changes net demand to $x^{h_0}(p) + dx^{h_0}(p)$
  - equilibrium price $p \rightsquigarrow p + dp$

Formal netting out

- **Pecuniary externalities:** (using Envelope Theorem)

$$dU^h = \lambda^h \left(-x^h \cdot dp \right)$$

- With transfers:

$$dU^h = \lambda^h \left(-x^h \cdot dp + dT^h \right)$$

- Hence require $dT^h = x^h \cdot dp$ for compensation.
- **Netting out:**

$$\sum_h dT^h = \sum_h x^h \cdot dp = 0$$

by market clearing + finiteness of goods.

Remarks

- The effect of dx^{h_0} is second order in dU^{h_0}, hence does not show up
- Result relies on (perfectly) **competitive equilibrium:**
  - maximizing households to apply Envelope Theorem
  - single budget constraint to solve for dT^h (complete markets)
  - prices not in additional constraints (e.g. borrowing constraints)
  - finite amount of goods

Section 2

Dynamic inefficiency

Gearing up ...

- This recitation focuses on two models from class:
  - **simple model** from beginning of lecture note 7
  - **canonical OLG model**
- Revisit briefly before getting into the weeds

Subsection 1

Simple model

Simple model

- $\mathbb{N}$ consumption goods $\mathbf{c} = (c_t)$
- $\mathbb{N}$ agents
- Agent $t \in \mathbb{N}$ endowed with 1 unit of t-th good
- Prices:
  - $\mathbf{p} = (p_t)_t$ for consumption goods; normalize $p_0 = 1$
- Preferences:

$$\max U^t(\mathbf{c}^t) = c_t^t + c_{t+1}^t$$

$$p_t c_t^t + p_{t+1} c_{t+1}^t \leq p_t$$

Simple model: Results

- Unique competitive equilibrium: $p_t = 1 \; \forall t$
- **Inefficient:** A transfer of 1 unit of good $t+1$ to agent t...
  - ...raises agent 0's utility...
  - without changing anyone else's!
- FWT does not apply: $\sum_{t=0}^{\infty} p_t \cdot 1 = \infty$
- **"Dynamic inefficiency"**
  - Any kind of pecuniary externality here?

Subsection 2

Canonical OLG model

The canonical OLG model

- Here: Example with $n = 0$ (no population growth)
- Agents $t = 0, 1, 2, \ldots$
- Agent t endowed with 1 unit of labor at time t, solves

$$\max U^t(c_t^t, c_{t+1}^t) = \log c_t^t + \beta \log c_{t+1}^t$$

$$c_t^t + k_{t+1} \leq w_t$$

$$c_{t+1}^t \leq R_{t+1} k_{t+1}$$

- Output $y_t = k_t^\alpha$, wages $w_t = (1-\alpha) k_t^\alpha$, interest rates $R_{t+1} = \alpha k_{t+1}^{\alpha - 1}$

Solution of canonical OLG

- Log preferences $\Rightarrow$

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)k_t^{\alpha}$$

- Unique, globally stable steady state

$$k^* = \left[\frac{\beta(1-\alpha)}{1+\beta}\right]^{1-\alpha} \qquad R^* = \frac{1+\beta}{\beta}\frac{\alpha}{1-\alpha}$$

- **BUT:** $R^* < 1$ if β suff large relative to α !
- **Dynamic inefficiency:** Permanent reduction $k^* \rightsquigarrow k^* - \Delta k$ raises output each period!
- **FWT** !?

The big puzzle

- Note: FWT proof fails because value of aggregate wealth $= \infty$ since $\sum_{t=0}^{\infty} R^{*-t} \to \infty$
  - but finite aggregate wealth is not a *necessary condition* for FWT...
- **But why intuitively does it fail?**

1. Production inefficiency?
2. Pecuniary externalities?
3. Incomplete markets?

Section 3

Why does the FWT fail?

Subsection 1

Production efficiency?

Check FWT

- To see whether the FWT applies, and if not, why not, we map the OLG model into our canonical GE economy...
- Arrow-Debreu world:
  - agents consume + rent labor endowments & capital at $t = 0$
  - firms produce output & **optimize allocation of capital for** $t > 0$
- Here: Combine all goods into a single huge representative firm
  - alternative: firms for each time period t that supply each other with capital
- **Production side = same as in NGM!**

Mapping into canonical GE economy

- $\mathbb{N}$ consumption goods $\mathbf{c} = (c_t)$, $\mathbb{N}$ labor goods $\mathbf{L} = (L_t)$, initial capital good
- $\mathbb{N} \cup \{-1\}$ agents.
- Agent -1 endowed with initial capital $k_0 > 0$.
- Agent $t \in \mathbb{N}$ endowed with 1 unit of t-th labor good.
- Preferences:

$$U^t(\mathbf{c}^t) = \log c_t^t + \beta \log c_{t+1}^t$$

- Prices:
  - $\mathbf{p} = (p_t)_t$ for consumption goods; normalize $p_0 = 1$
  - $p_t w_t$ for labor good t
  - R_0 for initial capital good

Technology

- Firms solve

$$\max \sum_{t=0}^{\infty} p_t \left(y_t - w_t L_t\right) - R_0 k_0$$

subject to

$$y_t = k_t^{\alpha} - k_{t+1}$$

- Euler: $p_{t-1} = R_t p_t$ and so

$$\sum_{t=0}^{\infty} p_t \left(y_t - w_t L_t\right) = \sum_{t=0}^{\infty} p_t \left(R_t k_t - k_{t+1}\right) =$$

$$R_0 k_0 - k_1 + \sum_{t=1}^{\infty} \left(p_{t-1} k_t - p_t k_{t+1}\right) = R_0 k_0$$

where $-k_1 + \sum_{t=1}^{\infty} \left(p_{t-1} k_t - p_t k_{t+1}\right)$ is a telescopic sum canceling to zero

- Hence **zero profits.... or not?**

Technology with dynamic inefficiency

- Assume we're in a dynamically inefficient steady state, $R^* < 1$
- Hence $p_t < p_{t+1}$
- Is the firms objective $\sum_{t=0}^{\infty} p_t \left(y_t - w_t L_t\right) - R_0 k_0$ still meaningful?
  - Maximized by $k_{t+1} = k^*$ for all t?

Technology with dynamic inefficiency (2)

- Compare to "golden rule"

$$k^{gold} = \arg\max \; k^{\alpha} - k$$

- If $k_0 = k^*$ but $k_{t+1} = k^{gold}$ thereafter, y_t **strictly rises in every single period.**
- Achieves **positive profits.** Profit maximization??
- $\rightarrow$ **This does not satisfy our definition of competitive equilibrium!**
  - competitive equilibrium still well-defined when using separate firms for each period
  - difference to "single representative firm" points to **production inefficiency**

Subsection 2

Pecuniary externalities?

Pecuniary externalities?

- Agent 0's savings: Causes pecuniary externality that does not net out?
- Consider change in savings dk_1. Affects future paths of prices and wages.
- Can show:

$$dU^t = \lambda^{(t)} \left\{ w^* \alpha^t \frac{dk_1}{k^*} \left[1 - \frac{\alpha}{R^*} \right] + R^{*t} dT_t \right\}$$

$$dU^0 = \lambda^{(0)} \left\{ (\alpha - 1) dk_1 + dT_0 \right\}$$

Pecuniary externalities

- Do they net out? For $t \geq 1$

$$dT_t = -w^* \left(\frac{\alpha}{R^*}\right)^t \frac{dk_1}{k^*} \left[1 - \frac{\alpha}{R^*}\right]$$

- Note: $\alpha / R^* < 1$ always. So:

$$\sum_{t=0}^{\infty} dT_t = dT_0 - \underbrace{\frac{w^*}{k^*} \frac{\alpha}{R^*}}_{1-\alpha} dk_1 = 0$$

Yes, they net out. So it's not the reason for dynamic inefficiency!?

  - we only considered change in savings by single generation
  - many generations: run into "order of summation" issues...
- What else could it be?

Section 4

Market incompleteness (very preliminary)

Naive market incompleteness

- Agents are not "alive" until their born – thus markets are incomplete?
    - limited market participation?
- **No**. Previous part shows: Agents fit canonical GE framework perfectly fine!
    - just have preferences over 2 specific goods
- Is this the end of market incompleteness as an explanation of dynamic inefficiency?

What if ...

- you can trade certain bundles of goods, in addition or instead of the other goods?
  - e.g. "new" goods $\mathbf{x}$ that are a linear combination of existing goods $\mathbf{c}$
- Usually, this is irrelevant, as long as the two representations have the same dimension
- **Here:** This might actually matter!
  - new kind of "market incompleteness"

Introducing composite goods

- Introduce new **composite goods**: $\mathbf{x} = (x_t)_t$
  - think of x_t as combination of -1 cons good at time t and 1 at time $t+1$
  - call $\mathbf{e}_t^x$ indicator for a single unit of composite good t
  - call $\mathbf{e}_t^c$ indicator for a single unit of consumption good t
- Assume each agent operates production technology

$$\mathbf{e}_t^x \leftrightarrow \mathbf{e}_{t+1}^c - \mathbf{e}_t^c$$

  and trades in composite goods.
- Normally, wouldn't expect this to do anything
  - after all, markets are already complete?

Revisiting the simple model

- Preferences are

$$U^t = c_t + c_{t+1} = (1 - x_t) + x_t$$

subject to feasibility

$$1 - x_t \geq 0$$

$$x_t \geq 0$$

- Dynamically inefficient equilibrium from before:
  - $x_t = 0$ for all t
  - composite goods: price 0

Revisiting dynamic inefficiency

- **Here**: This is not an equilibrium! Agent 0 could sell a bundle of composite goods $\sum_{t\geq 1} e_t^x$
- ... and convert them into a single unit of good 1

$$-\sum_{t\geq 1} e_t^x = -\sum_{t\geq 1} \left(\mathbf{e}_{t+1}^c - \mathbf{e}_t^c\right) = \mathbf{e}_1^c$$

- This lets agent 1 increases his consumption!
- Exactly what the planner did.
- Remarks:
  - **Caveat: Needs to be done a lot more carefully**
  - Conjecture: Goes through even for canonical OLG model

Section 5

Bubbles

A bubbly asset

- In both examples: If initial agent could consume more, get efficiency
- Suppose there is an asset in unit supply with value V owned by agent 0
- Asset has no cash flows (fundamental value of zero)
- **Claim:** There is an equilibrium where each agent t receives the bubbly asset from agent $t-1$ and pays agent $t+1$ with it

Bubbly equilibria in simple model

- Budget constraints for $t \geq 1$

$$p_t c_t^t + p_{t+1} c_{t+1}^t + \underbrace{V}_{\text{buy bubble}} \leq p_t + \underbrace{V}_{\text{sell bubble}}$$

budget constraint for $t = 0$

$$p_0 c_0^0 + p_1 c_1^0 \leq p_0 + \underbrace{V}_{\text{sell bubble}}$$

- Hence: For any $V \in [0, 1]$ there is an equilibrium where
  - $t \geq 1 : c_t^t = 1 - V, \; c_{t+1}^t = V$
  - $t = 0$: $c_0^0 = 1, \; c_0^1 = V$
- **Efficient if $V = 1$!**

Canonical OLG model

- Works similar in canonical OLG model: Tirole (1985)

Happy Thanksgiving!