# *Actuators & the Coil Gun*

## Outline

Review of Electrostatic Actuators

- -Energy method for Electrostatics

Rail Gun

- -Energy method for Magnetostatics

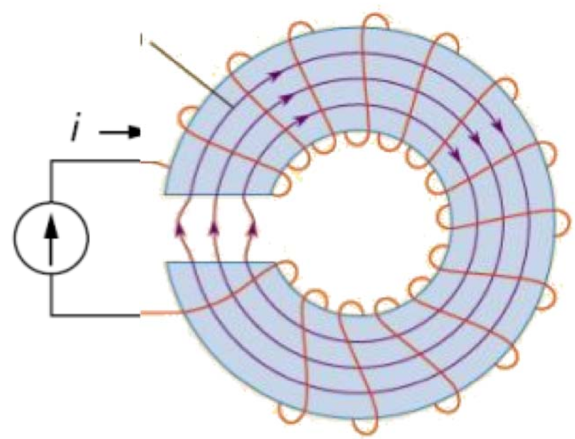
Coil Gun

$$f_x = i\frac{\partial \lambda}{\partial x}$$

# TRUE or FALSE?

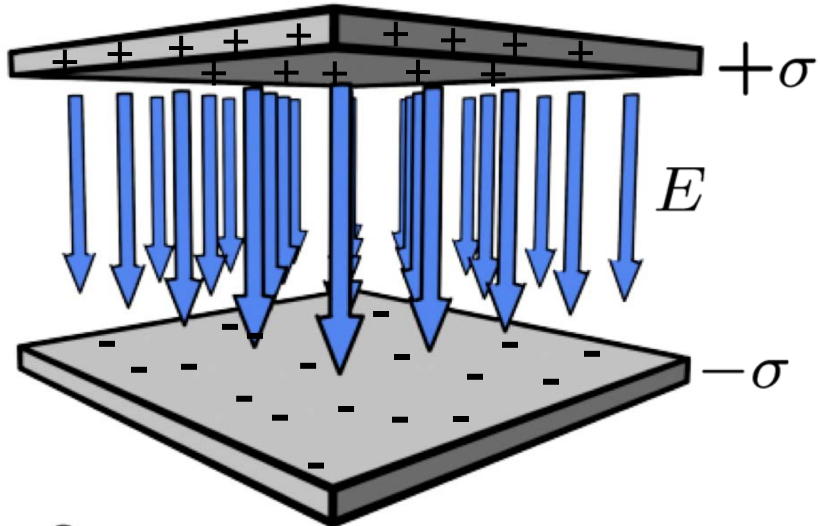
1. Since $W = \frac{1}{2} CV^2$, the force always acts to decrease capacitance to go to lower energy. ____

2. If the gap is decreased, the magnetic flux will increase. ____




3. The energy stored in the electric field is $\frac{1}{2} \varepsilon_o E^2$ and has units of Joules. ____

# *Attractive Force Between Parallel Plates*

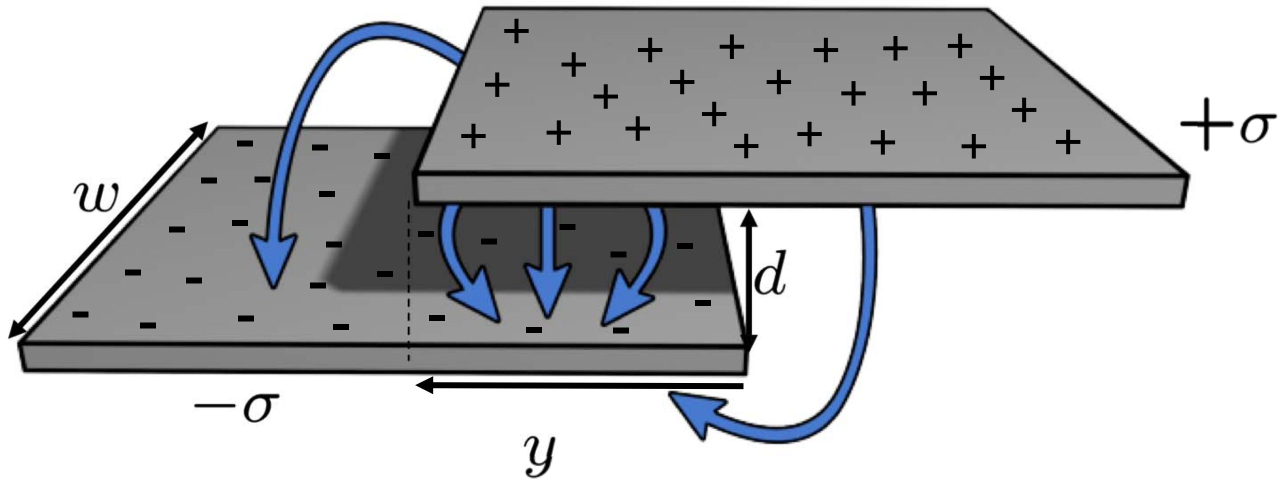



$$f = \frac{v^2}{2}\frac{dC}{dx} \qquad C(x) = \frac{\epsilon_o A}{x}$$

**CAPACITOR PLATES ARE PULLED TOWARDS EACH OTHER (FORCE ACTS TO INCREASE C)**

$f =$ [ ]

# *Linear Electrostatic Actuator*

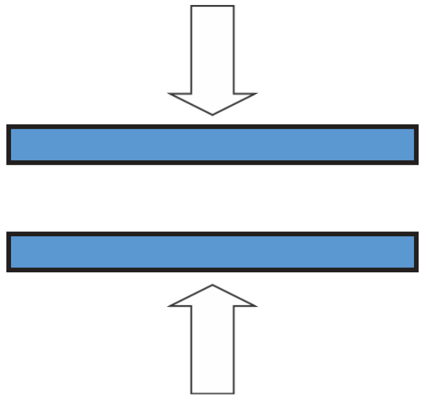


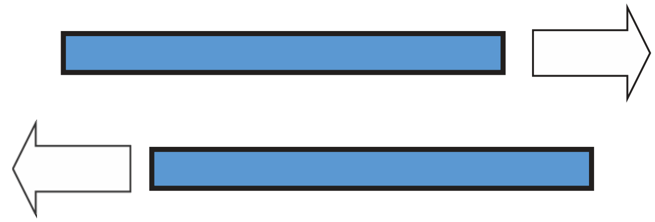

$$C(y) = \frac{\epsilon_o(yw)}{d}$$

$f =$ [ ]

**CONSTANT FORCE ALONG DIRECTION OF MOTION**
**(FORCE ACTS TO INCREASE C)**

# Gap Closing Electrostatic Actuators





$$C(x) = \frac{\epsilon_o A}{x}$$

$$f = -\frac{v^2}{2}\frac{\epsilon_o A}{x^2}$$

Moderate force

$$\sim \frac{1}{x^2}$$

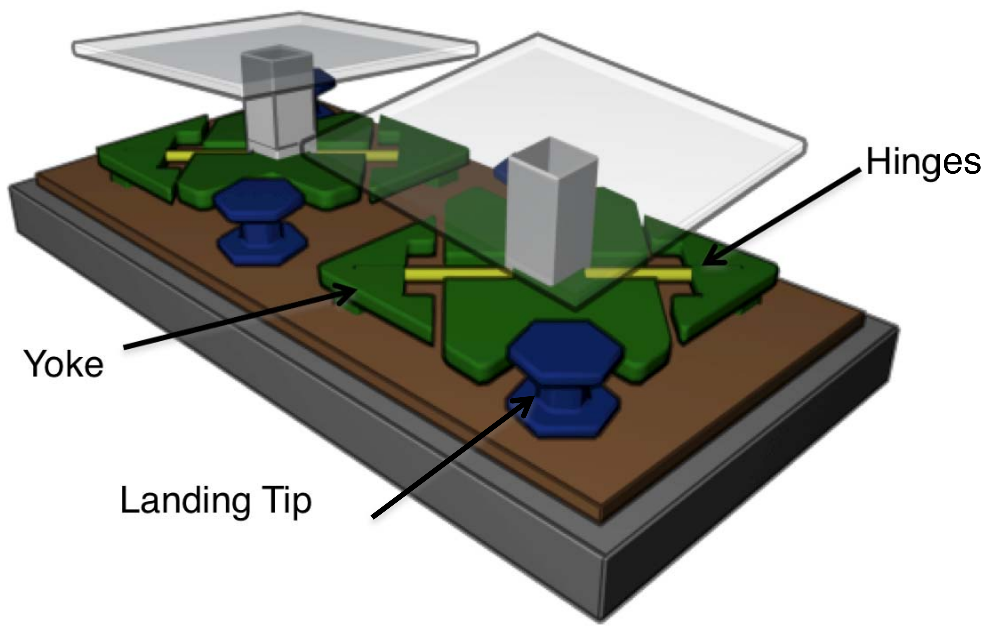
$$C(y) = \frac{\epsilon_o (yw)}{d}$$

$$f(y) = \frac{\epsilon_o}{2}\frac{Q^2}{C^2}\frac{w}{d}$$

Weak force

$$\sim \frac{1}{d}$$

# MEMS Actuator Mirror



# *Rail Gun Force ... Using Energy Conservation*

Perfectly conducting plates and slider

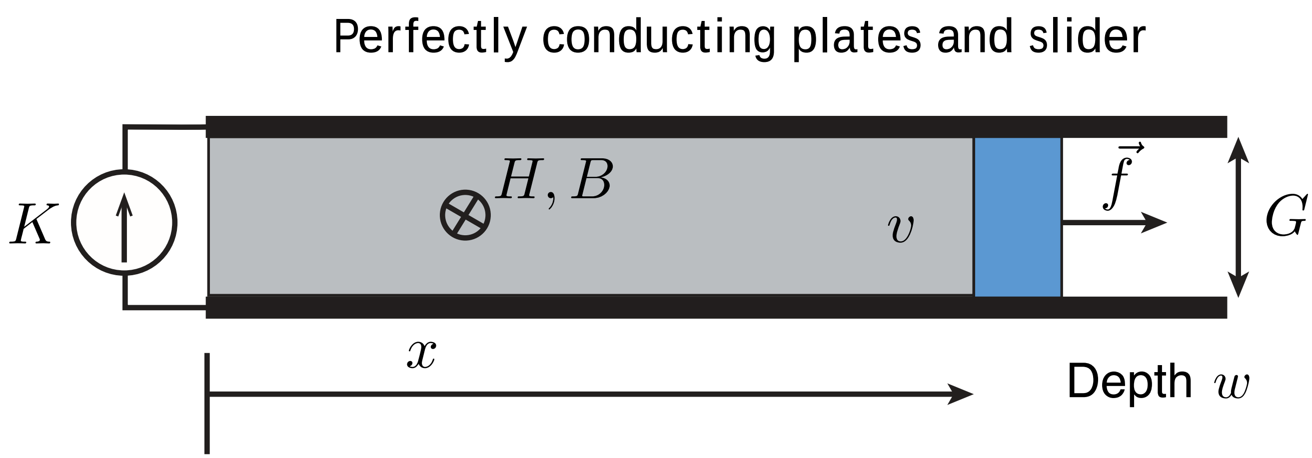

Depth $w$

$L(x) = \mu_o G x / w$ ... one-turn solenoid

$i = Kw$

$$f = \frac{1}{2} i^2 \frac{\partial L}{\partial x} = \frac{1}{2} \mu_o K^2 G w$$

# Rail Gun Force -Energy Conservation Again




$$W = \int \frac{1}{2}\mu_o H^2 dV = \int \frac{1}{2} B^2/\mu_o dV; B = \mu_o H$$

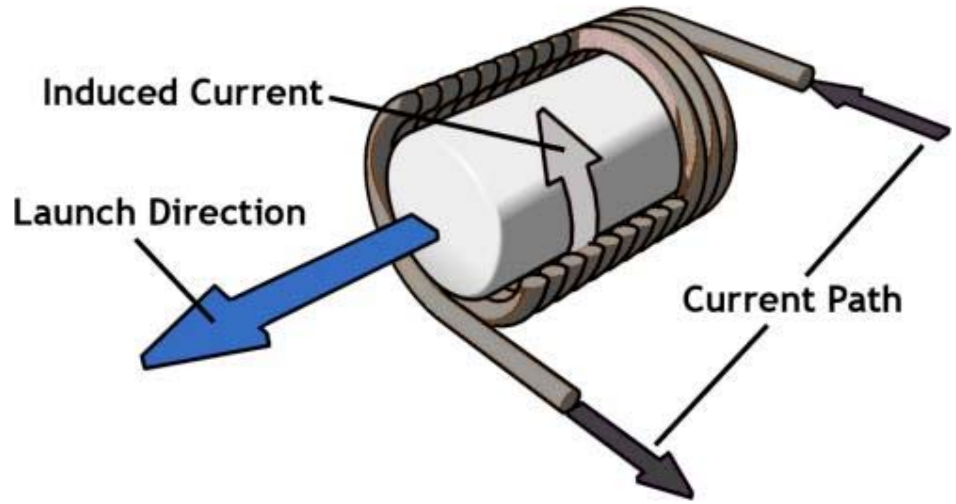
$$= \frac{1}{2}\mu_o K^2 x G w$$ (Recall that $H = K$)

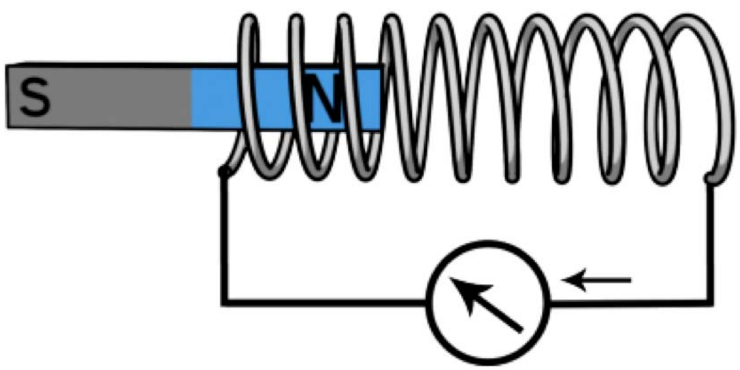
$$= \frac{1}{2}\lambda^2/(\mu_o x G)$$ ... (Keep $\lambda$ constant)

$$f = \frac{-\partial W}{\partial x} = \frac{1}{2}\mu_o K^2 G w$$

# *Coil Gun*

How can we use a solenoid to propel a "bullet" if the force is always into the solenoid?

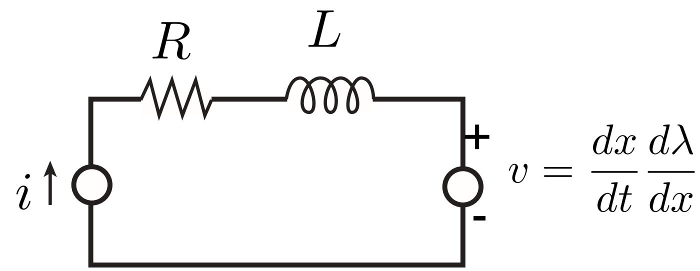



Our Plan: Two magnets repel
(or attract). We should be able to get a coil to repel a magnet !

Consider this energy model:

'Back voltage' v is due to motion



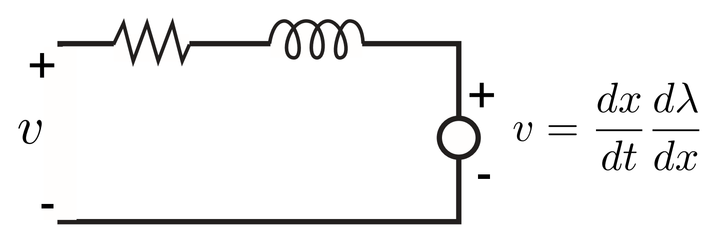

$$P_m = f_m u = f_m \frac{dx}{dt} = vi = \frac{dx}{dt} i \frac{d\lambda}{dx}$$

Then, force is simply: $$f_m = i\frac{d\lambda}{dx}$$

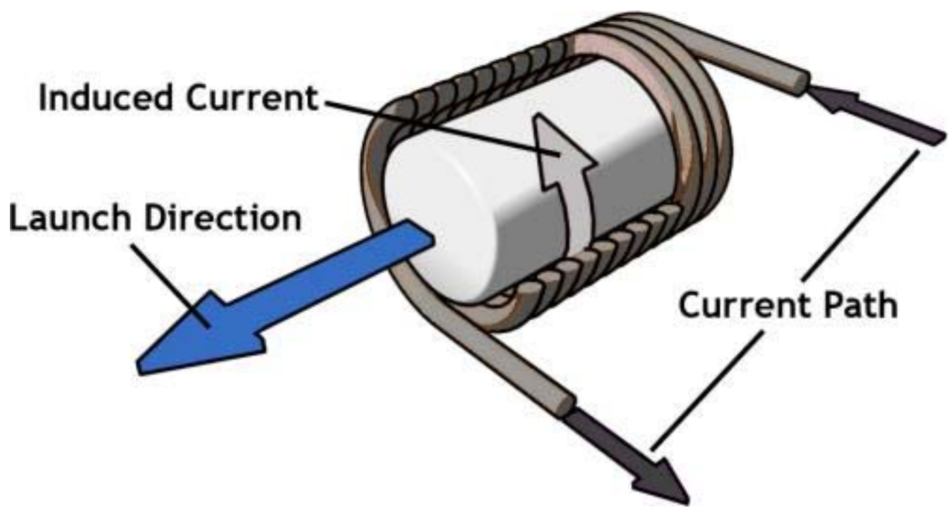
Some caution here: this is the force due to the interaction with the coil -- It doesn't mean there may not be other forces!

To find back voltage: run the system open circuited

This will give the right back voltage; If you know how fast the magnet is going, you get flux change



# *Coil with Permanent Magnet*




$$W_s = \int i(\lambda) d\lambda = \int_0^{\lambda_f} i \frac{\partial \lambda}{\partial x} dx$$

$$= \int_0^{\lambda_f} f_x dx$$

Instantaneous force on the magnet…

$$f_m = i \frac{d\lambda}{dx}$$

# Key Takeaways

Energy method for calculating Forces calculated at constant flux linkage

$$f_r = -\frac{\partial W_s}{\partial r}$$

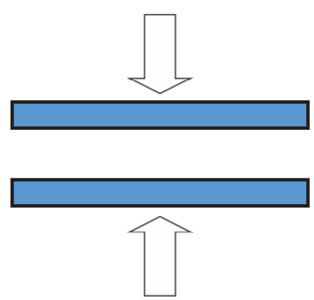
Coil Gun

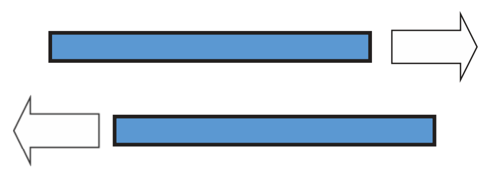
$$f_x = i\frac{\partial \lambda}{\partial x} \qquad W_{mechanical} = \int f_m dx = Mgh_{throw}$$

Force acts to increase Capacitance

$$C(x) = \frac{\epsilon_o A}{x}$$

$$f = -\frac{v^2}{2}\frac{\epsilon_o A}{x^2}$$





$$C(y) = \frac{\epsilon_o (yw)}{d}$$

$$f(y) = \frac{\epsilon_o}{2}\frac{Q^2}{C^2}\frac{w}{d}$$

Moderate force $\sim \frac{1}{x^2}$

Weak force $\sim \frac{1}{d}$

6.007 Electromagnetic Energy: From Motors to Lasers
Spring 2011