Life Cycle Labor Supply (Revised 9-26-17)

Josh Angrist

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A   The LCLS Setup

Live long and prosper

Utility is a function of the lifetime stream of consumption $(c_{i0},..., c_{iT})$ and leisure $(l_{i0},..., l_{iT})$, where $h_{it} = \tau - l_{it}$ is hours worked in a period.

- At the beginning of adult life, we make a plan to max

$$U(c_{i0}, c_{i1},..., c_{iT}; l_{i0}, l_{i1},..., l_{iT})$$

  assuming known paths for wages and prices

- This is intractable; try this instead:

$$U(c_{i0}, c_{i1},..., c_{iT}; l_{i0}, l_{i1},..., l_{iT}) = \sum_{t=0}^{T} U_t(c_{it}, l_{it}),$$

  a restriction called "intertemporal additivity"

- Further simplification buys us stronger implications. It's customary to model within-period utilities as the same but for discounting:

$$U(c_{i0}, c_{i1},..., c_{iT}; l_{i0}, l_{i1},..., l_{iT}) = \sum_{t=0}^{T} \left(\frac{1}{1+\rho}\right)^t U(c_{it}, l_{it}) \qquad (1)$$

  This implies that at fixed consumption and leisure choices, the MRS across adjacent periods is $\frac{1}{1+\rho}$

- We often simplify yet further, invoking "within-period additivity":

$$U(c_{it}, l_{it}) = u(c_{it}) + v(l_{it})$$

Go out as you came in

The lifetime budget constraint is

$$\sum_{t=0}^{T}\left(\frac{1}{1+r}\right)^t \left[p_t c_{it} \quad w_{it}(\tau \quad l_{it})\right]=0 \tag{2}$$

You speak of chance, grasshopper ...

- We don't *know* future wages and prices

- Here's the formal structure: In each period, we max expected utility looking forward, confident we'll do the right thing down the road

$$\max E_t \sum_{s=t}^{T}\left(\frac{1}{1+\rho}\right)^{s-t} U(c_{is}, l_{is}) \tag{3}$$

$$\text{s.t. } A_{it+1} = (1+r)A_{it} + (1+r)(w_{it}h_{it} \quad p_t c_{it})$$
$$A_{iT} = 0$$

- Formal modeling of uncertainty changes the lessons of LCLS theory remarkably little: to understand the consequences of uncertainty, we can solve the certainty model and consider consequences of shocks to the marginal utility of wealth (later)

... as if such a thing were sure to exist

Let's solve for the certainty plan, initially under intertemporal additivity without restricting preferences further.

Under certainty, we max the RHS of (1), while constrained to die with a clean slate, as described by (3).

- *Recall the LCH/PIH*: what's the key insight? Keep your eyes peeled for something similar

- Note that the Lagrange multiplier we require here has an i on it. FOCs for period t choices go like this:

$$\left(\frac{1}{1+\rho}\right)^t U_c(c_{it}, l_{it}) = \left(\frac{1}{1+r}\right)^t {}_i p_t \tag{4}$$

$$\left(\frac{1}{1+\rho}\right)^t U_l(c_{it}, l_{it}) = \left(\frac{1}{1+r}\right)^t {}_i w_{it} \tag{5}$$

- Rewriting these as

$$U_c(c_{it}, l_{it}) = \left(\frac{1+\rho}{1+r}\right)^t {}_i p_t \tag{6}$$

$$U_l(c_{it}, l_{it}) = \left(\frac{1+\rho}{1+r}\right)^t {}_i w_{it} \tag{7}$$

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Note that there's a third (unwritten) FOC that pins down the multiplier as well (where does that come from?)

- We've gone far enough to learn something useful:

$$[c_{it}, h_{it}] = f(r, \rho, \lambda_i, p_t, w_{it})$$

- We also have
$$\lambda_i = f(r; \rho, ; p_0, ..., p_T; w_{i0}, ..., w_{iT})$$

  - This reminds us of the PIH for consumption: how does consumption behave in the benchmark case? (Be sure you can show this in a simple model)

- **This suggests an interesting prediction:** holding MU(wealth) $= \lambda_i$ fixed, raising w_{it} increases h_{it}

  - Higher w_{it} on RHS of (8) means we must raise MU of leisure; we can do that by deceasing leisure and we do *that* by increasing hours worked

  - This is a hand-waving argument since we know that utility functions need only be *quasi-concave* while here I'm using old-fashioned concavity, and, implicitly, within-period additivity

  - Browning, Deaton, and Irish (1985) give an elegant duality-type argument for the result that with MU(wealth) fixed, $\frac{\partial h_{it}}{\partial w_{it}} > 0$; this is discussed briefly below

Optimizing in an uncertain world

- Maximizing (4) instead of (1), leaves us with the same FOCs, except the MU(wealth) is now time-varying, so we write λ_{it}

  - Time-varying MU(wealth) is a random variable; we don't know today what tomorrow will bring, and how this news affects our lifetime budget. Even so, we predict this, without bias:

$$\lambda_{it} = E_t \left[\frac{\lambda_{it+1}}{1+r} \right], \tag{8}$$

  where E_t denotes expectation conditional on all thats known at time t (see Browning, Deaton, and Irish, 1985; Altonji, 1986, for details)

Dual roads to a full life

Recall that expenditure minimization is the dual to utility maximization in a static model. Browning, Deaton, and Irish (1985) show that that *LCLS dual* is profit maximization: imagine yourself as a "utility factory," producing output valued at the price at which you implicitly buy it. What should this price be?

Since λ_i is the marginal utility of a dollar of wealth, wealth required to buy u_0 utils (a dollar value) is $q_i \equiv (1/\lambda_i)u_0$. Profit is defined as the difference between the value of the utility we produce and the amount spent (on leisure and consumption) to get it.

- Under additive separability, profit over all periods is maximized by solving within-period profit-maximization problems: your utility factory produces efficiently when it's efficient every period

- Under certainty (using my notation above), profit is maximized by solving

$$\max_{u,c_{it},h_{it}} \left\{ q_i \frac{u}{(1+\rho)^t} - \frac{w_{it}(T - h_{it}) + p_t c_{it}}{(1+r)^t}; U(c_{it}, T - h_{it}) = u \right. \qquad (9)$$

for each t. This has FOCs again given by (5) and (6), but here the marginal utility of wealth, $q_i = \lambda_i^{-1}$, is a *parameter*

  - Solving the profit max problem directly yields the *Frisch* labor supply function, $h(w_{it}, p_t, \lambda_i)$, also known in the vernacular as "λ *constant*" labor supply

- **A duality-type type argument shows that the derivative of Frisch hours worked with respect to wages must be positive**

  1. Define the *profit function* to be the maximized maximand associated with (10); write this as $\pi(q_i, w_{it}, p_t)$. A Shephard's-Lemma-type relation implies that

  $$\frac{\partial \pi(q_i, w_{it}, p_t)}{\partial w_{it}} = h(w_{it}, p_t, \lambda_i)$$

  2. As in conventional producer theory, the consumer/worker profit function is convex in prices, so

  $$\frac{\partial^2 \pi(q_i, w_{it}, p_t)}{\partial w_{it}^2} = \frac{\partial h(w_{it}, p_t, \lambda_i)}{\partial w_{it}} > 0$$

  (We think of wages as the price of market labor; you sell this plus utility to cover the cost of consumption inputs)

- The LCLS hours equation makes the utility price into a parameter, instead of unearned or full income (as in the Marshallian problem) or a reference (lifetime) utility level (as for Hicks)

  - The Frisch labor supply elasticity, also called the intertemporal substitution elasticity (ISE), exceeds Hicks', unless income effects are zero; the difference between them (Hicks minus Frisch) is proportional to the product of the relevant income effect (a negative number for hours) and Frisch's "income flexibility," $\partial \ln q / \partial \ln A_T$, where A_T is unearned income (set to zero in my version, but generally a parameter). This derivative is positive, since MU(wealth) declines as wealth increases (see BDI, p. 509 for the general formula)

- In an uncertain world, MU is time-varying, so we add (9) as before; the solution is otherwise unchanged

B   Empirical LCLS

- Following Heckman and MaCurdy (1980), MaCurdy (1981) assumes within-period additivity of the form:

$$U(c_{it}, h_{it}) = \gamma_{1i} c_{it}^{\sigma_1} - \gamma_{2i} h_{it}^{\sigma_2}$$

  where we switch to utility in terms of hours worked

- In addition to providing parametric specificity, this generates an important simplification of (7) and (8). From the FOC for hours, we get

$$\gamma_{2i}\,\sigma_2 h_{it}^{\sigma_2 - 1} = \left(\frac{1+\rho}{1+r}\right)^{t} \lambda_i w_{it}$$

  Futzing and putzing, this yields a linear-in-logs labor supply function:

$$\ln h_{it} = \left[\frac{\ln \lambda_i - \ln \gamma_{2i} - \ln \sigma_2}{\sigma_2 - 1}\right] + \frac{t}{\sigma_2 - 1}\ln\left(\frac{1+\rho}{1+r}\right) + \frac{1}{\sigma_2 - 1}\ln w_{it}$$

- Approximating this gives us Heck-MaC labor supply as usually written:

$$\ln h_{it} = \mu_i + \delta(\rho - r)t + \delta \ln w_{it} + u_{it} \tag{10}$$

  where $\delta = \frac{1}{\sigma_2 - 1}$ is the Heck-Mac ISE and u_{it} is a "tacked-on" error (Note that $\sigma_2 > 1$ [why?])

Understanding the ISE

The parameter δ, the Heck-Mac ISE, interests us greatly

- In general, the ISE (Frisch elasticity) is positive and (weakly) larger than a traditional Hicksian substitution elasticity, which of course, exceeds the Marshallian uncompensated elasticity

- The ISE describes labor supply responses that hold the marginal utility of wealth fixed

- Recall that λ_i is a function of the entire path of wages and prices (obtained by substituting solutions like (11) back into the budget set). What sort of "experiments" generate such wealth-constant effects?

  – Consider my lifetime work plan: My wage profile is known; my marginal utility of wealth is fixed. But I work harder at age 30 than 25. How come and how much? The ISE answer this question; it describes how I allocate my hours over my lifetime to best exploit the low-hanging fruit on offer when wages are high, binge-watching *GoT* and riding single track when my time is cheap

- Cab drivers who anticipate trip demand over days of the week and hours of the day make the same calculation: evolutionary wage changes need not play out in "evolutionary time"

- Well, we don't all drive cabs (not yet, anyway). The ISE also approximates the response to any short-run or small change that changes changes lifetime wealth little. The ISE therefore looms large in macro: cyclical variation that is either anticipated or modest enough to leave λ_i unchanged generates an ISE-mediated supply response (Lucas and Rapping's (in)famous explanation of cyclical unemployment: leisure!)

- The ISE doesn't explain the response to changes in (lifetime) wealth. The Heck-MaC model produces a simplified version of that idea. Labor supply responds to a shock of amount $\triangle_t$ by:

$$\frac{\partial h_{it}}{\partial \triangle_t} = \frac{d \ln w_{it}}{d \ln \triangle_t} + \frac{d \ln \lambda_i}{d \ln \triangle_t}$$

A shift in the entire life-cycle wage profile, for example, reduces and has a dominating negative wealth effect on hours.

   - **Proof that higher wages lower λ ...**
      * I gave a hand-waving argument in the previous section. For rigorous proof we can solve the comparative statics problem that asks how all endogenous variables, *including the marginal utility of wealth*, change as a function of changes in prices. This can be done by brute force by differentiating first order conditions, including the budget line, and solving the relevant comparative statics problem using Cramer's Rule as in Varian 1978, Sections 3.9 and 1.14). Quasi-concavity of the utility function - which signs the determinant of the bordered Hessian and principal minors involved in this - implies the result.

- The ISE concept isn't tied to Heck-MaC preferences; any LCLS model has one. Labor economists tend to see the ISE through the lens of the Heck-Mac model since it provides powerful simplification, and we like to think of the ISE as a parameter (hence, we label it in Greek). In practice, of course, individuals react differently to changes in wages. Our empirical models implicitly target an average ISE or an economy-wide effect

- More important than the ISE parameter generated by particular preferences is the ISH (for hypothesis): make hay while the sun shines!

A sense of smoothness

- The LCLS framework presumes perfect credit: workers borrow and lend freely at parametric interest rates, *frictionlessly* exploiting the fact that

to everything there is a season and a time to every purpose under heaven
... when wages are high, it's time to work!

- Workers who can't take advantage of seasonal advantage are said to be liquidity constrained

- Liquidity constraints and simple myopia look similar: both generate strong within-period wealth effects (as we'll soon discuss further)

- How *relevant* is the ISE? Well, how big are your wealth effects? On this, reasonable labor economists can disagree (See, e.g., Card, 1994)

MaCurdy Metrics (up next)

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