Hypothesis testing

6.011, Spring 2018

Lec 22

Choosing between H=H_0 and H=H_1 with minimum P(error)

$$P(H_0 \text{ is true}) = P(H = H_0) = P(H_0) = p_0$$

$$P(H_1 \text{ is true}) = P(H = H_1) = P(H_1) = p_1$$

$\Rightarrow$ choose more probable hypothesis for min $P(\text{error})$

$$P(H_1) \underset{\text{`}H_0\text{'}}{\overset{\text{`}H_1\text{'}}{\gtrless}} P(H_0)$$

Again choosing between $H=H_0$ and $H=H_1$ but now given $R=r$, for min $P(\text{error} \mid R=r)$

$$P(H_1|R=r) \underset{\text{`}H_0\text{'}}{\overset{\text{`}H_1\text{'}}{\gtrless}} P(H_0|R=r)$$

Pick whichever hypothesis has maximum *a posteriori* probability

Implementing the maximum *a posteriori* (MAP) rule

$$P(H_1|R=r) \underset{\text{`}H_0\text{'}}{\overset{\text{`}H_1\text{'}}{\gtrless}} P(H_0|R=r)$$

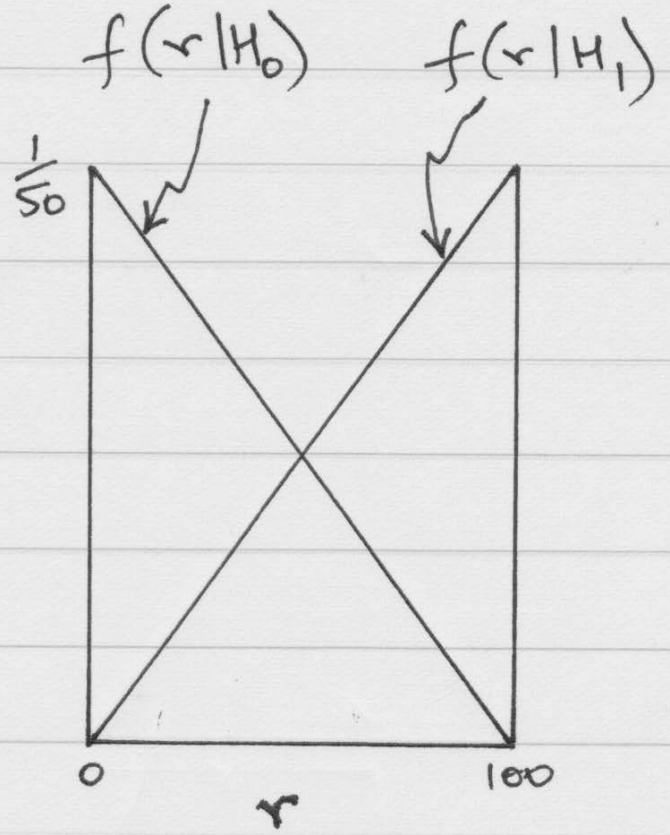
$$p_1 . f_{R|H}(r|H_1) \underset{\text{`}H_0\text{'}}{\overset{\text{`}H_1\text{'}}{\gtrless}} p_0 . f_{R|H}(r|H_0)$$

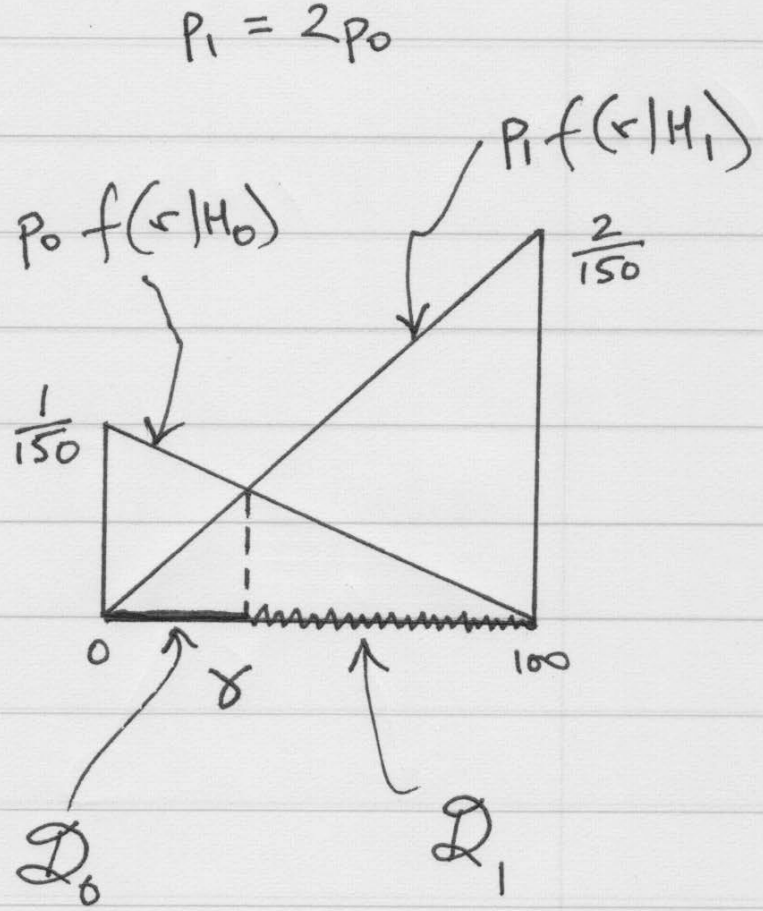
Likelihood ratio test (LRT) implemention of MAP rule

$$p_1 . f_{R|H}(r|H_1) \underset{\text{`}H_0\text{'}}{\overset{\text{`}H_1\text{'}}{\gtrless}} p_0 . f_{R|H}(r|H_0)$$

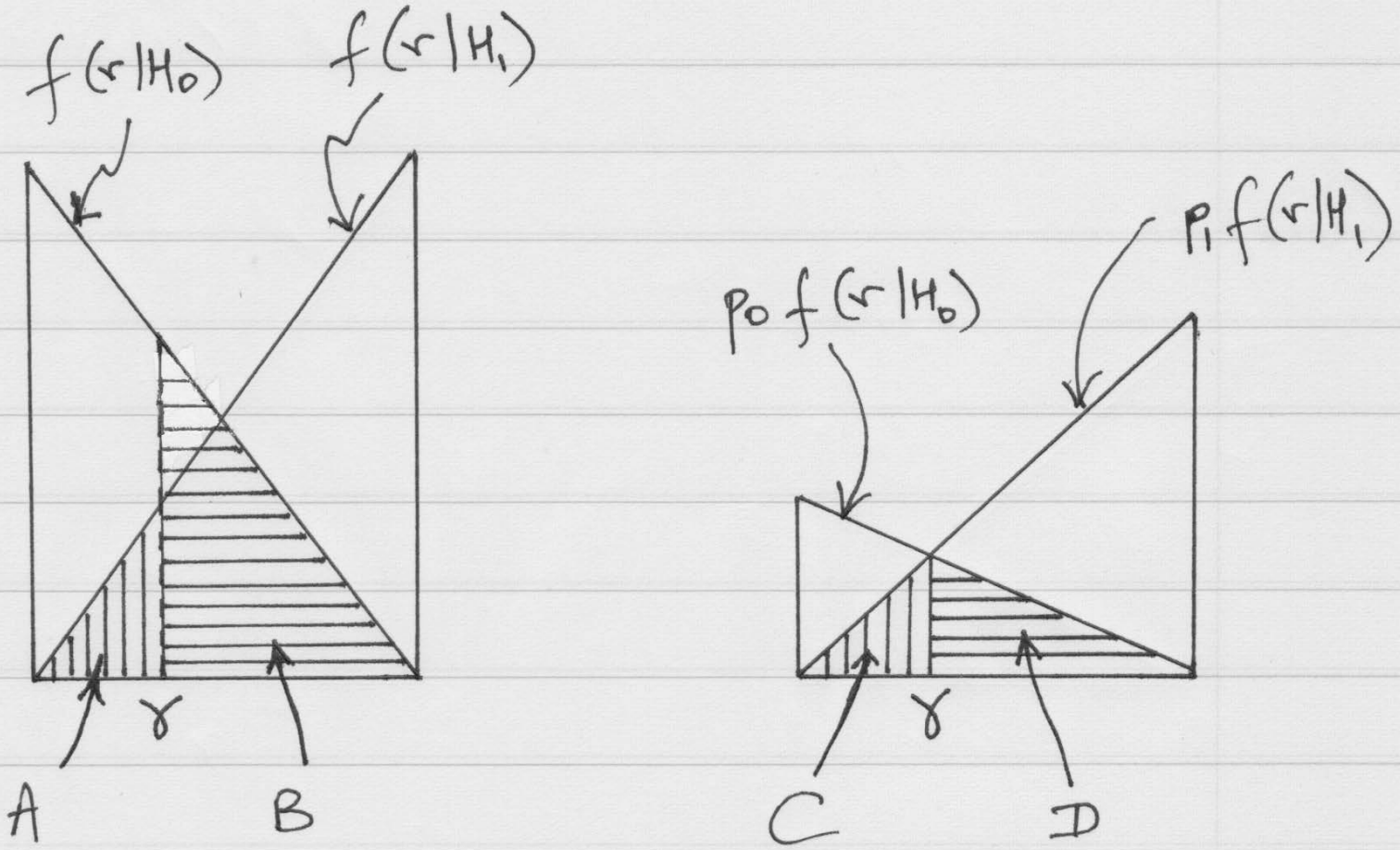
$$\Lambda(r) = \frac{f_{R|H}(r|H_1)}{f_{R|H}(r|H_0)} \underset{\text{`}H_0\text{'}}{\overset{\text{`}H_1\text{'}}{\gtrless}} \frac{p_0}{p_1} = \eta$$

Binary hypothesis testing (example)

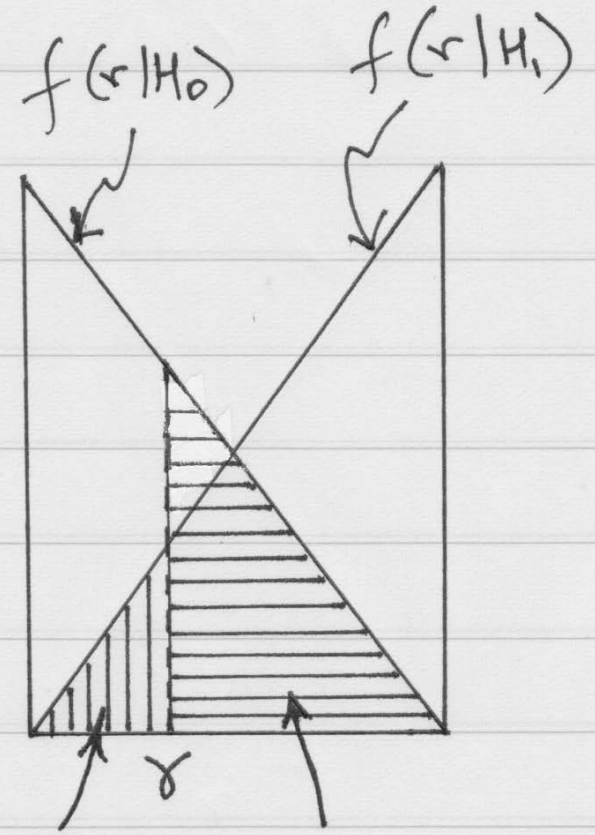




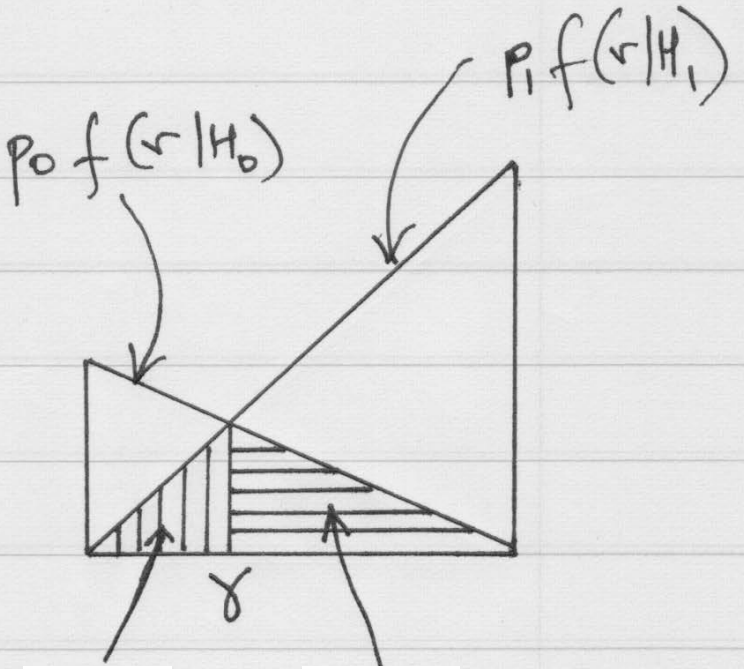
Binary hypothesis testing (example)



Binary hypothesis testing (example)



P_M P_{FA}



p_1P_M + p_0P_{FA} = P(error)

Terminology

- prevalence (p_1)
- (conditional) probability of detection, sensitivity, true positive rate, recall
- specificity, true negative rate
- (conditional) probability of false alarm, false positive rate (= 1– specificity)
- (conditional) probability of a miss, false negative rate (= 1 – sensitivity)
- positive predictive value, precision
- negative predictive value

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