Good morning. Today we move in the direction that takes a big turn from the direction we have been going in so far. All the devices we have had up until now, resistors and voltage sources, and even your digital devices like the AND gate or the inverter and so on had a very specific property. We didn't dwell on that property, but that property was that these were not what are called memory devices. In other words, the outputs at any given time are a function of the inputs alone. In other words, if you took your inverter or your NAND gate for that matter and you build a circuit comprising 50 NAND gates connected in structures that we have talked about, you apply an input and boom you get an output.

And your output is a function of the inputs alone, right? The same thing with your resistors and voltage sources. At any given point in time your output VO of T was some function of the input VI of T.

What we are going to do today is discuss a new element which will introduce a whole new class of fun stuff for all of us to deal with. And that is called storage.

In other words, the output of a circuit is now going to depend not just on the inputs but it is going to depend on the background or it is going to depend on where the circuit has been in the past. So past is going to matter.

It is a very fundamental difference.

And what I would like to do is start by giving you folks a little bit of a surprise. I am going to do a little demo taking two of your inverter circuits.

I am going to start by taking a couple of inverters.

Remember, I am using this structure here as an inverter.

And I am going to couple this to another inverter and take an output C, some VS, some load resistance RL, my B terminal and my A terminal.

So I'm going to apply some input between ground and my A terminal. And for fun I want to apply a square wave at the input. A square wave between zero and 5 volts. And this is how my time goes.

Let's assume that VS is 5 volts.

So what I am going to do is plot for you the behavior of this inverter. I am going to plot for you A, which would look like this. I am going to plot for you B, which would be the inverted wave form.

And then plot C, which would be a wave form that looks like this again. Let me do a plot here.

So this is A.

-- and so on. Time goes this way.

And let's say this is between zero and 5 volts.

And B should be an inverted wave form that should look like this.

If all that we believe of the world so far is true then this is how things should behave, so C should look like this.

This is what the world should look like and if everything that you learned about is true and correct and all of the good stuff. Let me show you a little demo and see if I can try to pull the rug out from under all that you have learned so far and show you some surprising stuff.

Here are the three wave forms that I showed you up here.

This is my A. This is my A wave form.

This is the B wave form. Notice that B, as you expect, is an inverted form of A.

And this is C. We all expect this, correct? But what I am going to do is let me expand the time scale on this so that I can look at these transitions a little bit more carefully.

I am just going to expand the time scale.

There you go. All I have done is expanded the time scale and spread that out a little bit.

And what you see there is quite different from what you expect.

A is a square wave as expected, but B is stunningly different.

It is a zero as expected because this is a one.

But here I get some really strange behavior, behavior that is like nothing on earth.

Like nothing you have seen before.

And then, of course, it becomes a one eventually, but there's some really, really shady stuff going on here. And so far you are not prepared to deal with this. We have not given you the facility to deal with his issue. What is the problem with this?

We could say who cares? What is the problem with this?

Let's look at the result. I am looking at this, I am focusing on this piece here.

And notice that instead of being a sharp rise it looks like this. It is going up a little bit more slowly. What kind of problem would that create? The problem that it creates is the following. Let me play around with this graph a little bit more. What I am going to do is just take this output here, the C output and line it up against the A output. And so I am going to line up the C wave form on top of the A wave form.

So you can see for yourself if something really, really strange and nasty is happening, I am just going to move up the C wave form and line it up.

What is happening out there? If you look carefully, what you observe is that the C wave form transitions just ever so slightly later than the A wave form.

Look here. And I claim that it is because of this. Because of this, the C wave form falls just a little bit later, and that little thing we see out there is a delay.

So nothing you have learned so far prepares you for this.

Suddenly, instead of the output exactly following the input, my output is following the input but a little bit later.

And it is this fact of life that things happen a little bit later, is really the reason why each of you and all of us needs to buy new computers every couple of years.

This simple basic fact. If this fact of life didn't exist, you would buy one computer and be done with it for life. Intel would make gobs of money one year, and so would Dell and Gateway and so on, and then no more. That's it.

This is it. But because of this a little itty-bitty difference here the entire semiconductor technology is charging along trying to do something about that.

You buy newer and newer computers each year.

It turns out this little itty-bitty thing here, that is called the delay, the inverter delay.

And it happens because of a specific element that has been introduced here that we have not shown you so far.

And a large part of the semiconductor industry and follow-on courses and design and so on focuses on how could I make my delay smaller, how can I get to be faster and faster and faster? This relates to how fast we can clock your Pentium IV. Remember it came all the way to 1.3 gigahertz? What's the fasted Pentium money can buy today? What is the fastest P4?

Oh, 3.2 have come out? I don't know.

Ken claims 3.2. But, yeah, there you go, 3.2 gigahertz. It all has to do with this little itty-bitty thing. You saw it for the first time here. When some of you become CTOs at Intel and so on, just remember that it all began on October 16th with this little rinky-dink thing here.

What you are going to learn now is some really cool stuff that has huge implications for life. So why does that happen?

Why did this transition happen just a little bit later?

The reason is that remember when this wave form reaches VT, the threshold voltage of this MOSFET, this guy is going to switch, right? So because of the slower rise of the voltage, the VT is going to be reached a small amount of time later. So I am going to hit VT slightly later. And because of that this guy is going to transition just a bit later because this intermediate wave form B is slower. It hits VT just a little bit later than if it would have made an instantaneous transition.

And therefore my output falls just a little bit later and this gives rise to my delay in the inverter.

We can call that d if you would like, some delay.

In your course notes, this material is covered in Chapters 9 and 10. That was to kind of motivate why we are going to be doing all that you we will be doing.

Don't anybody come within a foot of this even by mistake.

I mean it. It is pretty deadly stuff.

Today we will talk about the capacitor.

And in the next couple of lectures I am going to tie it all together and show you how this relates to that.

I will show you exactly how the delay happens.

You can compute it based on some simple principles that you will learn about in the next couple of lectures.

What I am going to do is first of all show you, I claim that that delay happens because of the presence of a capacitor somewhere in there. What I will do now is take you into a closer look, take a closer look at the MOSFET and show you were the capacitor is.

This is the MOSFET that you have seen so far, drain, gate and source. This is called an n-channel MOSFET. And what I am going to do is dissect this and show you what is actually happening, what this looks like on silicon.

So here is my slab of silicon. It is very thin.

And let's say this is, I won't go into details here.

You will learn a lot more about this in future device classes like 301 and so on, but suffice it to say I will just introduce it here to give you a sense of where the capacitor is. This is p-type silicon.

And the way you build a MOSFET is you create a couple of tubs in which you dope to be n-type. The basic silicon is dope p-type. And this guy here is n-type.

And what you do is a thin oxide layer is placed on top of that and then on top of that a thin metal layer.

This is a metal layer. This is a thin piece of oxide, silicon dioxide. And this is my P substrate.

Now this is a little metal layer that is really a wire on top of the silicone. This metal layer could be some sort of a wire that meanders around on the surface of silicone. And this is a wire that connects to the gate. This is the gate of my MOSFET.

And this guy here is the drain. And this guy here is the source. And this is my gate.

So there is a little piece of metal here.

This is this piece of metal here.

And there is a piece of oxide and then my silicone substrate.

Notice that this is my oxide. When I apply a positive voltage to the gate here with respect to the substrate, what happens is that I draw up negative charges.

I draw up electrons here into this channel region and I have corresponding plus type out here so that I get a view here that looks like a couple of plates. And I end up with an oxide in the middle. There is no connection.

Two plates separated by a small distance with plus q and minus q on the plates. And, because of that, what ends up happening here is that this piece behaves like a capacitor. So a capacitor has two plates with a thin insulating material in the middle with some permittivity epsilon. And so I get a little piece of a capacitor here. That is the capacitor that is forming. I did not set out to build that capacitor, but there is a capacitor nonetheless.

So when I apply a positive voltage at the gate, negative electrons are pulled up here which forms a channel, and then a current can then flow.

And that is how the MOSFET turns on.

So n-type electrons back to n-type, and I get electron flow here and that gives me my channel.

This is just kind of devices in four minutes or less.

You will do an entire course on this, if you like, if you take 301. What we do is to be able to capture the behavior that we just saw, the funny delayed behavior, we have to augment our model.

We have to introduce a new element.

So what we do is here is a MOSFET, gate, drain and source. And notice here we model this by putting a little capacitor, CGS between our gate and the source. So this becomes a simple model for our MOSFET device which is the good old gate drain source device from the past with a little capacitor CGS having some value for CGS in maybe ten to the minus 14 or thereabouts farads. So that is a little capacitor that has come about in this device that we fabricated here.

It is that capacitor that is at between node B and ground because it is between the gate and the source of the second inverter. And it is that capacitor that is playing the games that we saw out there.

So let's look at some of the behavior of an ideal linear capacitor. A capacitor, as I said, has a couple of plates.

There are a couple of plates. Between the plates is some dieletric, permittivity epsilon. Let's say the area of the plates is A, and let's say the plates are separated by a distance D. I get some charge here, let's say q. So q and minus q on the capacitor. And the capacitance C is given by epsilon A divided by D. Epsilon, as I said, is the productivity of the dielectric.

So if it is free space then it would be epsilon zero which is the permittivity of free space. That is the capacitance in farads. And the symbol looks like this.

Capacitor C. Voltage v.

Current i. So this, much like the resistor, voltage source and so on, this now becomes a primitive element in your tool chest of elements like the voltage source and so onn. Capacitance with the voltage v across it and a current i. And I have assigned the associated variables here according to the associated variable discipline. A question to ask ourselves is remember we said we are all now in a playground from all of nature, in this playground where the lumped matter discipline holds? And also remember that we said that for the lumped matter discipline to hold we have to make a couple of assumptions. One of those assumptions was that dq/dt, for all their elements should be zero for all time. So right now what about the capacitor? It has got some charge q.

So charge must have built up somehow.

Does that mean that I lied all along, that we are no longer in this playground, that we have been ejected from the playground because of the capacitor, or are we still in the circuits playground in which the lumped matter discipline holds and all good things happen and so on?

It seems like a contradiction, doesn't it?

I took you from Maxwell's playgrounds to the EECS playground where I said the lumped matter discipline holds.

And one of the foundations of the LMD was that dq/dt should be zero for all time inside the elements that we are going to deal with. And right now boom, it's not four weeks into the course and Agarwal introduces an element and it has q in it. It turns out that the capacitor also adheres to the lumped matter discipline.

Remember the discipline says that dq/dt is zero for all time within elements. So I am going to be clever.

What I am going to do is I want to choose element boundaries in a very cleaver way. Notice that if I have q here on this plate then I get minus q on the other plate.

So if I take the whole element, the element as a whole, if I am careful in terms of how I package my boundaries, if I put both my plates inside my element boundary then I still do get the net charge being zero.

So dq/dt is indeed zero for all time provided I make sure that my element has both the plates. Therefore, if you come across somebody else that gives you an element that says I have an idea. Let's create a new branch of electrical engineering in which we model the capacitor not as one element for two plates, but let's build a capacitor by combining two new elements, two garbage elements called G1 and G2. G1 is like the top plate.

G2 is the bottom plate. I put them together and I get a capacitor. But notice if I just pick one plate then the element G1 will not adhere to the LMD.

It adheres to the LMD because I choose my element boundaries in a way that both plates come within it.

So it is very fundamental and key.

And you can read a lot more about it in the course notes.

I purposely dwelt on that simple point because I think it is foundational and important. And you really need to understand that the capacitor does satisfy LMD.

We are still in the good old playground.

A few simple facts here. These are in the notes.

And you have also seen this before, I am sure.

I can relate the charge to the capacitance and the voltage as q is equal to Cv. And q is in coulombs, this is in farads and this is in volts.

So there is some charge q stored on the capacitor and it is in coulombs and q is equal to Cv.

So I can differentiate this with respect to time to get the current, and that becomes $i=dq/dt$.

So the current at any given time is dq/dt.

And so I substitute for q in terms of Cv here.

That is what I get. So the current $i=d(Cv)/dt$.

A 6.002 assumption, capacitance in general can be time-varying. I can get time-varying capacitors. In fact, there are some sensors which are capacitive. And, as I talk, my sound waves can change the pressure on the top plate of the capacitor. And move the top plate of the capacitor, thereby changing the capacitance by moving the plate.

Remember d here, as the plate moves closer I get a higher capacitance. So we won't be dealing, unless explicitly said so, with time-varying capacitances.

So what we can do is 6.002 allows us to write Cdv/dt.

So my current source capacitor is Cdv/dt.

I can also write down the energy, capacitors store energy.

$E=1/2Cv^2$. I am sure you have seen all this before in physics and so on.

That is the amount of energy stored in the capacitor if it is holding a charge q. Let me do a little demonstration for you. They don't make glasses like they used to. Our friend Lorenzo has charged up this capacitor. It is a huge capacitor.

It is a 250 volt capacitor so it is nasty.

He has charged it up and has kept it there.

And to show you that it does contain stored charges it has been sitting there holding charge.

Maybe the first row should go backwards, just step back for a second. I think you guys would be safe but I just don't want to take any chances.

This is holding a bunch of charge.

It is kind of sitting there. If I short the terminals it should try to say oh, I've got a path, let me get my charge out. All right.

Let's do it. This is always a scary moment for me. And I say a little prayer before I do this.

Good? OK. Gee, you guys would love to see me getting fried, huh? All right.

Let's see.

So it did contain charge.

So there is a reason why Lorenzo puts one hand inside his pocket when he shorts it, because there is a natural tendency to hold the wire with both hands, and la, la, la, la, la and put it across the capacitor.

By doing this you are guaranteed that you will just be touching it with one hand. Hopefully you folks will remember for life that a capacitor can sit around and hold its charge for a while. All right.

That is enough of fun and games.

Let's get on with our business of building circuits.

What I am going to do is, as I promised you, I am going to close the loop on that example by halfway through the next lecture. I'm going take you on a bit of a journey involving capacitors and resistors and involving some analysis, and then we will close it all up for you at about the middle of next lecture. What I would like to do next is here is a new element. And let's do some fun stuff with elements. Well, you know about voltage sources, you know about resistors, let's put them together and see how they behave.

Let's have a capacitor here, C, vc(t) and some current i.

What I am going to do, in general, whenever I have something new or something strange, let's say like a capacitor or some other device. It is interesting to model the rest of the circuit behind it if it contains only resistors and voltages and linear elements as a Thevenin equivalent.

So let me do that. This is R and this is vi.

This stuff in the back is my standard pattern, voltage source in series with a resistor, and I connect that across my capacitor. But remember, although you saw those funny wave forms and so on, the capacitor is a linear device.

Because you can see from here that the current relates to dv/dt. That is a linear operation.

You don't see V squareds and Vis and things like that in there. It's is a linear device.

Let's go back to our trusty old method, the node method.

If you just blindly apply the node method and simply grunge through a bunch of math, you should be able to get to the answer, that is for some voltage v or some form of voltage vi, I should be able to figure out what vc looks like.

So let's do that. This is the node that is of interest here with the unknown node voltage vc.

So let me apply the node method.

(vc-vi)/R is the current going this way.

That plus the current through the capacitor should equal zero.

And what is the current through the capacitor?

The node method tells me that, get the current in terms of the element values. We know that the current is given by CdvC/dt.=O. Just shuffling things around a little bit, I can write RC dvc/dt+vc=vi.

We are writing the node equation and then getting the equation that characterizes this little circuit.

Notice here that this has units of volts.

And since I have time here, this also must have units of time.

Let's go about solving this little circuit and understanding how it behaves. The specific example that we will look at looks like this. Let's say the capacitor voltage at time T=0 is V0. This is given.

So at time T=0, I am telling you that the capacitor contains a charge. And because of that there is a voltage V0 across it. That capacitor had a voltage of 250 volts across it and most of the devices we deal with in laptops and so on today, like the Pentium IV, voltages are on the order of 1.5 volts, very small voltages.

So that is the value in the capacitor, the voltage.

That is called a state. This is called the state, capacitor state. It is the state of the capacitor. And I also give you that vi(t)=VI. So my voltage is VI.

And somehow, I am not telling you how, but some how it arranged to have the capacitor voltage be V0 at time T=0. Now I want to look to the solution to this for t greater than or equal to zero.

And in that time my voltage vi is at some capital VI, some DC voltage VI. So I am going to solve the differential equation RC dvc/dt+vc=vi given these two values. Input is DC voltage VI and VC0 is V0, the initial charge in the capacitor.

So from now until almost to the end of the lecture, it is just going to be math by solving this very simple first order differential equation. And the key here will be that throughout 6.002 we will be following one method to solve these. There are many methods to solving differential equations, and we will follow one method.

That method is called the method of homogenous and particular solutions. In 1802, I believe, you would have learned maybe this, and certainly other methods. You can use any method to solve it. We will just stick to one method. And this is also used in the course notes. In this method what we do is take the solution VC by finding two other components.

One is called the homogenous solution.

And summing that up with the particular solution.

And that is the total solution. So total solution is the sum of the homogenous and the particular solutions.

And the method has three steps. As I said before, we will be using this method again and again with every differential equation that we encounter in this course.

And you won't encounter a while lot.

The first step we find the particular solution.

The second step, find the homogenous solution.

The total solution is the sum of the two.

And then find --- There will be some unknown constants depending on the equation that you have. And in the end we simply find the unknown constants by applying the initial conditions that we have. Boom, boom, boom.

Particular. Homogenous.

Find constants. Three things.

So let's go about solving this equation and apply those three conditions. Again, remember, what I am doing now for the next 10 minutes or 15 minutes is using math that you know about to simply solve this first order of differential equations. There is nothing really new that I am going to talk about here.

One is to find the particular solution vCP, which will then be added into the vCH to get me the solution.

So the way you find the vCP is you find any solution that satisfies this equation. This is the equation.

You find any solution that satisfies it.

And find the simplest possible solution that money can buy.

Find it. That's the particular solution.

Any solution is fine. In this case, a really simple one would be vCP equals VI.

Let's see if a constant works. One thing you will realize in differential equations is that they are actually much simpler than they seem. And the reason is that almost every time you have to assume you know the answer, and then you are checking to see what you assumed was correct. Assume the answer is this like you are really smart, and then check it out and say oh, yeah, that must have been the answer.

So here we assume that I think VI is going to work so let's try it out. Substituting in here.

RC dvc/dt is 0. vi is a constant.

So I get vi equals vi, so therefore this is a particular solution. Done.

I substitute vi here. So dvi/dt=0.

This vanishes and vi=VI. Bingo.

Therefore, VI is a solution to this equation.

So I am done with my vCP.

And in general what you have to do is use trial and error.

By trial and error try out a bunch of solutions until you get lucky. In general, again, in all of 6.002 for many of the excitations a simple constant usually suffices. Our second step is to find the homogenous solution. And we can also do that very quickly. And to do that we have to find a general solution to the homogenous equation.

The homogenous equation is the same differential equation but with the drive set to zero.

We want to follow a set pattern to solve the differential equations here, and the set pattern is find vCP, vCH, find constants. And to find vCH we are also going to follow a set pattern to find the homogenous solution.

So we set the drive to zero, so vi is set to be zero.

And I need to find a general solution to this.

As I promised earlier, diff equations are really, really simple because the way we are going to solve them is we are going to assume we know the answer and then go check it.

So let's try Ae^st. Let's try and see if this can solve this particular equation for some values of A and S.

I am telling you that the solution is going to be of this form. Assume it.

And then simply go ahead and find me A and S, and do that by substituting it back into the equation and find out the corresponding As and Ss. So let's go ahead and do that.

I get RC. I substitute this back up so I get dAe^(st)/dt+Ae^st=0. And let me plug that in and see what comes. I get RCAse^st+Ae^st=0.

I want to discard the trivial solution of A being 0.

That is a trivial solution so I will discard that.

And what I will do is cancel out the As from here, assuming A is not zero, and cancel e^st here.

And what is left is RCs+1=0. What this is saying is that if I can find an S such that this is true then Aest is a general solution to my homogenous equation.

This is easy enough. And so S=-1/RC.

If I choose my S to be -1/RC then the simple math that I have gone through shows me that this must be the solution to the homogenous equation. Or in other words vCH=Ae^(-t/RC). All this is saying is that Ae^(-t/RC) is a solution to my homogenous equation.

A is an unknown constant. A is some constant.

I don't know what that is yet. Notice RC has popped up again.

And the cool thing about RC is that, this is time, this also has units of time. We commonly represent RC as some time constant tau, as units of time.

Associated with that circuit is the time constant tau, which is simply RC. I commonly write this as Ae^(-t/tau).

I am very the end here. I have the particular solution here. I have got the homogenous solution there. I need to tell you about something else. The way I found the homogenous solution was in four steps. I assumed a solution of the form Ae^st. I created this equation here in S. This is called the characteristic equation for that circuit.

We will see this time and time again for RC and other forms of circuits. Assume a solution of this form.

Construct the characteristic equation.

Find the roots of the characteristic equation.

In this case it is an equation in S.

So this is the root. And then form the solution based on that root. Four steps.

Ae^st, characteristic equation, root and then write down the general homogenous solution. Four steps there.

And finally I want to write down the total solution.

And the total solution is simply vCP+vCH.

And vCP was VI and vCH was Ae^(-t/tau).

tau was simply RC. That is my solution.

Now, remember the last step. The last step was form the total solution and find out the remaining constants.

Find out the remaining constants by using my initial conditions. At t=0, I know that vC=V0.

I know that. And so therefore I can substitute t=0 to find the constant.

So I know that VO=VI+A. t=0, this thing becomes 1, and so I get this equation from which I get A=V0-Vi.

In other words, my solution vC is simply VI+(VO-VI) e^(-t/tau). So the last 15 minutes have just been math. No electrical engineering here, but electrical engineering stopped at the point where you wrote this differential equation down, went through a bunch of math and came up with a solution.

Purely mathematically. So here I simply used math to get you the solution. And, as I have been promising you throughout this course, in the next lecture I will give you an intuitive EE method of doing it.

Real electrical engineers, real EECS folks don't do it this way. Real EECS folks do it intuitively. And I will show you how to do it in four easy seconds in the next lecture.

But you need to understand the foundations of how this comes about, and so this is the answer.

You can also get the current iC is simply Cdvc/dt.

I won't do that for you, but you can simply differentiate it and get the current.

So I can plot for you vC, time t, vC.

The intuitive way of looking at this is I have VI which is the final value of the voltage. When t is infinity this part goes to zero so the vC is simply VI.

And then there is a component V0-VI which decays according to this starting out at an initial value of V0.

Notice when t is zero vC is V0, you can see that in the equation, and so it starts out at V0 and ends up at VI.

I start here, I end up here.

And this portion V0-VI decays out over time like this.

And this decay is governed by the RC time constant or tau.

I am going to show you very quickly a couple of examples of wave forms, one that goes like this and one that looks like this. This is when I start with some value V0 and I don't apply any input, it should decay down to zero, t, t, vC, vC.

If I apply zero for VI then this should simply decay down to nothing over time. And if I apply some VI but there is no state in the capacitor then that same equation is going to look like this.

You can go and confirm for yourselves that when I apply some input but the capacitor has zero state, I start at zero, I finish up at VI and my wave form looks like this.

There you go. That's the first one.

The second one where I have 5 volts on the capacitor and no input. Assume that at time equals zero I take away an input, short the input voltage to ground for example, apply zero volts.

You will see the decay from 5 volts to 0 volts.

And in the first case I start with zero volts in my capacitor, I apply input of 5 volts, and notice that at t=0 the capacitor rises up to that level.

So notice that these circuits with capacitor and resistors are typified by wave forms that are exponential rises and exponential decays. We will see more of that next time.