Fall 2009 14.64:
Problem Set Four Solutions

Amanda Pallais

December 9, 2009

Borjas Question 7-2

(a) Indexing the minimum wage to inflation would weakly decrease inequality. It would "pull up" the wages at the very bottom of the distribution.

(b) Increasing the benefit level paid to welfare recipients may not have any effect on wage inequality; it does not directly affect their wages at all. However, if the increase in benefits induces many welfare recipients to leave the market, then it could raise wages.

(c) Increasing wage subsidies to firms that hire low wage workers could lead such firms to hire many more such workers, which would raise wages and thereby decrease inequality.

(d) To the extent that you believe that illegal immigrants lower the wages of the unskilled natives, then kicking those immigrants out should raise native wages (at the bottom). If you believe that such immigrants have no effect on the wages of skilled natives, then this policy would decrease inequality.

Borjas Question 7-9

Suddenly the number of skilled workers goes from 4 million to 5 million, an increase of 25%. So the change in wages for skilled workers is $0.25/-0.4 = -0.625$. Similarly, the change in wages for unskilled workers is $0.50/-0.10 = -5$. Using the hint in Borjas, the wage ratio goes up by $-0.625-(-5) = 4.375$. That means that the wage ratio becomes $\frac{x-2.5}{2.5} = 4.375 \Rightarrow 13.44$.

Borjas Question 9-6

(a) If the firm does not discriminate, then it would only hire black workers. In particular, it would hire

$$\frac{500}{\sqrt{E_b}} = 10 \Rightarrow E_b = 2500.$$

This firm earns profit

$$100 \times 10\sqrt{2500} - 10 \times 2500 = 25000.$$

(b) The discriminating firm hires black workers based on

$$\frac{500}{\sqrt{E_w + E_b}} = 10(1 + 0.25),$$

and white workers based on

$$\frac{500}{\sqrt{E_w + E_b}} = 20.$$

Clearly, this firm is not "racist enough" to want to hire only whites. In that case, it hires blacks only, and hires 1600 of them. Its profits are

$$100 \times 10\sqrt{1600} - 10 \times 1600 = 24,000.$$

(Notice that these are actual profits, ignoring the utility cost of hiring black workers.)

(c) This firm now has discrimination coefficient 1.25. Clearly, he will only hire whites, as at the current wages, $W_b = 10(1 + 1.25) > 20 = W_w$. He will hire

$$\frac{500}{\sqrt{E_w}} = 20 \Rightarrow E_w = 625.$$

Profits will be

$$100 \times 10\sqrt{625} - 20 \times 625 = 12,500.$$

Notice that all firms are paying for their bigotry.

Borjas Question 9-11

(a) Five percent of black drivers are drunk and five percent of white drivers are drunk.

(b) Of the 5,000 cars observed, $0.20 \times 5000 = 1,000$ are driven by black drivers. Then $0.20 \times 0.05 \times 5000 = 50$ are drunk. Meanwhile, there are 4,000 cars driven by whites, and $0.05 \times 4000 = 200$ of these are drunk. Blacks are $50/(50+200) = 0.20$ of the drunk driver population. (We already knew that.)

(c) First off, how many black drivers will be pulled over? Well, 50 black drivers are drunk, $0.10 \times 50 = 5$ of these will swerve and be pulled over. Of the non-swerving drunk black drivers, $0.50 \times (50 - 5) = 22.5$ will be pulled over and subsequently arrested.

Meanwhile, of 200 white drunk drivers, $0.10 \times 200 = 20$ swerve and are arrested. Otherwise, there are no white drivers pulled over. Consequently, the share of drunk drivers arrested that are black is

$$\frac{5 + 22.5}{5 + 22.5 + 20} = 0.579.$$

Borjas Question 10-1

If we maximize the utility of the union subject to the firm's demand function, we are solving

$$\max_E E \times (20 - 0.01E),$$

which has first order condition

$$20 - 0.02E = 0,$$

and thus $E = 1000$, $w = 10$.

Borjas Question 10-2

We now solve

$$\max_E E \times (20 - 0.01E - w^*).$$

This leads to the first order condition

$$20 - 0.02E - w^* = 0,$$

and thus to $E = (20 - w^*)/(0.02)$. If $w^* = 8$ then this is 600, and $w = 14$.

We now assume that unions care about how high the wage is relative to the competitive wage. In that case, they will bargain for much higher wages, leading to lower employment (since we stay on the firm's demand curve).

Borjas Question 10-10

Clearly, employment in the union sector is given by $L_u = 1,000,000 - 20 \times 30,000 = 400,000$. This means that $600,000$ employees flood the non-union sector, leading to a wage there of $600,000 = 1,000,000 - 20w \Rightarrow w = 20,000$. So the union effectively lowers wages in the non-union sector, leading to a union wage gap of 10,000 dollars.

B. Analytical and discussion problems

1. The Hicks model models firms' and unions' accepted wage increase from a new contract as a function of the length of a strike. The union's accepted wage increase decreases in the length of a strike, while the firm's accepted wage increase increases in the length of a strike. Under this set-up, the strike will end when the accepted wage increase of the two parties is equal. However, if: 1) each party knows how the other's offer will change as the strike progresses, and 2) strikes are expensive for each side, then each party should know what wage increase will be agreed upon before negotiation even begins - and the parties should agree to this increase from the beginning. Hence, the Hicks model of strike activity suggests that "strikes are mistakes" (given that these two conditions are true) caused by mis-estimating how the other party's accepted wage increase changes as a function of strike length.

   Strikes are undoubtedly costly to each party (firms lose production, workers lose wages). Despite this, strikes may still occur for a few reasons:

   1) parties have imperfect information about how the other's wage offer will change as the strike progresses
   2) unions want the possibility of striking to remain as a credible threat in future negotiations. If the firm believes that the union won't actually strike, then the union's bargaining power is substantially reduced. To retain this threat as credible, unions may strike from time to time to demonstrate to firms that striking remains a credible threat.
   3) even if union *leaders* understand the firm's ability and willingness to bargain (for instance, they may be more informed about the firms' finances and profitability), their rank-and-file may not necessarily be as well informed. Union leaders might realize that their rank-and-file demands are unreasonable based on the firm's constraints, but if their union membership doesn't believe this, then they could lose their leadership role. As a result, union leaders may authorize a strike in order to retain their leadership and pacify the rank-and-file. (This is the Ashenfelter/Johnson argument).

2. From figure 10.7 in the text, the union and firm will bargain to a wage/employment combination such that the union's utility curve is tangent to the firm's isoprofit line. All combinations of points for which these curves are tangent define the contract curve – within some boundaries. First, the wage/employment combination must at least leave the firm with 0 profits. Second, the offered wage must at least be at the competitive market level w* (for which the firm demands some amount of labor E*). So, to be more precise, the following defines the set of wage/employment combinations that make up the contract curve between the firm and union:

$$\frac{\partial \pi}{\partial w} \Big/ \frac{\partial \pi}{\partial E} = -\frac{\partial U}{\partial w} \Big/ \frac{\partial U}{\partial E}$$

Such that: $\pi(w,E) \geq 0$, $w \geq w^*$, $E \geq E^*$

Where $\pi(w,E)$ denotes the firm's profits as a function of wage and employment, $U(w,E)$ denotes the union's utility as a function of wages and employment, w* denotes the competitive wage, and E* denotes the amount of labor that the firm will hire at the competitive wage w*.

Extra credit: the easiest way to do this to specify a generic profit function: $\pi(w,E) = f(E) - wE$. From the question, we know that $U(w,E) = (w - w^*)E$. So simply applying the above condition:

$$\frac{\partial \pi}{\partial w} \Big/ \frac{\partial \pi}{\partial E} = -\frac{\partial U}{\partial w} \Big/ \frac{\partial U}{\partial E} \Rightarrow \frac{-E}{f'(E) - w} = -\frac{E}{w - w^*} \Rightarrow f'(E) - w = w - w^* \Rightarrow f'(E) = w^*$$

implying that employment *E** is set at the competitive level. So employment is set at the competitive level (i.e. the level that would prevail if the firm had to pay the market wage), and positive profit (or rents) exist which are then split between the union and the firm by bargaining over the wage. If the firm has total bargaining power, then it will keep all profits for itself and only pay the competitive wage w*. If the union has total bargaining power, then it will choose a wage *w* that leaves the firm with zero profits at an employment level of *E**. There is no reason to necessarily believe firms trade off wages and employment in this manner (in particular, it seems that unionized firms or industries often employ *too many* people, suggesting that unions push for employment above competitive levels), but if it is true then the firm/union bargaining process *does not* result in deadweight loss – because employment is set at the competitive level.

B. Question 3

Production Function: $Y = AK^{\beta}L^{1-\beta}$

The elasticity of substitution $= \frac{\partial \ln(K/L)}{\partial \ln(w/r)}$ where w is the wage (marginal product of labor) and r is the rental rate of capital (the marginal product of capital).

$$r = \frac{\partial Y}{\partial K} = A\beta K^{\beta-1}L^{1-\beta}$$

$$w = \frac{\partial Y}{\partial L} = A(1-\beta)K^{\beta}L^{-\beta}$$

$$\frac{w}{r} = \frac{A(1-\beta)K^{\beta}L^{-\beta}}{A\beta K^{\beta-1}L^{1-\beta}}$$

$$\frac{w}{r} = \frac{(1-\beta)K}{\beta L}$$

$$\frac{w}{r}(\frac{\beta}{1-\beta}) = \frac{K}{L}$$

Taking logs of both sides gives

$$\ln(\frac{K}{L}) = \ln(\frac{w}{r}) + \ln(\frac{\beta}{1-\beta})$$

We can see that $\frac{\partial \ln(K/L)}{\partial \ln(w/r)}$ =1.

Extra Credit: CES Production Function

Production Function: $Y = (A_K K^{\rho} + A_L L^{\rho})^{1/\rho}$

The elasticity of substitution $= \frac{\partial \ln(K/L)}{\partial \ln(w/r)}$

$$r = \frac{\partial Y}{\partial K} = (A_K K^{\rho} + A_L L^{\rho})^{\frac{1-\rho}{\rho}} \rho A_K K^{\rho-1}$$

$$w = \frac{\partial Y}{\partial L} = (A_K K^{\rho} + A_L L^{\rho})^{\frac{1-\rho}{\rho}} \rho A_L L^{\rho-1}$$

$$\frac{w}{r} = \frac{(A_K K^{\rho} + A_L L^{\rho})^{\frac{1-\rho}{\rho}} \rho A_L L^{\rho-1}}{(A_K K^{\rho} + A_L L^{\rho})^{\frac{1-\rho}{\rho}} \rho A_K K^{\rho-1}} = \frac{A_L L^{\rho-1}}{A_K K^{\rho-1}}$$

$$\frac{w}{r} = \frac{A_L K^{1-\rho}}{A_K L^{1-\rho}}$$

Taking logs of both sides gives

$$\ln(\frac{w}{r}) = \ln(\frac{A_L}{A_K}) + (1-\rho)\ln(\frac{K}{L})$$

$$\ln(\frac{K}{L}) = \frac{1}{1-\rho}\ln(\frac{w}{r}) - \frac{1}{1-\rho}\ln(\frac{A_L}{A_K})$$

So $\frac{\partial \ln(K/L)}{\partial \ln(w/r)} = \frac{1}{1-\rho}$.

C. Empirical problem

1.

Let's begin by comparing differences in means across union members (where by being a union member we mean "being in a union or being covered by a collective bargaining agreement"):

```
. reg lnwage unionst

------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     unionst |   .3366164   .0141834    23.73   0.000     .3088171    .3644157
       _cons |   2.151269   .0028168   763.73   0.000     2.145749     2.15679
------------------------------------------------------------------------------
```

So the unconditional union wage gap is approximately 34%.

Adding in our regular human capital controls (education, gender, potential exp, potential exp squared, and dummies for race):

```
. xi: reg lnwage unionst school exp exp2 i.sex i.race
------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     unionst |   .2139796   .0124967    17.12   0.000     .1894861    .2384731
      school |   .0970703   .0009165   105.91   0.000     .0952739    .0988666
         exp |   .0400893    .000671    59.75   0.000     .0387742    .0414044
        exp2 |   -.000606   .0000155   -39.21   0.000    -.0006363   -.0005757
      _Isex_2 |  -.3080682   .0048637   -63.34   0.000     -.317601   -.2985354
  _Irace_200 |   -.086464   .0086055   -10.05   0.000    -.1033307   -.0695972
  _Irace_300 |  -.1600247   .0256013    -6.25   0.000    -.2102032   -.1098462
  _Irace_650 |   .0312534   .0146575     2.13   0.033     .0025247    .0599821
  _Irace_700 |   -.000251   .0426905    -0.01   0.995    -.0839242    .0834221
       _cons |   .6125708   .0136741    44.80   0.000     .5857695     .639372
```

Our union wage gap lowers to 21%. However, part of this union wage differential could be because some industries are more unionized that others, and even in the absence of being in a union, the wages of people in more heavily unionized industries could be higher. That is, part of the union wage gap could simply be measuring that union members are more likely to be working in higher-wage industries or occupations. To test this, we'll include both occupational and industry fixed effects (i.e. dummies for occupation and industry):

```
. xi: reg lnwage unionst school exp exp2 i.sex i.race i.occ i.ind
------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     unionst |   .1961587   .0119271    16.45   0.000     .1727817    .2195357
      school |   .0635339   .0010598    59.95   0.000     .0614567    .0656111
         exp |   .0311121   .0006498    47.88   0.000     .0298384    .0323858
        exp2 |  -.0004661   .0000148   -31.45   0.000    -.0004951    -.000437
     _Isex_2 |  -.2539577   .0053252   -47.69   0.000    -.2643951   -.2435204
  _Irace_200 |  -.0491833   .0082228    -5.98   0.000       -.0653   -.0330666
  _Irace_300 |  -.1347235    .024304    -5.54   0.000    -.1823593   -.0870878
  _Irace_650 |    .057066   .0139206     4.10   0.000     .0297818    .0843503
  _Irace_700 |    .016311   .0404924     0.40   0.687     -.063054    .0956759
     _Iocc_1 |  -1.007299    .025905   -38.88   0.000    -1.058073   -.9565255
```

```
     _Iocc_2 |   .0304322   .0091801     3.32   0.001     .0124392    .0484252
     _Iocc_3 |  -.2456943   .0085312   -28.80   0.000    -.2624153   -.2289733
     _Iocc_4 |  -.1503918   .0119167   -12.62   0.000    -.1737485   -.1270352
     _Iocc_5 |  -.1797576   .0103996   -17.29   0.000    -.2001407   -.1593744
     _Iocc_6 |  -.3781419   .0103212   -36.64   0.000    -.3983714   -.3579125
     _Iocc_7 |   -.449987   .0094648   -47.54   0.000    -.4685379   -.4314361
     _Iocc_8 |   -.848989   .0258858   -32.80   0.000    -.8997252   -.7982528
     _Iocc_9 |   -.449596   .0133347   -33.72   0.000     -.475732   -.4234599
     _Iind_2 |   .0572847   .0128442     4.46   0.000     .0321102    .0824593
     _Iind_3 |   .1600732   .0105142    15.22   0.000     .1394654     .180681
     _Iind_4 |    .170743   .0107448    15.89   0.000     .1496833    .1918028
     _Iind_5 |  -.1127282   .0112888    -9.99   0.000    -.1348541   -.0906022
     _Iind_6 |  -.2622741   .0093344   -28.10   0.000    -.2805695   -.2439788
     _Iind_7 |  -.0907257   .0090346   -10.04   0.000    -.1084334    -.073018
     _Iind_8 |  -.0835284   .0089414    -9.34   0.000    -.1010536   -.0660032
     _Iind_9 |   .0507926   .0116318     4.37   0.000     .0279942    .0735909
       _cons |   1.384629   .0195534    70.81   0.000     1.346305    1.422954
```

Interestingly, the union wage gap falls only slightly, so differences in wage levels between highly unionized industries/occupations and less unionized industries/occupations isn’t an explanation.

Note: sticking in only industry and not occupational controls lowers the conditional union wage gap to around 17%; sticking in just occupational and not industry controls lowers the conditional union wage gap to around 22%. Hence, it appears that the unconditional union wage gap is better explained by differences in industry unionization rates than differences in occupation unionization rates.

Usually we don’t like to include occupation and industry along with education in a wage regression – as we’ve discussed, one of the ways that education can increase one’s wages is through choice of industry or occupation. However, what we’re interested in here is the effects of being unionized, rather than the effects of education – and so we don’t mind so much that the coefficient on education wouldn’t represent an accurate estimate of returns to education. We want to include human capital variables like age, gender, education, and race because we think some of these things might be correlated with being unionized so that, if we exclude them, the union wage gap is picking up the effects of these confounding variables rather than the true effects of being unionized.

So the union wage gap in this sample is quite substantial: at least 17%.

2. Let’s start with a regular returns to education wage regression:

```
. xi: reg lnwage school exp exp2 i.sex i.race
------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      school |   .0974948    .000918   106.20   0.000     .0956955    .0992941
         exp |    .040655   .0006715    60.54   0.000     .0393388    .0419711
        exp2 |  -.0006151   .0000155   -39.74   0.000    -.0006454   -.0005847
     _Isex_2 |  -.3113114   .0048698   -63.93   0.000    -.3208562   -.3017667
   _Irace_200 |  -.0838186   .0086214    -9.72   0.000    -.1007166   -.0669207
   _Irace_300 |  -.1602684   .0256529    -6.25   0.000    -.2105479   -.1099889
   _Irace_650 |   .0317539    .014687     2.16   0.031     .0029675    .0605404
   _Irace_700 |   .0004096   .0427764     0.01   0.992    -.0834319    .0842511
       _cons |   .6108669   .0137013    44.58   0.000     .5840124    .6377213
```

Returns to education are initially 10%. However, it's possible that this estimate is too high if highly educated workers are more likely to participate in unions, and unions have positive wage effects: because in this case, highly educated workers are earning more not just due to their education but also because they're more likely to be unionized. Or, our measurement of returns to education could be too low if low educated workers are more likely to participate in unions, because then the wage gap between high and low educated workers is smaller than it otherwise should be (of course if education also directly impacts the probability of being unionized, then it's not clear that we also want to control for education: this is just like why we don't want to control for occupation or industry if interested in returns to education, because one of the effects of education on wages could be through its effect on unionization status). Nevertheless, let's see what we get:

```
. xi: reg lnwage school exp exp2 i.sex i.race unionst
------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      school |   .0970703   .0009165   105.91   0.000     .0952739    .0988666
         exp |   .0400893    .000671    59.75   0.000     .0387742    .0414044
        exp2 |   -.000606   .0000155   -39.21   0.000    -.0006363   -.0005757
     _Isex_2 |  -.3080682   .0048637   -63.34   0.000     -.317601   -.2985354
  _Irace_200 |   -.086464   .0086055   -10.05   0.000    -.1033307   -.0695972
  _Irace_300 |  -.1600247   .0256013    -6.25   0.000    -.2102032   -.1098462
  _Irace_650 |   .0312534   .0146575     2.13   0.033     .0025247    .0599821
  _Irace_700 |   -.000251   .0426905    -0.01   0.995    -.0839242    .0834221
     unionst |   .2139796   .0124967    17.12   0.000     .1894861    .2384731
       _cons |   .6125708   .0136741    44.80   0.000     .5857695     .639372
```

There's virtually no change in the schooling coefficient, so the observed returns to education are not explainable by the fact that highly educated workers are more or less likely to be unionized.

But there's another question we can ask: how do returns to education vary depending on union status? This is an interesting question, because one of the effects that we think unions have is that they compress the wage distribution (either across the economy or within a firm). On the lower (left-hand) tail this is because unions push up wages for the low-wage earners. They might also pull in the upper (right-hand) tail for higher wage workers to promote equality in earnings within the union. One way that they could do this is by demanding fixed compensation rather than compensation based on individual performance (i.e. unions may be less likely to accept piece-rate payment schemes or "pay for performance").

So if unions try to compress the wage schedule within a firm, then we might expect returns to education to be less for union members than non-members because unions try to compress the wage distribution. The regression that tests this will include a dummy for union status, our years of schooling variable (we call these "main effects") and an interaction term between schooling and union status (i.e. multiplying union status dummy by union status). i.e.:

$$\ln wage = \alpha + \beta_1 school + \beta_2 EXP + \beta_3 EXP^2 + \beta_4 UNION + \beta_5 UNION * school + \beta_6 X + \varepsilon$$

where X includes gender and race controls.

Returns to education for a non-union member (when union=0) are β_1. Returns to education for a union member (when union=1) are $\beta_1 + \beta_5$. So β_5 represent the incremental effect of schooling for union members. Here's that regression:

```
. xi: reg lnwage school unionst unionschool exp exp2 i.sex i.race
i.sex             _Isex_1-2           (naturally coded; _Isex_1 omitted)
i.race            _Irace_100-700      (naturally coded; _Irace_100 omitted)


------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      school |   .0982075   .0009332   105.24   0.000     .0963786    .1000365
     unionst |   .5954406   .0606105     9.82   0.000     .4766442     .714237
 unionschool |  -.0287149   .0044646    -6.43   0.000    -.0374655   -.0199644
         exp |     .04013   .0006708    59.82   0.000     .0388152    .0414448
        exp2 |  -.0006071   .0000155   -39.29   0.000    -.0006374   -.0005769
     _Isex_2 |  -.3069464   .0048654   -63.09   0.000    -.3164826   -.2974101
  _Irace_200 |  -.0865796   .0086031   -10.06   0.000    -.1034417   -.0697175
  _Irace_300 |  -.1608994   .0255946    -6.29   0.000    -.2110647   -.1107341
  _Irace_650 |   .0300744   .0146546     2.05   0.040     .0013514    .0587973
  _Irace_700 |   .0015341   .0426795     0.04   0.971    -.0821176    .0851857
       _cons |   .5970871   .0138807    43.02   0.000      .569881    .6242931
```

Indeed, the interaction term is negative! Returns to education for non-union members are around 10%. Returns to education for union members are around 7% (.098-.029). So, since returns to education are lower within firms with unions, it appears that unions may compress earnings within unionized firms – and hence, unions may compress the wage distribution (reduce wage inequality) for the economy as a whole. So throughout the 1970s through 1990s, unionization rates fell and inequality increased – given these facts and our regression results, it seems plausible that the declining unionization rates contributed somewhat to increasing inequality.

14.64 Labor Economics and Public Policy
Fall 2009