MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.007 – Electromagnetic Energy: From Motors to Lasers
Spring 2011

Problem Set 1: Energy Conversion
Due Wednesday,February 9, 2011

Problem 1.1 – Energy Conversion Between Majors

This problem develops your understanding of $Power = Through \times Across$.

List a few general devices that convert power between "majors," meaning from one type of power (e.g., mechanical) to another (e.g., electrical), and for at least one device, discuss briefly how the *Through* and *Across* variables of each type of power relate. Bonus points will be awarded for coming up with a device not mentioned by anyone else in the class. Set up a table like the one below. Use a DC motor as one of your devices.

Device	Input		Output	
	Through	Across	Through	Across
DC Motor				

Problem 1.2 – Energy Stored in a Toyota Prius

These are the specs for a typical Toyota Prius car and its battery

Output Voltage: 201.6 V
Capacity: 1310 Wh
Mass of Battery: 45 kg
Mass of Car (with battery): 1328 kg

(i) How many joules of energy are stored in the battery at full charge?

(ii) How high would your Prius rise if the battery's energy were used to propel it skyward?

(iii) How fast would the Prius be traveling when it reached the ground if launched in a vacuum? Why would it not reach this speed in air? Where is the potential energy going?

(iv) Batteries such as the one above make use of a chemical reaction involving nickel ions. Each of these ions account for roughly two electrons of current. Given the rated capacity of the battery above, what is the mass of nickel ions in this battery?

Molar mass of nickel: 58.71 $\frac{\text{g}}{\text{mol}}$
Electron charge: 1.60×10^{-19} C
Avogadro's number: 6.022×10^{23}

What percent of the battery's mass consists of the energy-storing ions? This should be shockingly small! The rest of the battery's mass is taken up by the anode and cathode that participate in the chemical reaction as well as safety devices and packaging.

(v) The hybrid Prius, employing both electric motor and gasoline combustion, has a top speed of 96 mph when the engine is producing 73 kW of power. Assume that all of the frictional and drag effects can be accounted for as a single friction force which is proportional to the square of velocity, v, so $F_{fric} = \mu v^2$. Calculate the effective friction constant μ. Note that 1 mph = 0.447 $\frac{\text{m}}{\text{s}}$

(vi) Assuming the same friction constant μ, how much power is needed to go at 50 mph?

(vii) Assume the engine has an efficiency of 27%, and that gasoline has an energy content of about 132 MJ/gallon. What gas mileage (miles/gallon) would you expect at a speed of 96 MPH? At 50 MPH?

Problem 1.3 – EMALS

The Electromagnetic Aircraft Launch System is a linear motor used to accelerate an aircraft to flying speed. The track is 100 meters long and the aircraft has a mass of 20 metric tons. Flying speed is 100 meters/second (about 200 nautical miles per hour).

(i) Assuming that the rate of acceleration is constant, what must that acceleration be to get the airplane to flying speed at the end of the track?

(ii) What force is required? Ignore all losses.

(iii) Now, assume there is a frictional force that is linearly proportional to the velocity. Still assuming the motor provides a force that is constant, write an equation of motion in terms of the time-dependent velocity $v(t)$.

A review of solving differential equations follows in Problem 1.5.

Problem 1.4 – Solar energy

This problem can be answered by performing information search on line.

(i) How much energy does the sun shine during one hour over an area of 1m^2 of the continental (48 contiguous) United States at midday? Please express your answer in units of kW-h.

(ii) You are given 15% efficient solar cells in order to convert solar energy into electricity. If the electricity will be used to power the electrical needs of the United States, how large would the area of your solar installation need to be (what percent of US land)? Compare this area to the area occupied by cities and roads in US. (Note that 40% of US land area is used for agriculture.)

Problem 1.5 – Math Review

This problem reviews the math you will encounter in 6.007.

Electromagnetics and quantum mechanics make extensive use of trigonometry, matrices, complex numbers, vector calculus, and differential equations. In completing the brief problems below, you should review your notes from 18.02 and 18.03 to prepare for applying your mathematical prowess in these areas later in class.

(i) *Trigonometry and complex numbers.* Geometric optics makes considerable use of trigonometry in calculating the paths of electromagnetic waves through space. Additionally, phasors (which we will cover in more detail later) are used to simplify linear differential equations involving waves by using Euler's formula.

(a) Simplify $\sqrt{2} - e^{-j\frac{\pi}{4}}$ to its polar form.

(*Hint*: Convert to rectangular coordinates first.)

(b) What are $\sin(x)$ and $\cos(x)$ in complex form?

(c) For $k = k_R + jk_I$ and $A = e^{-jkz}$, expand out A^* in rectangular form.

(ii) *Vector quantities and phasors.*

Wave optics is entirely vector based, and one of the representations of Maxwell's equations, as you learned in 8.02, is based on vector calculus.

(a) Recall the differential form of Faraday's law, which relates the electric field vector $\mathbf{E}$ to the magnetic field vector $\mathbf{H}$ as

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}. \tag{1}$$

If $\mathbf{E} = (\hat{x} + \hat{y})\, E_0 \sin(kz - \omega t)$, what is $\mathbf{H}$?

What if $\mathbf{E} = (\hat{x} + \hat{y})\, E_0 e^{jkz}$, where $k = k_R + jk_I$?

(b) The time-dependence of a vector $\mathbf{E}(t)$ with amplitude $\mathbf{E} = \mathbf{E}_R + j\mathbf{E}_I$ is given by $\mathbf{E}(t) = \mathrm{Re}\left[\mathbf{E}e^{-j\omega t}\right]$. Show that

$$\mathbf{E}(t) = \mathrm{Re}\left[\mathbf{E}e^{-j\omega t}\right] = \mathbf{E}_R \cos(\omega t) + \mathbf{E}_I \sin(\omega t). \quad (2)$$

Draw the motion of $\mathbf{E}(t)$ when $\mathbf{E}_R = \hat{x}A$ and $\mathbf{E}_I = \hat{y}B$ (A and B both real and positive).

(iii) *Differential equations.* First and second-order differential equations are common in both electromagnetics and quantum mechanics and will be used often in 6.007.

(a) What is the general solution to the differential equation

$$\frac{d^2 x(t)}{dt^2} + k^2 x(t) = 0? \quad (3)$$

(b) What is the particular solution when $x(0) = x(L) = 0$?

(c) For what values of k is this solution valid?

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