Examples of Uniform EM Plane Waves

Outline

Reminder of Wave Equation
Reminder of Relation Between E & H
Energy Transported by EM Waves (Poynting Vector)
Examples of Energy Transport by EM Waves

Coupling of Electric and Magnetic Fields

Maxwell's Equations couple H and E fields…

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\left(\int_S \vec{B} \cdot d\vec{A}\right)$$

$$\oint_C \vec{H} \cdot d\vec{l} = \oint_S \vec{J} \cdot d\vec{A} + \frac{d}{dt}\int \epsilon E \, dA$$

$$\frac{\partial E_y}{dz} = \frac{\partial B_x}{dt}$$

Source free
$J = 0$

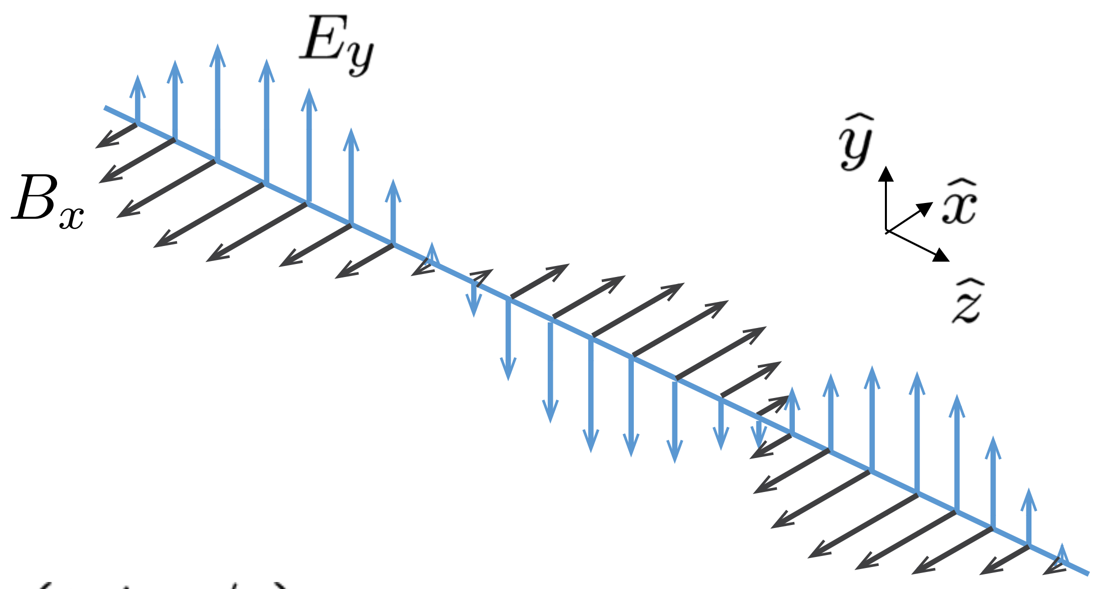
$$\frac{\partial B_x}{\partial z} = \epsilon\mu\frac{\partial E_y}{\partial t}$$

$$\frac{\partial^2 E_y}{\partial z^2} = \epsilon\mu\frac{\partial^2 E_y}{\partial t^2}$$

The Wave Equation

Magnetic Field in a Uniform Electromagnetic Plane Wave

$$\frac{\partial B_x(z_o)}{\partial z} = \epsilon_o \mu_o \frac{\partial E_y}{\partial t}$$



In free space ...

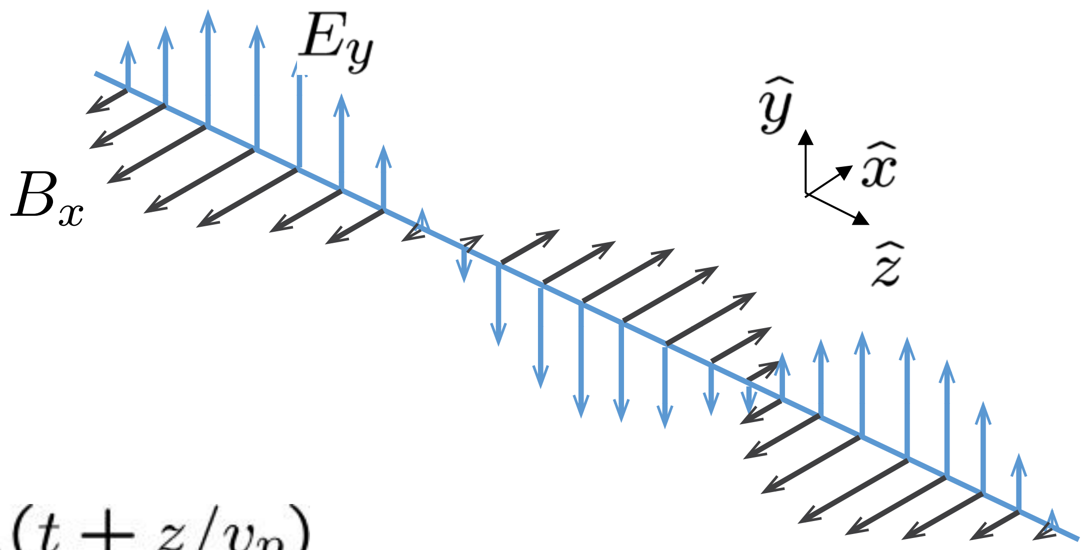
$$E_y = f_+(t - z/c) + f_-(t + z/c)$$

$$H_x = -\sqrt{\frac{\epsilon_o}{\mu_o}}\Big(f_+(t - z/c) - f_-(t + z/c)\Big)$$

... where $c = \frac{1}{\sqrt{\epsilon_o \mu_o}}$

Uniform Electromagnetic Plane Waves In Materials

$$\frac{\partial B_x(z_o)}{\partial z} = \epsilon\mu\frac{\partial E_y}{\partial t}$$



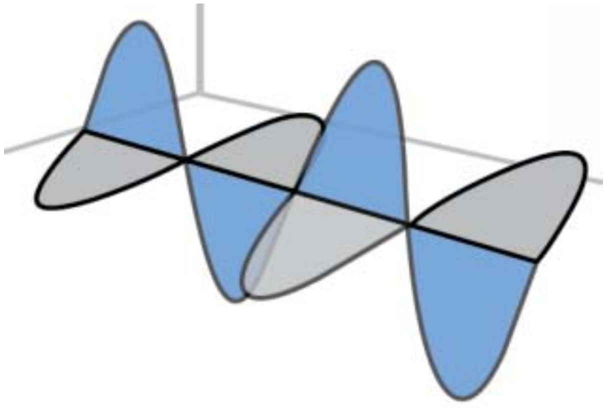
Inside a material ...

$$E_y = f_+(t - z/v_p) + f_-(t + z/v_p)$$

$$H_x = -\sqrt{\frac{\epsilon}{\mu}}\left(f_+(t - z/v_p) - f_-(t + z/v_p)\right)$$

... where $v_p = \frac{1}{\sqrt{\mu\epsilon}}$ is known as the phase velocity

The Characteristic Impedance



$$E_y = f_+(t - z/v_p) + f_-(t + z/v_p)$$

$$H_x = -\sqrt{\frac{\epsilon}{\mu}}\left(f_+(t - z/v_p) - f_-(t + z/v_p)\right)$$

$$\left[\Omega\,\frac{A}{m}\right] \quad \eta H_x = \eta H_{x+} + \eta H_{x-} = -E_{y+} + E_{y-} \quad \left[\frac{V}{m}\right]$$

- η is the *intrinsic impedance* of the medium given by

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

- Like the propagation velocity, the intrinsic impedance is independent of the source and is determined only by the properties of the medium.

$$\eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}} \approx 377 \text{ Ohms} \approx 120\,\pi \text{ Ohms}$$

Phasor Notation for Uniform Plane Waves

$$E_y = A_1 cos(\omega t - kz) + A_2 cos(\omega t + kz)$$

$$= \text{Re}\left(A_1 e^{j(\omega t - kz)}\right) + \text{Re}\left(A_2 e^{j(\omega t + kz)}\right)$$

$$H_x = -\frac{A_1}{\eta} cos(\omega t - kz) + \frac{A_2}{\eta} cos(\omega t + kz)$$

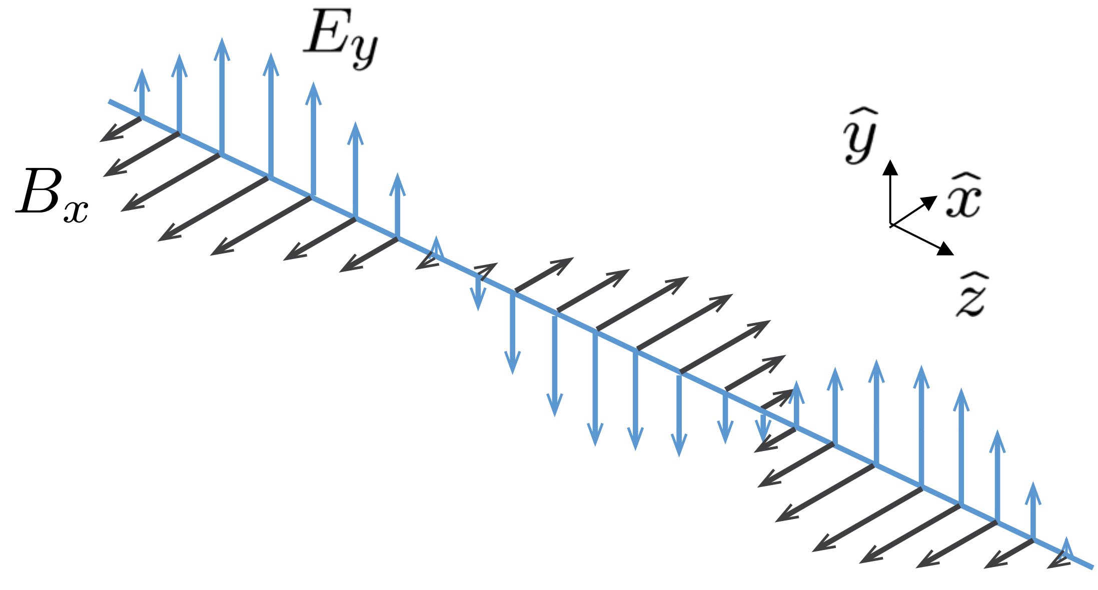
$$= \text{Re}\left(-\frac{A_1}{\eta} e^{j(\omega t - kz)}\right) + \text{Re}\left(\frac{A_2}{\eta} e^{j(\omega t + kz)}\right)$$

Imaginary numbers are noted differently in science and engineering disciplines. While scientists use I, engineers use j. The relation between the two is as follows:

$$i = -j$$

Both scientists and engineers think that their version of the imaginary number is equal to $\sqrt{-1}$ and independently, over time, they developed equations for identical physical relations that can now only be reconciled if *i* is set to be equal to *-j*

Sinusoidal Uniform Electromagnetic Plane Waves



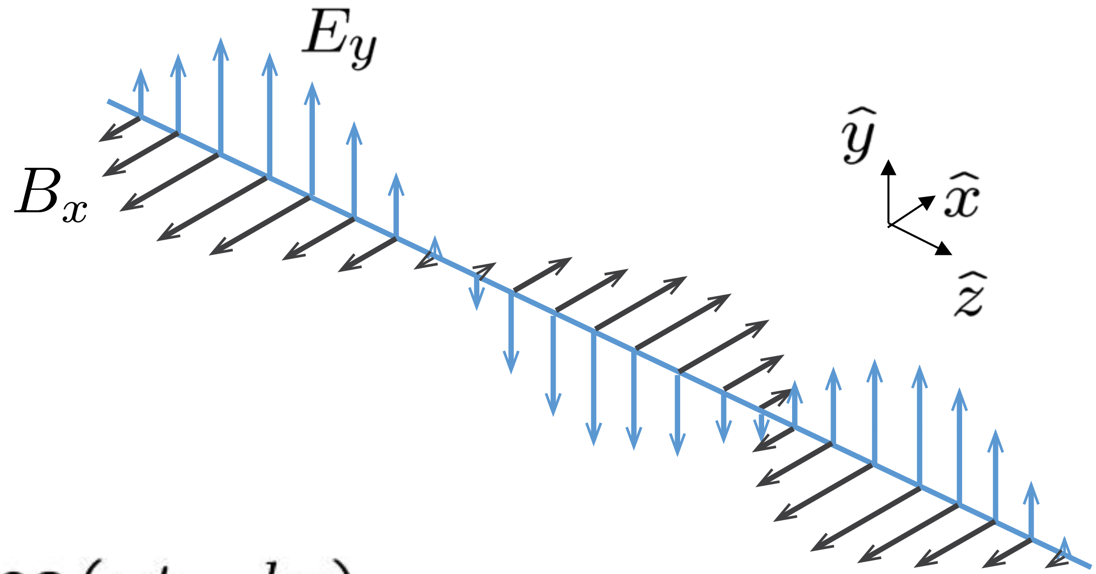
Define the wave number $k = \dfrac{\omega}{v_p}$

so that ...

$$f_+(t - z/v_p) = A\cos\left(\omega\left(t - z/v_p\right)\right) = A\cos\left(\omega t - kz\right)$$

$$f_-(t + z/v_p) = A\cos\left(\omega\left(t + z/v_p\right)\right) = A\cos\left(\omega t + kz\right)$$

Sinusoidal Uniform Electromagnetic Plane Waves



In free space ...

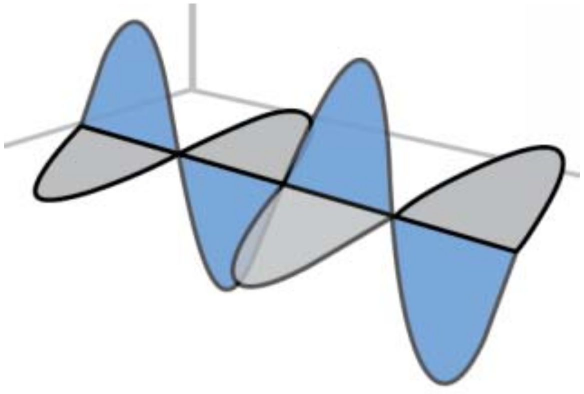
$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$f_+(t - z/c) = A\cos(\omega t - kz)$$

$$f_-(t + z/c) = A\cos(\omega t + kz)$$

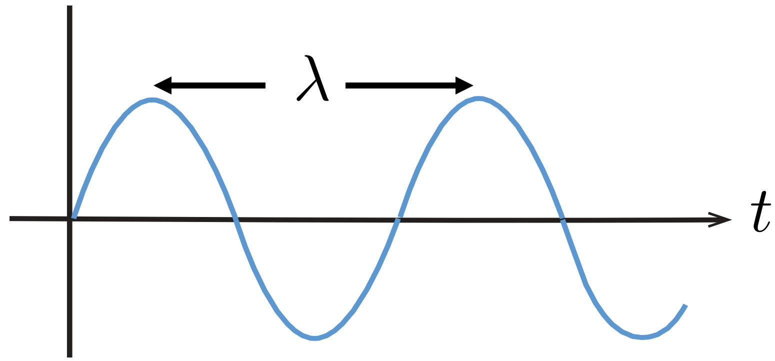
$$E_y = A_1 cos(\omega t - kz) + A_2 cos(\omega t + kz)$$

$$H_x = -\frac{A_1}{\eta} cos(\omega t - kz) + \frac{A_2}{\eta} cos(\omega t + kz)$$



Sinusoidal Uniform Plane Waves

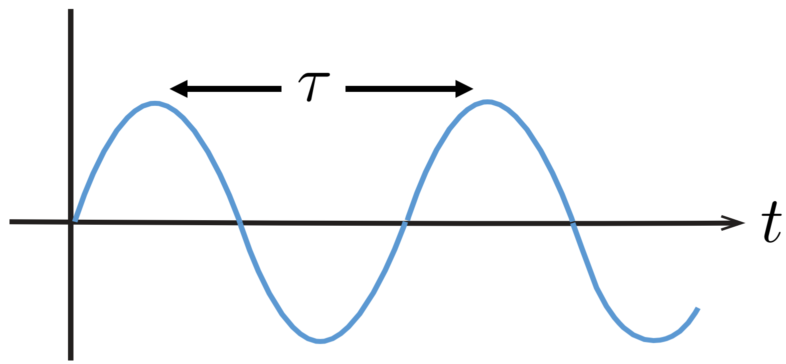
Spatial quantities:



$$k = \frac{\omega}{v_p}$$

$$k = \frac{2\pi}{\lambda}$$

Temporal quantities:



$$\omega = \frac{2\pi}{\tau}$$

$$E_y = A_1 \cos(\omega t - kz) + A_2 \cos(\omega t + kz)$$

$$H_x = -\frac{A_1}{\eta} \cos(\omega t - kz) + \frac{A_2}{\eta} \cos(\omega t + kz)$$

Energy Density of a Uniform Plane Wave

Magnetic

$$\frac{W_s}{V} = \frac{1}{2}\mu_o H \cdot H$$

Electric

$$\frac{W_s}{V} = \frac{1}{2}\epsilon_o E \cdot E$$

$$E_y = f_+(t - z/c) + f_-(t + z/c)$$

$$H_x = -\sqrt{\frac{\epsilon_o}{\mu_o}}\Big(f_+(t - z/c) - f_-(t + z/c)\Big)$$

$$\frac{W_{s_{electric}}}{V} = \frac{W_{s_{magnetic}}}{V}$$

Using less cumbersome notation →

$$\frac{W_E}{V} = \frac{W_M}{V}$$

Power Flow of a Uniform Plane Wave

$$\frac{P}{A} = c\left(\frac{W_E}{V} + \frac{W_M}{V}\right)$$

$$= c\frac{2W_M}{V}$$

$$= |\vec{E}| \cdot |\vec{H}|$$

$$\boxed{\frac{\vec{P}}{A} = \vec{E} \times \vec{H}}$$

← Poynting Vector, named after John Henry Poynting (1852-1914)

Poynting's Theorem

Ampere

$$\vec{E} \cdot (\underbrace{\nabla \times \vec{H} = \vec{J} + \frac{d}{dt}\epsilon_o \vec{E}}_{\text{Ampere}})$$

Faraday

$$\vec{H} \cdot (\underbrace{\nabla \times \vec{E} = -\frac{d}{dt}\mu_o \vec{H}}_{\text{Faraday}})$$

Σ

Vector identities
Vector manipulations

$$0 = \int_S \underbrace{\vec{E} \times \vec{H}}_{} \cdot dS + \int_V \underbrace{\vec{J} \cdot \vec{E}}_{} \, dV + \frac{d}{dt} \int_V \underbrace{(\frac{1}{2}\epsilon_o \vec{E} \cdot \vec{E} + \frac{1}{2}\mu_o \vec{H} \cdot \vec{H})}_{} \, dV$$

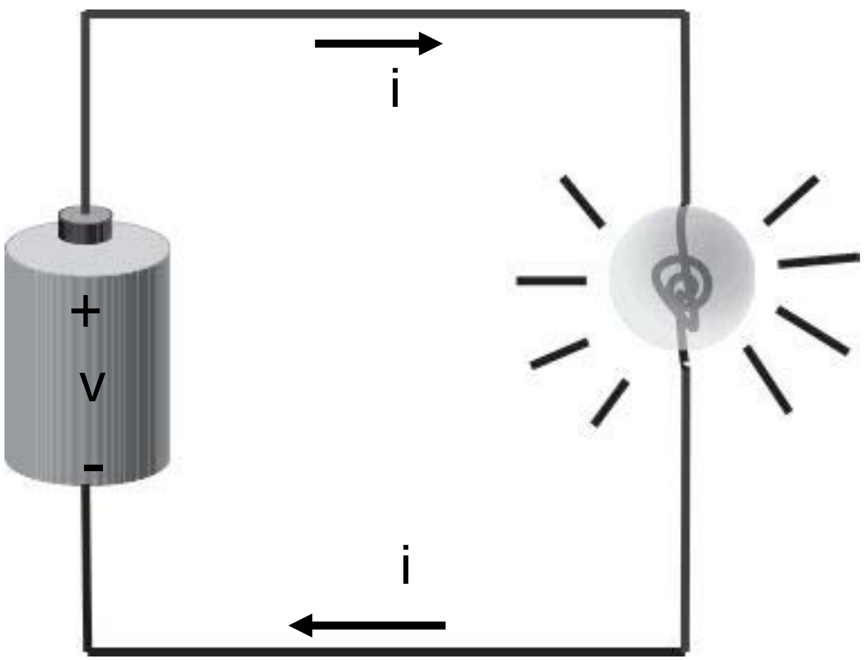
Wave transport of energy from V through S

Energy conversion within V

Energy storage within V

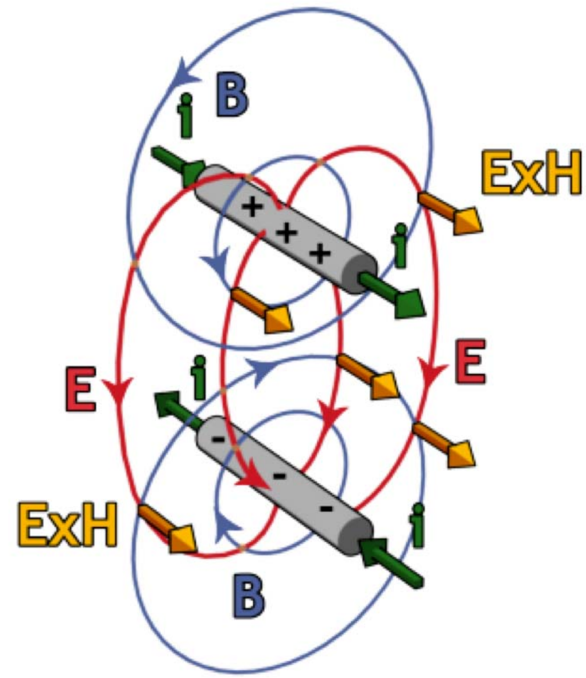
EM Power Flow

Q: How does power flow from the battery to the light bulb?



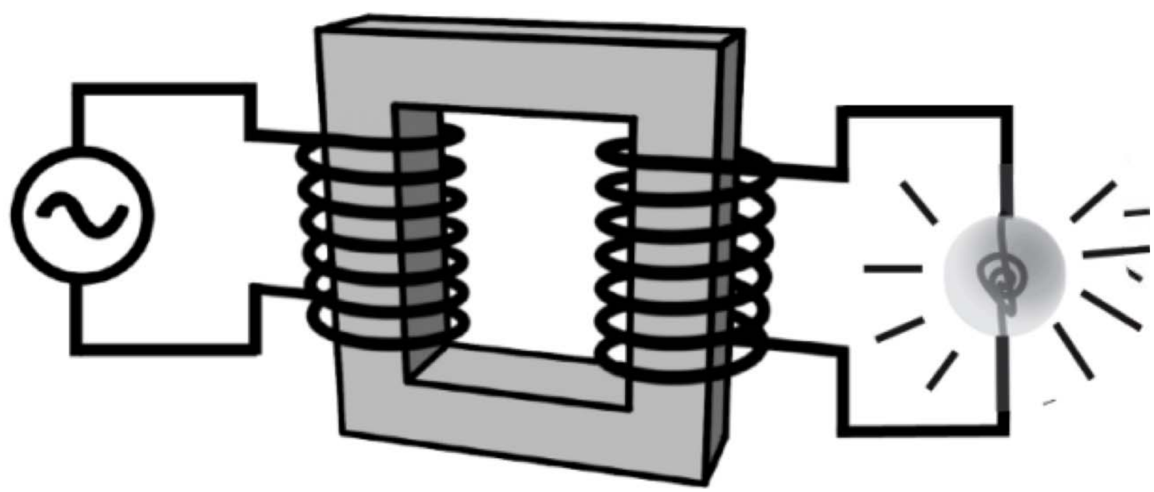
The wires serve only to guide the fields.

A: Through the EM fields, which are guided by the wires.



EM Power Flow

Poynting's Theorem also explains how electrical energy flows from the source through the transformer to the light bulb in the circuit below.



Amplitude & Intensity

How *bright* is the light?

	Amplitude, A	**Intensity, I**
Sound wave: (loudness)	**peak differential pressure p_o**	**power transmitted/area**
EM wave: (brightness)	**peak electric field E_o**	**power transmitted/area**

Power transmitted is proportional to the square of the amplitude.

$$Intensity = |\vec{E}| \cdot |\vec{H}| = \frac{E^2}{\eta} = \eta H^2$$

Superposition of EM Waves of the Same Polarization

Two E_y –polarized EM waves are incident on the same surface.
EM Wave #2 has four times the peak intensity of EM Wave #1, i.e., $I_2 = 4I_1$.

1. What is the maximum intensity, I_{max} ?

(a) $4I_1$

(b) $5I_1$

(c) $9I_1$

$$A_2 = \sqrt{I_2} = \sqrt{4I_1} = 2\sqrt{I_1} = 2A_1$$

$$I_{tot} = (A_{tot})^2 = (A_1 + A_2)^2 = (A_1 + 2A_1)^2 = 9A_1^2 = 9I_1$$

2. What is the minimum intensity, I_{min} ?

(a) 0

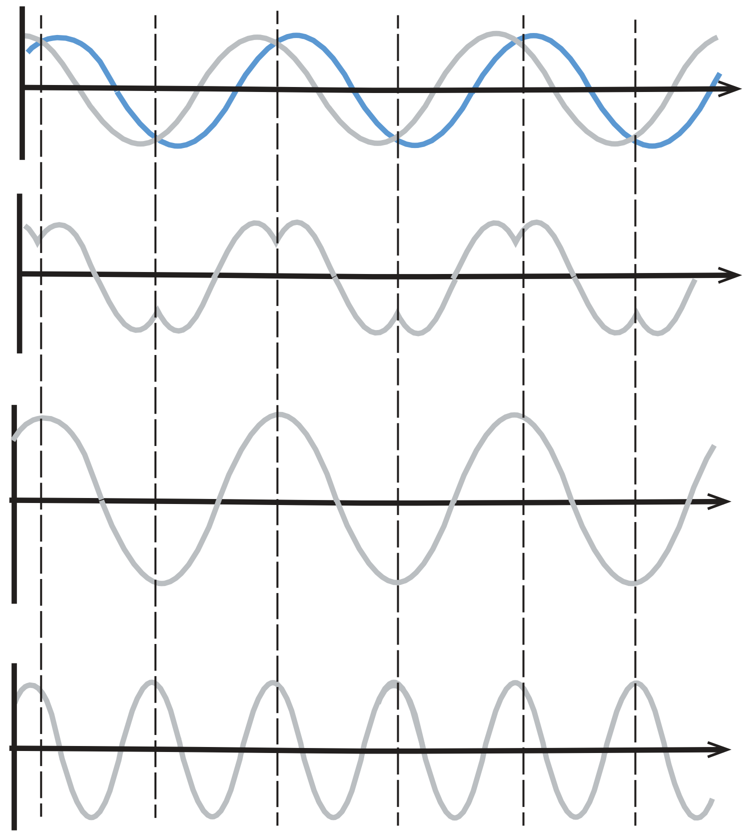
(b) I_1

(c) $3I_1$

$$I_{tot} = (A_{tot})^2 = (A_1 - A_2)^2 = (A_1 - 2A_1)^2 = A_1^2 = I_1$$

Pop - Question

If you added the two sinusoidal waves shown in the top plot, what would the result look like ?



(a)

(b)

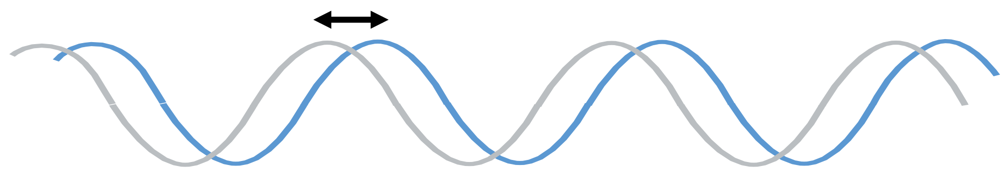
(c)

Adding Waves with Different Phases

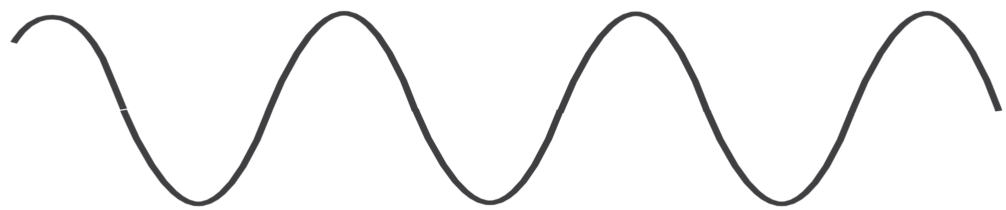
Example: Suppose we have two waves with the same amplitude A_1 and angular frequency ω. Then their wavevectors k are also the same. Suppose that they differ only in phase ϕ :

$$y_1 = A_1 cos(\omega t - kz) \quad \text{and} \quad y_2 = A_1 cos(\omega t - kz + \phi)$$

Spatial dependence of 2 waves at t=0:



Resultant wave:



Trig identity:

$$A_1(\cos\alpha + \cos\beta) = 2A_1 \cos\left(\frac{\beta+\alpha}{2}\right)\cos\left(\frac{\beta+\alpha}{2}\right)$$

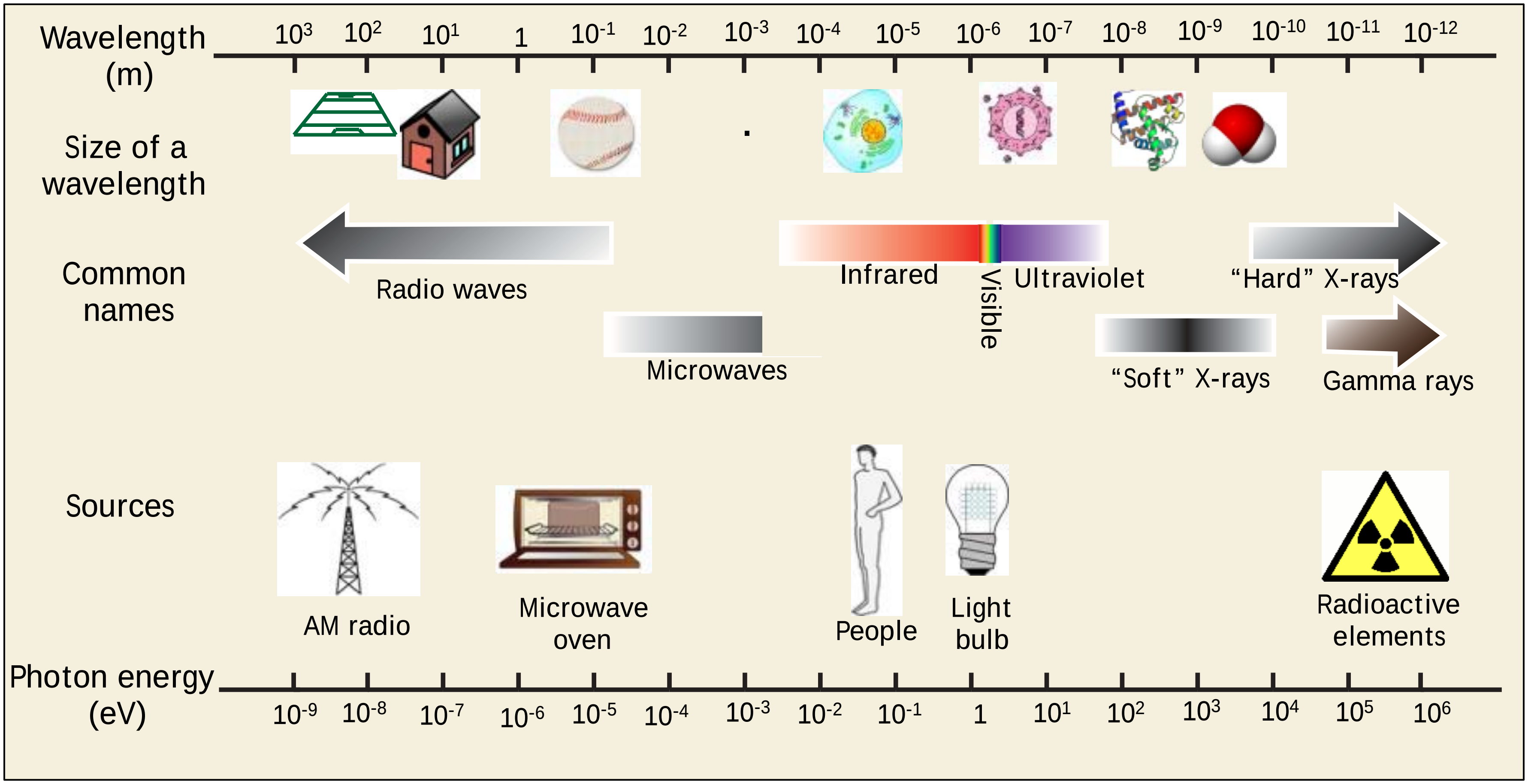
$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

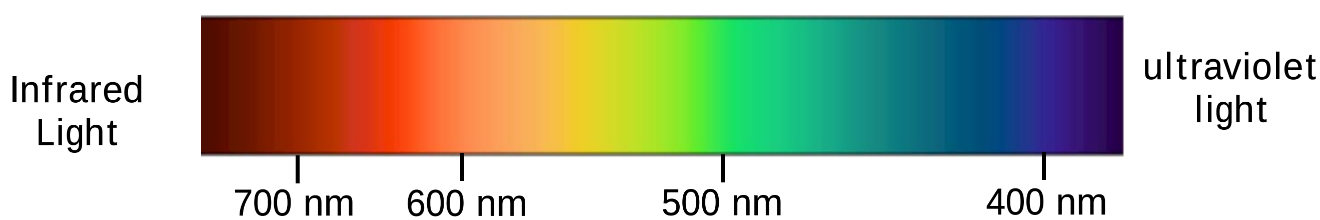
$$y_1 + y_2 \qquad (\omega t - kz + \frac{\phi}{2}) \qquad \frac{\phi}{2}$$

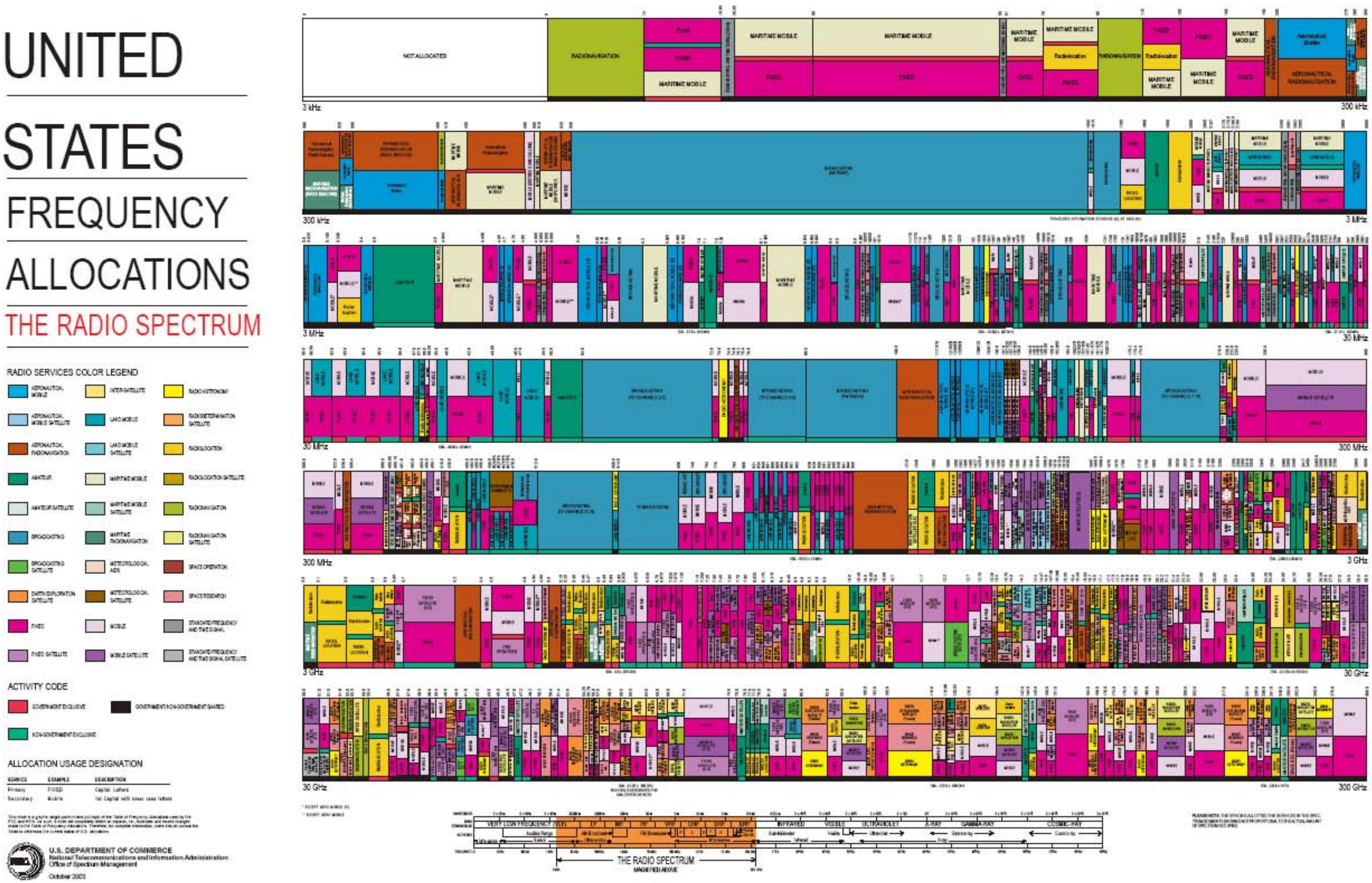
$$\boxed{y \;=\; 2A_1\cos(\phi/2)\;\;\cos(\omega t - kz + \phi/2)}$$

Amplitude **Oscillation**

The Electromagnetic Spectrum







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Electromagnetic Fields in Laser Drilling

What are intensity I, f, E, H for a 1000-J laser emitting a 1 nsec pulse at λ = 1 μm when focused on a 1 mm^2 spot?

How many cycles in the pulse?

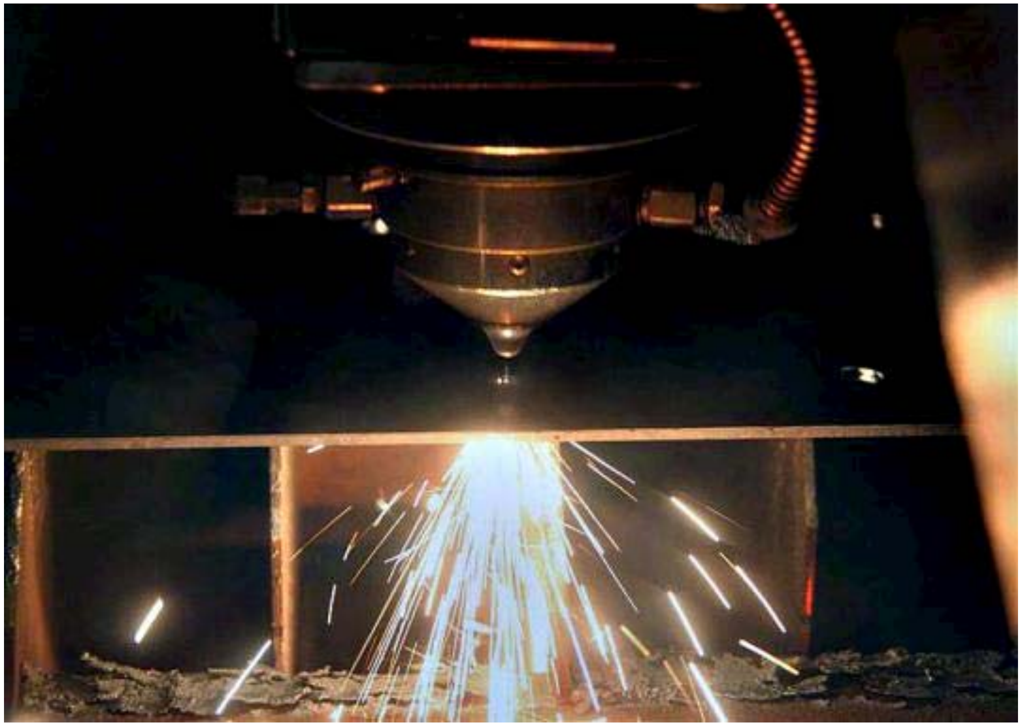


Image by Jasper84 (Metaveld BV) <http://commons.wikimedia.org/wiki/File:Lasersnijden_laserkop.jpg> on Wikimedia, also used for <http://www.metaveld.com>

What is approximate *E* required to spontaneously ionize an atom within one cycle?

{we want energy transferred $qE \cdot d$ to roughly exceed electron binding energy within nominal orbital diameter d}

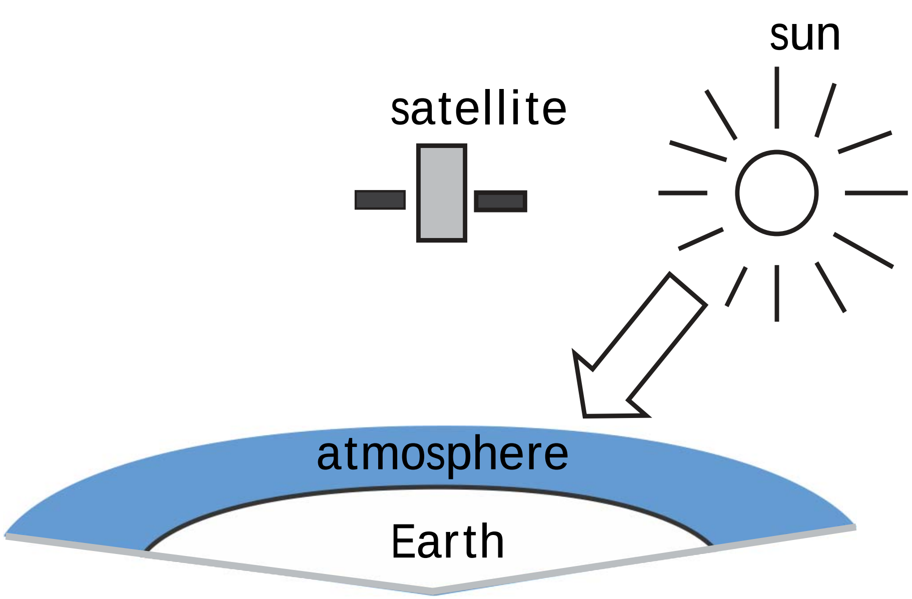
Electromagnetic Fields from a Cell Phone and the Sun

$$Intensity = \frac{Power}{Area} = |\vec{E}| \cdot |\vec{H}|$$

What are I, f, E, H for a cell phone radiating 1-watt of power over 1 m^2 of Uniform Plane Waves at 1 GHz?

The energy intensity of sunlight shining on Earth is on the order of ~1000 W/m^2. What is the amplitude corresponding E field ?

If we are going to power the World with solar-generated energy, how much land area has to be covered with Solar Cells?



World Land Area =149 million km^2
World Land/Water Area =510 million km^2

US Land Area (48 states) 7.5 million km^2

World Consumed Energy
at the rate of 15.8 TW in 2006

US consumed Energy
at the rate of 3TW

sun at ~37º: air mass 1.5 (AM1.5) 844 W/m^2 ← Terrestrial Solar cell standard

Solar spectrum outside atmosphere:
air mass 0 (AM0) 1353 W/m^2

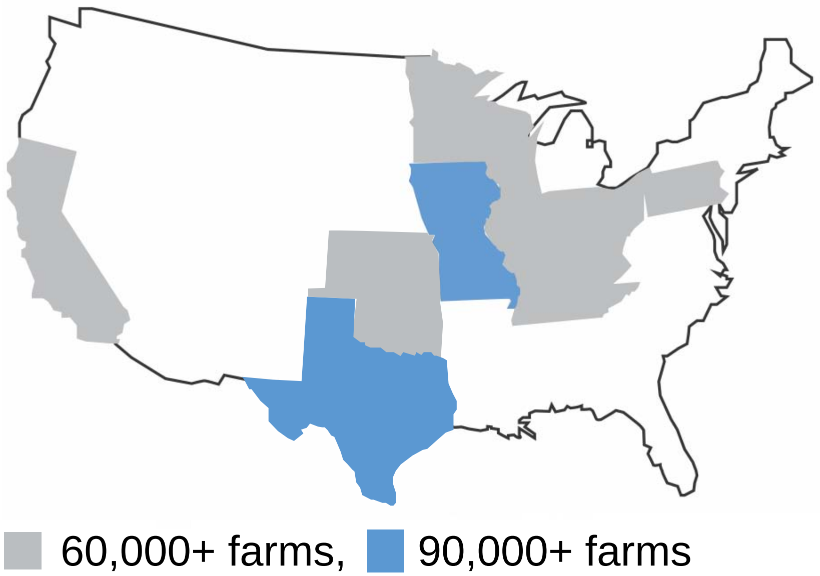
Photovoltaics

-The average power incident upon the surface of the Earth from sunlight is ~10,000 times the average global power consumption.

- The average power incident upon the continental United States is ~500 times our national consumption (total energy consumption, not just electricity).

If ~2% of the continental United States is covered with PV systems with a net efficiency of 10% we would be able to supply all the US energy needs

(Note: This is an overestimate. We need only 0.35% of US land for PV electricity generation)
(Note: 40% of our land is allocated to producing food)



Earth and solar panel images are in the public domain

Summary of Properties of a Uniform Plane Waves

1. Propagation velocity $v_p = \dfrac{1}{\sqrt{\mu\epsilon}}$ with $c = \dfrac{1}{\sqrt{\epsilon_o \mu_o}}$
2. No Electric of Magnetic field in direction of propagation
3. Electric field normal to magnetic field
4. Value of electric field is *η* times that of magnetic field at each instant
5. Direction of propagation given by $\vec{E} \times \vec{H}$
6. Energy stored in electric field per unit volume at any instant at any point is equal to energy stored in magnetic field
7. Instantaneous value of the Pointing vector given by $\dfrac{E^2}{\eta} = \eta H^2 = |\vec{E} \times \vec{H}|$
8. Superposition of EM Plane waves of same frequency and phase adds their electric fields.

$$Intensity = \frac{Power}{Area} = |\vec{E}| \cdot |\vec{H}|$$