14.452. Topic 3. RBCs

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1. Motivation, and organization

- Looked at Ramsey model, with productivity shocks. Replicated fairly well co-movements in output, consumption, and investment.
- Next step, before we can really assess it, is to allow for variations in employment: a labor/leisure choice.
- Class of models known as the RBC model. Initially due to Prescott.
- As we shall see, can do well at explaining co-movements in output, consumption, investment, employment. But three major issues:

- Labor supply elasticity? Major issue. A problem common to all the models we shall see.
- Productivity shocks? Implausible that they would be large at high frequency. Major issue.
- Other shocks seem to matter. In particular, monetary policy. Major issue.

Organization

- Central planning problem.
- FOCs. Derivation and interpretation
- Balanced growth conditions
- Back to FOCs
- The effects of productivity shocks

- Labor supply elasticity?
- Evidence on technological progress. Use and misuse of Solow residuals. Dynamic effects of technological shocks.

2. The optimization problem

Again look at a planning problem:

$$\max E[\sum_{0}^{\infty} \beta^i \, U(C_{t+i}, L_{t+i})|\Omega_t]$$

subject to:

$$N_{t+i} + L_{t+i} = 1$$

$$C_{t+i} + S_{t+i} = Z_{t+i} F(K_{t+i}, N_{t+i})$$

$$K_{t+i+1} = (1 - \delta) K_{t+i} + S_{t+i}$$

L is leisure and N is work. By normalization, total time is equal to one. Z_t are productivity shocks. Utility is a function of both consumption and leisure.

Ignore growth. If growth, then the production function would have Harrod-neutral technological progress , so $Z_t F(K_t, A_t N_t)$, with $A_t = A^t, A > 1$ for example. Work with efficiency units.

2. The first order conditions

Use Lagrange multipliers. Put the three constraints together to get:

$$K_{t+i+1} = (1-\delta)K_{t+i} + Z_{t+i}F(K_{t+i}, 1 - L_{t+i}) - C_{t+i}$$

Associate $\beta^i \lambda_{t+i}$ with the constraint at time t:

$$E[U(C_t, L_t) - \lambda_t(K_{t+1} - (1-\delta)K_t - Z_t F(K_t, 1 - L_t) + C_t) + \beta U(C_{t+1}, L_{t+1}) - \beta\lambda_{t+1}(K_{t+2} - (1-\delta)K_{t+1} - Z_{t+1}F(K_{t+1}, 1 - L_{t+1}) + C_{t+1}) + ... \mid \Omega_t]$$

The first order conditions are therefore given by:

$$C_t \ : \ U_C(C_t, L_t) = \lambda_t$$

$$L_t \ : \ U_L(C_t, L_t) = \lambda_t \ Z_t \ F_N(K_t, 1 - L_t)$$

$$K_{t+1} \ : \ \lambda_t = E[\beta\lambda_{t+1}(1 - \delta + Z_{t+1}F_K(K_{t+1}, 1 - L_{t+1}) \mid \Omega_t]$$

Define, as before, $R_{t+1} \equiv 1 - \delta + Z_{t+1}F_K(K_{t+1}, 1 - L_{t+1})$ and define $W_t = Z_tF_N(K_t, 1 - L_t)$, so:

$$U_C(C_t, L_t) = \lambda_t$$

$$U_L(C_t, L_t) = \lambda_t W_t$$

$$\lambda_t = E[\beta \lambda_{t+1} R_{t+1} \mid \Omega_t]$$

Interpretation (Optimization problem, consumers in the decentralized economy, taking the wage and the interest rate as given). Combining the first two:

An **intratemporal condition**:

$$U_L(C_t, L_t) = W_t U_C(C_t, L_t)$$

And an **intertemporal condition**:

$$U_C(C_t, L_t) = E[\beta R_{t+1} U_C(C_{t+1}, L_{t+1}) \mid \Omega_t]$$

Variational argument

$$U_L(C_t, L_t) = W_t U_C(C_t, L_t)$$

- Increase work by Δ, so decrease in utility of $U_L(C_t, L_t)\ \Delta$
- Increase consumption by $W_t\Delta$, so increase in utility of $W_t U_C(C_t, L_t)\ \Delta$

$$U_C(C_t, L_t) = E[\beta R_{t+1} U_C(C_{t+1}, L_{t+1}) \mid \Omega_t]$$

- Decrease consumption by Δ, so decrease in utility of $U_C(C_t, L_t)\Delta$
- Save, and get R_{t+1} next period, so an increase in expected utility of $E[\beta U_C(C_{t+1}, L_{t+1})\ R_{t+1}|\Omega_t]$.

Still: not easy to draw implications for general $U(C, L)$.

3. Balanced growth path restrictions on utility?

Caveats.

A clear trend in hours

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Could be due to increasing taxes. (Prescott, for Europe). But going back to early 20th century (pre income/payroll tax).

Hours	1909	1919	1929	(1940)
≤ 48	7.9	48.6	46.0	(92.1)
49-59	52.9	39.3	46.5	(4.9)
≥ 60	39.2	12.1	7.5	(3.0)

(Employed males, manufacturing. (from Pencavel, Handbook of Labor economics, Chapter 1, Table 1-7))

Is it reasonable to use balanced growth restrictions?

- Additive separability in time implies strong short run implications. Will be clear throughout course.

  Put another way. Many specifications of preferences with same long run implications, different short run implications

$$U(C_1 + C_2, L_1 + L_2) + \beta^2 U(C_3 + C_4, L_3 + L_4) + ...$$

- Home production versus leisure.

Derivation

Production side: Know we need Harrod neutral technological progress, say A^t, $A > 1$. (Remember we suppressed A_t just for notational convenience.

Focus on **utility side.**

- If balanced growth. In steady state, leisure, L, is constant. Consumption and the wage increase at rate A, so, from the intratemporal condition:

$$\frac{U_L(CA^t, L)}{U_C(CA^t, L)} = WA^t$$

where C, L and W are constant over time, and A increases. This is true for any A^t, so in particular, for $t = 0$ so $A^t = 1$, so

$$\frac{U_L(C, L)}{U_C(C, L)} = W$$

Using the two relations to eliminate the wage, we can write:

$$\frac{U_L(CA^t, L)}{U_C(CA^t, L)} = A^t \frac{U_L(C, L)}{U_C(C, L)}$$

The MRS between consumption and leisure must increase at rate A.

- This relation holds for any value of the term A^t So use for example $A^t = 1/C$:

$$\frac{U_L(1, L)}{U_C(1, L)} = \frac{1}{C} \frac{U_L(C, L)}{U_C(C, L)}$$

  Or, rearranging:

$$\frac{U_L(C, L)}{U_C(C, L)} = C[\frac{U_L(1, L)}{U_C(1, L)}]$$

  The MRS must be equal to C times the term in brackets, which is a function only of L. For this to hold, the utility function must be of the form:

$$u(C\tilde{v}(L))$$

- Now turn to the intertemporal condition. Write it as:

$$U_C(CA^t, L) = (\beta R)U_C(CA^{t+1}, L)$$

Or, given the restrictions above:

$$\frac{u'(CA^t\tilde{v}(L))}{u'(CA^{t+1}\tilde{v}(L))} = \beta R$$

For the LHS to be constant, $u(.)$ must be of the constant elasticity form:

$$u(C\tilde{v}(L)) = \frac{\sigma}{\sigma - 1}(C\tilde{v}(L))^{(\sigma-1)/\sigma}$$

If $\sigma = 1$, then:

$$U(C, L) = \log(C) + v(L)$$

where $v(L) \equiv \log(\tilde{v}(L))$

If assume separability of leisure and consumption (really no good reason to do that), then the form above is the only one consistent with the existence of a steady state.

Two special (and often used cases):

- Prescott's preferred specification. Unit elasticity of leisure to the wage, given MU of wealth.

$$\log(C) + \phi \log(L)$$

- Preferred New-Keynesian specification. Constant elasticity of labor supply to the wage, given MU of wealth. (more in line with micro empirical work).

$$\log(C) - \frac{\psi}{1+\phi} N^{1+\phi}$$

4. Back to the FOCs

Use the specification $U(C, L) = \log(C) + v(L)$ and return to the first order conditions:

The **intratemporal condition** becomes:

$$v'(L_t) = W_t / C_t$$

The **intertemporal condition** becomes:

$$E[\beta R_{t+1} \frac{C_t}{C_{t+1}} \mid \Omega_t] = 1$$

Interpretation. (Note that $U_C = 1/C$ is the marginal value of wealth). So equalize marginal utility of leisure to the wage times the marginal value of wealth. And the Ramsey-Keynes condition for consumption.

Effects of a favorable technological shock? First pass. It increases W and R, both current and prospective.

- Two effects on **consumption**. Smoothing (consumption up) and tilting (consumption down). On net, plausibly up.

- Turn to **leisure/work**. Two effects.

  A substitution effect: Higher W_t leads people to work harder.

  An income/wealth effect. Higher C_t works the other way. As people feel richer ($1/C$ is the marginal value of wealth), they want to consume more and enjoy more leisure.

  Net effect depends on the strength of the two effects. Substitution (elasticity), and wealth (persistence).

  In NK specification, using logs, $n = (1/\phi)(w - c)$.

- The more transitory the shock, the smaller the increase in C, and so the stronger the substitution effect.

- If the shock is permanent, the stronger the wealth effect. C_t could increase by more than W_t (but less than $Z_t F(K_t, N_t)$). (Permanent shock to technology, plus capital accumulation). Employment could decrease.

Another way of looking at the employment effects:

An intertemporal condition for leisure (this is the way Lucas and Rapping looked at it):

Replace consumption by its expression from the intratemporal condition. And, just for convenience, use $v(L) = \phi \log(L)$, so $v'(L) = \phi/L$. Then:

$$\phi C_t = W_t L_t$$

So, replacing in the intertemporal condition:

$$E[\beta(R_{t+1}\frac{W_t}{W_{t+1}})\frac{L_t}{L_{t+1}} \mid \Omega_t] = 1$$

What is relevant for the leisure decision is the rate of return "in wage units".

- A transitory shock, so W_t increases but W_{t+1} does not change much. Then L_t/L_{t+1} will decrease sharply. Strong increase in employment.

- A permanent shock: Then W_t/W_{t+1} is roughly constant, and so is L_t/L_{t+1}. (ignoring movements in R). No movement in employment.

5. Solving the model

Special cases The same as before. Assume Cobb Douglas production, assume log–log utility. Assume full depreciation.

$$K_{t+1} = Z_t K_t^{\alpha} (1 - L_t)^{1-\alpha} - C_t$$

and

$$U(C_t, L_t) = \log C_t + \phi \log L_t$$

Then, can solve explicitly. And the solution actually is identical to that of the benchmark model. N is always constant, not by assumption, but by implication now. Substitution and income effects cancel.

$$C_t = (1 - \alpha\beta) Y_t, \qquad I_t = \alpha\beta Y_t$$

$$N \mid \frac{\phi}{1 - N} = \frac{1 - \alpha}{1 - \alpha\beta} \frac{1}{N}$$

So, nice, but not useful if we want to think about fluctuations in employment...

So need to go to **numerical simulations**. SDP, or log linearization. Campbell: Full analytical characterization for log linearized model; Otherwise, use explicit solution for log-linear system (use RBC.m, based on Uhlig, or use Dynare.)

The effects of different persistence parameters for the technological shocks. See figures from RBC.m for three values of ρ.

(Could do the same for different elasticities of labor supply, or different intertemporal elasticities. But in these two cases, you need to modify the matrices in RBC.m a bit. You may want to do it.)

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Table removed due to copyright restrictions.
Table 1: Business Cycle Statistics for the U.S. Economy on page 938.
King, R., and S. Rebelo. “Resuscitating Real Business Cycles.” Chapter 14 in *Handbook of Macroeconomics*.
Vol. 1B. Edited by J. Taylor and M. Woodford. New York, NY: Elsevier, 1999. pp. 927-1007. ISBN: 9780444501578.

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Table 3: Business Cycle Statistics for Basic RBC Model on page 957.
King, R., and S. Rebelo. "Resuscitating Real Business Cycles." Chapter 14 in *Handbook of Macroeconomics*.
Vol. 1B. Edited by J. Taylor and M. Woodford. New York, NY: Elsevier, 1999. pp. 927-1007. ISBN: 9780444501578.

Figure removed due to copyright restrictions.
Figure 7: Basic Model: Simulated Business Cycles on page 959.
King, R., and S. Rebelo. “Resuscitating Real Business Cycles.” Chapter 14 in *Handbook of Macroeconomics*.
Vol. 1B. Edited by J. Taylor and M. Woodford. New York, NY: Elsevier, 1999. pp. 927-1007. ISBN: 9780444501578.

Summary

- Intertemporal and intratemporal conditions.
- Productivity shocks, consumption, and employment
- Too good to be true?