14.452 Macroeconomic Theory II
Instructor: **Olivier Blanchard**

Problem Set 4

Due Date: **May 11th**
We encourage you to work together, as long as you write your own solutions.
No late solutions will be accepted this time....

Money as a Factor of Production (Based on Dornbusch and Frenkel, 1973)

The shortcut used by Dornbusch and Frenkel to introduce money in the economy is that they assume that output available is equal to a fraction of production ($G(K, 1$), where the fraction is an increasing function of real money balances:

$$\left(1 - L\left(\frac{M}{P}\right)\right) G(K, 1$$

$L($. satisfies these properties: $L(\infty \ = 0, L(0 \ = 1, L'() \ < 0, L''() \ > 0$.
The households maximize:

$$Max \sum_{i=0}^{i=\infty} \beta^i U(C_{t+i}$$

$s.t.$

$$P_{t+i}C_{t+i} + M_{t+i+1} + P_{t+i}K_{t+1+i} = P_{t+i}\left(1 - L\left(\frac{M_{t+i}}{P_{t+i}}\right)\right)G(K_{t+i}, 1$$
$$+ X_{t+i} + M_{t+i} + P_{t+i}(1 - \delta \ K_{t+i}$$

where C, M, K, and X are consumption, money balances, capital holdings, and government transfers. There is no uncertainty.

1. Show that the budget constraint can be written in real terms as:
$$C_{t+i} + (1 + \pi_{t+1+i} \ \frac{M_{t+i+1}}{P_{t+i+1}} + K_{t+1} = \left(1 - L\left(\frac{M_{t+i}}{P_{t+i}}\right)\right)G(K_{t+i}, 1$$
$$+ \frac{X_{t+i}}{P_{t+i}} + \frac{M_{t+i}}{P_{t+i}} + (1 - \delta \ K_{t+i},$$
where $1 + \pi_{t+1+i} = \frac{P_{t+i+1}}{P_{t+i}}$.
2. Derive the FOCs of this problem. Characterize the solution to the problem using an intertemporal and an intratemporal condition.
3. Characterize the steady state. Is money neutral? Superneutral?
4. What are the basic differences of this approach with respect to including money in the utility function or a cash in advance constraint?

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Money in the Utility Function (Final, 2005)

Assume that consumer's utility in period t depends on consumption and leisure:

$$U(C_t \quad + V(L - aN_t$$

where L is the total time available for leisure and for trips to the banks,N_t is the number of trips to the bank, and a is time spent on each trip, so $L - aN_t$ is leisure.

If consumers decide to spend P_tC_t on consumption in period t, they do this at a constant rate within the period. They need money to buy goods. They get this money by going to the bank to exchange bonds for money. They can take N_t such trips. If $N_t = 1$, they go to the bank once, at the start of the period, take $M_t = P_tC_t$ out, and spend it over the period. Their average money balances for the period are therefore equal to $\bar{M}_t = \frac{M_t}{2} = \frac{P_tC_t}{2}$.

1. Derive average money balances as a function of spending, P_tC_t and the number of trips taken, N_t.
2. Replace N_t by its expression in terms of average real money balances and consumption in the utility function.
3. Discuss: "Putting money in the utility function is just a short cut for capturing the idea that having larger real money balances saves on trips to the bank."
4. In light of this exercise, does it make sense to assume that utility is separable in consumption and in real money balances?
5. Would money be neutral/super-neutral in this setup?

Seigniorage

This question asks you to extend the discussion on "Money growth, inflation, and seigniorage" in lecture to include rational expectations. Consider the basic setup presented by Olivier in class. Money demand is given by:

$$\frac{M}{P} = \exp(-\alpha\pi^e \quad .$$

We are going to use the equation in discrete time, where it takes the form (in logs):

$$m - p = -\alpha(E_tp_{t+1} - p_t \quad .$$

1. Show that with rational expectations, the current price level is a weighted average of expected next period prices and current money supply.
2. Replace forward to show that p_t is a function of expected future money supply.
3. Assume that $m_t - m_{t-1} = \theta_0$. Find the evolution of the price level.

A strange model of money (Waiver, 2005)

Take an economy with a continuum of individuals, indexed by i, and maximizing:

$$\sum_{o}^{\infty}(1 + \theta \ ^{-t}U(C_t^i$$

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subject to:

$$P_t C_t^i + M_t^i = P_t Y_t^i + M_{t-1}^i + T_{t-1}^i.$$

Individuals take Y_t^i as given. For the economy as a whole, output is exogenous, constant, perishable, and equal to $\overline{Y}$. The total nominal stock is constant and equal to $\overline{M}$.

1. Derive and interpret the first order conditions for individual i.
2. Using equilibrium conditions, derive the rate of inflation in the economy. Explain in words.
3. What is the rate of money growth? Why is the rate of inflation different from the rate of money growth?
4. Would a cash in advance give rise to different results? How would you introduce it? And what would the implications on the rate of inflation?

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