14.471: Fall 2012: Recitation I: Exact consumer surplus

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Part I

Outline/Take-aways

1. **Goal:** Measuring welfare loss/excess burden from taxation in an "exact way" in a empircally implementable way.
2. **marshallian demand:** In the first class we drew triangles. marshallian Excess Burden (EB) is appealing because it is easy and observable. But marshallian surplus is an ad-hoc measure and suffers from path-dependence.
   - Syndrom: path dependence
   - Conceptual issues
3. **Hicksian demand**: Solution: Introduce Hicksian EB and expenditure function to translate the utility loss into dollars
   - BUT: Hicksian demand is not observable and depends on utility measure $h(q, u)$
   - At what utility to measure Hicksian EB? 2 candidates:
     - Pre-tax utility: CV
     - Post-Tax utility: EV
4. **Hicksian Excess burden** measures without conceptual problems (e.g. Diamond-Mc Fadden)

Part II

Content

1 Goal

Measuring EB from taxes:

- Rigorous, exact way consistent with micro-theory
- Empirically implementable

2 marshallian surplus: Pros and Cons

2.1 Reminder: Definitions of marshallian/uncompensated demand.

Individual solves

$$\max_c u(c)$$

s.t.

$$q \cdot c \leq Z$$

where $q = p + t$. With λ being the multiplier on the budget constraint, the FOC in c_i is

$$u_{c_i} = \lambda q_i$$

These condition implicitly define:

- $x_i(q, Z)$: Marshallian (uncompensated) demand function
- $v(q, Z)$: the indirect utility function

2.2 Pro: Measuring EB with marshallian surplus is simple

- EB is a simple triangular surface:$EB = \frac{1}{2} dQ d\tau$
- Per-unit tax rate τ and (taking the total derivative of the) market equilibrium $D(p + \tau) = S(p)$ leads to the following change in the producer price:

$$\frac{dp}{d\tau} = \frac{\eta_D}{\frac{p+\tau}{p}\eta_S - \eta_D} \tag{1}$$

- Using that initial taxes are zero and (1) the ratio between the Harberger triangle DWL and raised revenues R equals:

$$\frac{DWL}{R} = \frac{1}{2} \frac{\eta_S \eta_D}{\eta_S - \eta_D} \left(\frac{\tau}{p} \right) \tag{2}$$

2.3 Con: Path dependence problem when more than one price changes

2.3.1 Illustration with taxes on 2 goods A and B

Let us compare:

- The change in consumer surplus when we move:
  - first from $\left(q_A^0, q_B^0\right)$ to $\left(q_A^1, q_B^0\right)$
  - and then from $\left(q_A^1, q_B^0\right)$ to $\left(q_A^1, q_B^1\right)$

$$CS_{top} = \int_{q_A^0}^{q_A^1} c_A(q_A, q_B^0, Z) dq_A + \int_{q_B^0}^{q_B^1} c_B(q_A^1, q_B, Z) dq_B$$

with

- The change in consumer surplus when we move:
  - first from $\left(q_A^0, q_B^0\right)$ to $\left(q_A^0, q_B^1\right)$
  - and then from $\left(q_A^0, q_B^1\right)$ to $\left(q_A^1, q_B^1\right)$

$$CS_{top} = \int_{q_B^0}^{q_B^1} c_B(q_A^0, q_B, Z) dq_B + \int_{q_A^0}^{q_A^1} c_A(q_A, q_B^1, Z) dq_A$$

Class Question: Who knows under which condition they are equal?

For $CS_1 = CS_2$, we need equal cross-partials $\frac{dc_A}{dq_B} = \frac{dc_B}{dq_A}$. This will not be satisfied for marshallian demand functions unless there are no income effects (b/c income effects and initial consumption levels differ across goods).

But they are equal for Hicksian (compensated) demand. Indeed, the Slutsky Matrix- containing the derivatives of the Hicksian demand function which holds utility constant- is symmetric.

$$s_{AB} = \frac{\partial^2 e}{\partial q_A \partial q_B}$$

2.3.2 Why do we have a price dependence issue? Conceptual problems

- Path-dependence problem reflects the fact that consumer surplus is an ad-hoc measure.
  - It is not derived from utility function or a welfare measure.
- Question of interest: How much utility is lost because of tax distortions beyond the revenue transferred to government?
- Need units to measure "utility loss"

3 Solution

3.1 Introduce expenditure function to translate the utility loss into dollars (money metric)

- Hicksian/compensated demand $h_A(q, u)$ ("Demand of a consumer over a bundle of goods that minimizes their expenditure while delivering a fixed level of utility") is the solution to the problem where you minimize costs:

$$e(q, u) = \min_h q \cdot h$$

s.t.

$$U(h) \geq U$$

- The value of the minimum is the expenditure function $e(q, U)$
- Now define individual's loss from tax increase as:

$$e(q^1, u) - e(q^0, u)$$

- $e(q^1, u) - e(q^0, u)$ is a single-valued function -> coherent measure of welfare cost, no path dependence issue

3.2 But where should u be measured?

3.2.1 Measure utility at initial price level (before tax change): CV

The compensating variation is the cost to a consumer of a shift in prices from q^0 to q^1 where the expenditure function $e(q^1, u^0)$ is evaluated at the old utility level u^0:

$$CV = e(q^1, u^0) - e(q^0, u^0)$$

Intuition: How much compensation is needed to reach original utility level at new prices?

3.2.2 Measure utility at new price level (after tax change): EV

The equivalent variation is the cost to a consumer of a shift in prices from q^0 to q^1 where the expenditure function $e(q^1, u^1)$ is evaluated at the new utility level u^1:

$$EV = e(q^1, u^1) - e(q^0, u^1)$$

Intuition: Lump sum amount agent willing to pay to avoid tax?

3.3 Can we derive empirically implementable formulas Hicksian demand as we did graphically with marshallian demand?

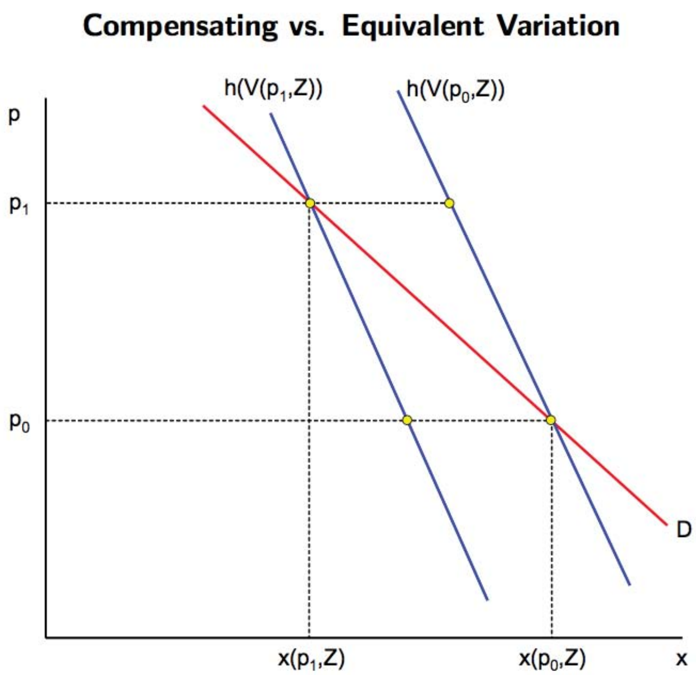
3.3.1 Build some intuition on graphical representation of Hicksian demand

- marshallian demand is more elastic than Hicksian because a drop in prices q
  - 1) substitution: pushes x up

– 2) income (if normal): pushes x up

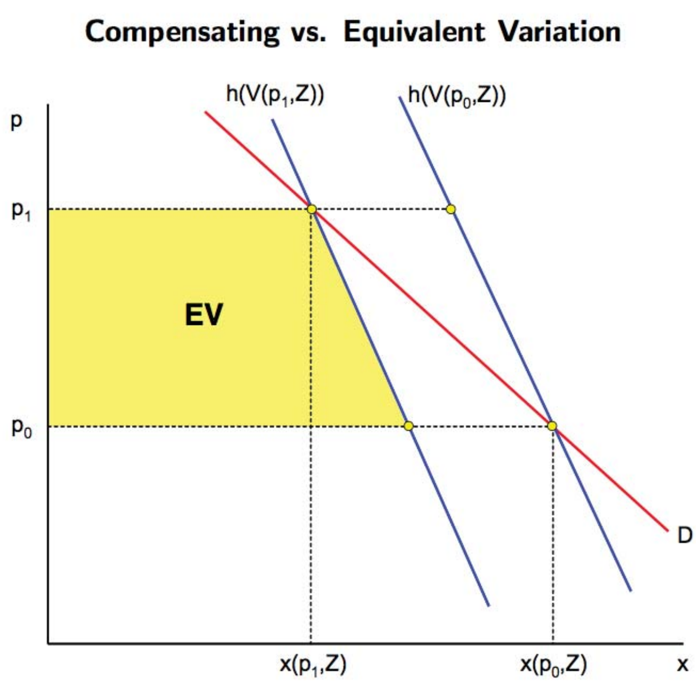
Thus marshallian reaction of quantities to prices is bigger than Hicksian effect restricted to substitution only.

Figure 1: marshallian demand is more elastic than Hicksian demand



- EV is the area to the left of the Hicksian Demand Curve related to NEW utility.

Figure 2: EV graphical

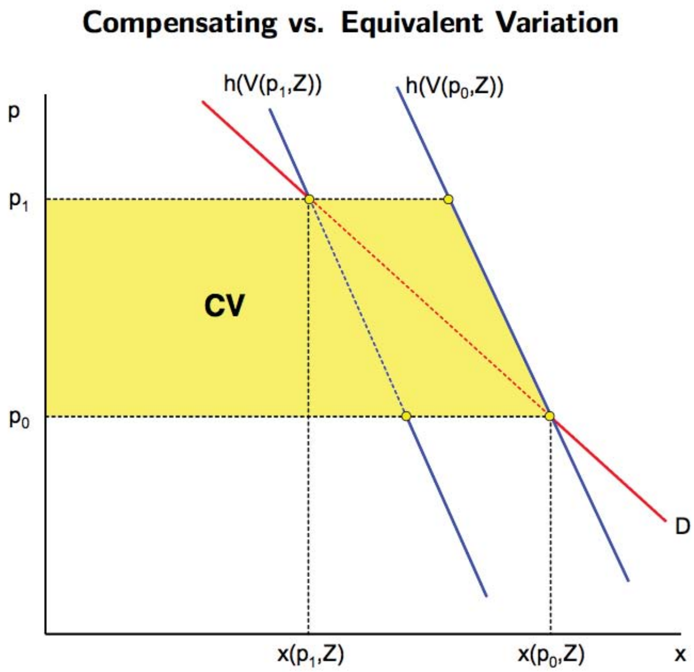


- CV is the area to the left of the Hicksian Demand Curve related to OLD utility. Sketch of the proof:
  - $CV = e(q^1, u^0) - e(q^0, u^0)$
  - Suppose only q_1 changes. The integral from q^0 to q^1 of $\int_{q^0}^{q^1} h(u^0) dq_1$
  - Use now that

$$h = \frac{\partial e}{\partial q_1}$$

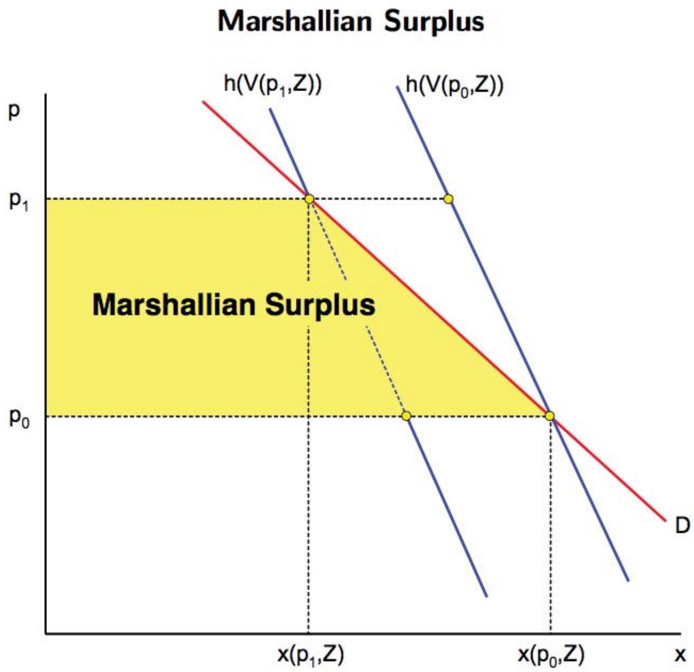
  - to get that $\int_{q^0}^{q^1} h(u^0) dq_1 = e(q^1, u^0) - e(q^0, u^0) = CV$

Figure 3: CV graphical



- With one price change $EV < MarsSurplus < CV$ but this is not true in general.

Figure 4: marshallian surplus graphical



4 Precise EB

- Deadweight burden: change in consumer surplus less taxes paid
- Equals what is lost in excess of taxes paid ("net cost measure of intervention")
- Diamond and McFadden (1974) measure the deadweight burden of a tax-induced price change from q_0 to $q_1 = q_0 + \tau$ as follows by subtracting compensated revenue (not observed):

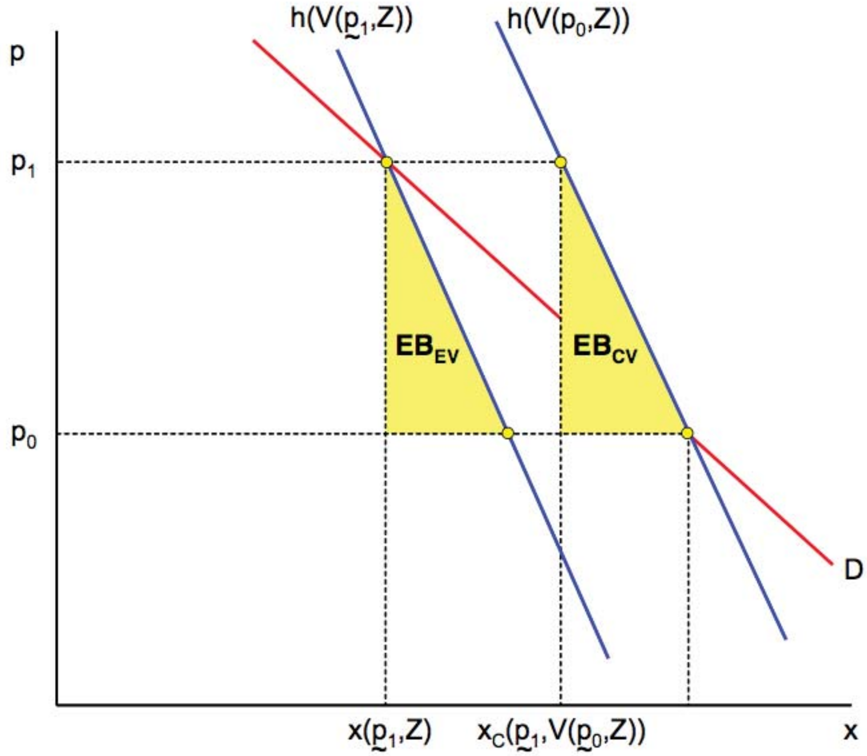
$$EB(u^0) = CV - (q^1 - q^0)h(q^1, u^0)$$

- Mohring (1971) works at new utility levels:

$$EB(u^1) = EV - (q^1 - q^0)h(q^1, u^1)$$

- In general, CV and EV measures of EB will differ (equal when quasilinear utility)
  - CV is useful if you have multiple alternatives because you always look at old utility.

Figure 5: CV and EV measures of EB can differ



- marshallian measure overstates excess burden because it includes income effects
  - Income effects are not a distortion in transactions
  - Buying less of a good due to having less income is not an e!ciency loss; no surplus foregone b/c of transactions that do not occur
- Thus: Hicksian EB is appealing because no path dependence. But unappealing because not observable and depends on utility measure $h(q, u)$

Part III

Appendices:

4.1 Symmetry Slutsky

- Hicksian demand $h_A(q,u)$ is the solution to the problem where you minimize cost:

$$e(q,u) = \min_h q \cdot h$$

$$U(h) \geq U$$

- The value of the minimum is the expenditure function $e(q,U)$
- By envelope (the change in the value function is given by the partial derivative of the Lagrangian with respect to choice variable) Hicksian demand equals

$$h_A = \frac{\partial e}{\partial q_A}$$

- now the Slutsky matrix is defined as the price derivative of hicksian demand

$$s_{AB} = \frac{\partial h_A}{\partial q_B}$$

- If we now use (**??**), we get the symmetry of the Slutsky result:

$$s_{AB} = \frac{\partial^2 e}{\partial q_A \partial q_B}$$

4.2 Reminder: Slutsky & Roy identity

- Slutsky based on deriving $h(q,U) = g(q, e(q,u))$ wrt q into a direct and indirect effect.

$$\frac{\partial h_A}{\partial q_B} = \frac{\partial g_A}{\partial q_B} + \frac{\partial e}{\partial q_B}\frac{\partial g_A}{\partial e} = \frac{\partial g_A}{\partial q_B} + h_B \frac{\partial g_A}{\partial R}$$

- Roy's identity:

$$g_i = -\frac{\frac{\partial V}{\partial q_i}}{\frac{\partial V}{\partial R}}$$

is based on taking ratio of

$$\frac{\partial V}{\partial R} = \lambda$$

and

$$\frac{\partial V}{\partial R} = -\lambda g_i$$

4.3 References

- Auerbach (1985) on Path dependence problem
- Duality theory and welfare evaluation of economic changes:
  - Quick: Salanié, B., 2011. The Economics of Taxation, The MIT Press: Appendix A: "Some Basic Microeconomics"
  - Depth: MWG section 3.I "welfare evaluation of economic changes 3.1"
- Lecture notes Raj Chetty

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