14.462
Midterm exam
March 17, 2004
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Consider the following economy. There is continuum of workers with mass 1, each endowed with L units of labor, and a continuum of goods of mass N. They have the same utility given by

$$U = \int_0^N \frac{1 - e^{-bc_i}}{b} di,$$

where N is the number of goods, which is endogenous. Each differentiated good is produced by a monopoly. There is a fixed overhead cost equal to $\bar{l}$ units of labor. There is no variable cost (an arbitary large quantity of the good can be produced: these goods are like software, music, etc).

1. Show that if the price of good i is p_i, then the demand for good i by a consumer with income R is

$$c_i = \bar{c} - \frac{1}{b} \ln p_i,$$

where

$$\bar{c} = \frac{R + \frac{1}{b}\int_0^N p_i \ln p_i di}{\int_0^N p_i di}$$

2. Show that each firm will charge a price $p_i = p = e^{b\bar{c}-1}$, where $\bar{c}$ is defined as above and common to all workers.

N is endogenously determined by the free entry condition. We normalize the common price level to $p = 1$.

3. Compute the wage level w (as defined by the wage of 1 unit of labor, so that a worker's income is wL). How does it depend on the overhead labor cost $\bar{l}$? Explain why.

4. Compute the utility of a worker. How is it affected by total productivity (as measured by L) and overhead costs?

We now modify the model and assume that each worker is also endowed with q units of managerial quality. A firm employing a manager of quality q has a total overhead cost now equal to $\bar{l}/q$ (instead of just $\bar{l}$). q is uniformly distributed in the population over $[q_{\min}, q_{\max}]$, i.e. with c.d.f. $F(q) = \frac{q - q_{\min}}{q_{\max} - q_{\min}}$ and density $f(q) = F'(q) = \frac{1}{q_{\max} - q_{\min}}$. Each worker has to work either as a worker or a manager, and can't do both. There is free entry of firms which compete to hire managers. Let $\omega(q)$ be the wage paid to a manager with quality q in equilibrium.

5. Show that (with the same price normalization as before), one must have

$$\omega(q) = 1/b - w\bar{l}/q$$

6. Show that all workers with managerial quality $q > q^*$ become managers in equilibrium, where

$$LF(q^*) = \int_{q^*}^{q_{\max}} \frac{\bar{l}}{q} f(q) dq$$

7. Show that this condition defines a unique q^* such that both q^* and $\bar{l}/q^*$ go up when $\bar{l}$ rises.

8. Show that the equilibrium wage is

$$w = \frac{1}{b(L + \bar{l}/q^*)}$$

9. How does an increase in overhead costs $\bar{l}$ affect

(i) The absolute income level for production workers wL?

(ii) Their utility

(iii) The number of managers?

(iii) Inequality (as measured by income ratios) between production workers and low-quality managers?

(iv) Inequality between production workers and high-quality managers?

(v) Inequality between two managers who remain in that activity after the increase in $\bar{l}$

10. Same questions for a change in productivity L.