The 5 basic equations of semiconductor device physics:

We will in general be faced with finding 5 quantities:

$$n(x,t),\ p(x,t),\ J_e(x,t),\ J_h(x,t),\ \text{and}\ E(x,t),$$

and we have five independent equations that relate them:

(1)
$$J_e(x,t) = q\,\mu_e\,n(x,t)\,E(x,t) + q\,D_e\,\frac{\partial n(x,t)}{\partial x}$$

(2)
$$J_h(x,t) = q\,\mu_h\,p(x,t)\,E(x,t) - q\,D_h\,\frac{\partial p(x,t)}{\partial x}$$

(3)
$$\frac{\partial n(x,t)}{\partial t} - \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} = g_L(x,t) - \left[n(x,t)\,p(x,t) - n_i^2\right] r$$

(4)
$$\frac{\partial p(x,t)}{\partial t} + \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x} = g_L(x,t) - \left[n(x,t)\,p(x,t) - n_i^2\right] r$$

(5)
$$\varepsilon\,\frac{\partial E(x,t)}{\partial x} = q\left[p(x,t) - n(x,t) + N_d(x) - N_a(x)\right]$$

where the assumptions we made getting these equations are:

$$N_d^+(x) \approx N_d(x),\ N_a^-(x) \approx N_a(x),\ R(x,t) \approx n(x,t)\,p(x,t)\,r$$

r, ε, μ_e, μ_h, D_e, and D_h are assumed to be independent of position.

Temperature is assumed to be constant (isothermal).
(Note: n_i, r, μ_e, μ_h, D_e, and D_h all depend on temperature).

This is a set of 5 coupled, non-linear differential equations that are in general not solvable analytically....

BUT we know the solution in three special cases already....

1) <u>Uniform doping, thermal equilibrium</u>: n_o, p_o
   (See page 4 below for a discussion of this case)

2) <u>Drift</u>: $g = 0$, uniform doping
   (See page 4 below for a discussion of this case)

3) <u>Uniform Low-level Injection</u>: n', p', τ_{min}
   (See pages 5-6 below for a discussion of this case)

AND we are able to find APPROXIMATE ANALYTICAL solutions for two very important new situations....

1) <u>Doping Profile Problems</u>: non-uniformly doped material in thermal equilibrium (an important subset of these problems are solved using the <u>depletion approximation</u>)
   (See pages 7-10 below for a discussion of these problems)

2) <u>Flow Problems</u>: non-uniform injection of excess carriers into uniformly doped material
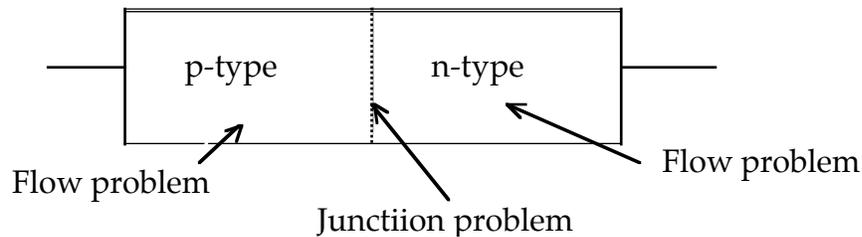   (See pages 11-14 below for a discussion of these problems)

<div align="center">********</div>

With an understanding of these solutions to the five equations we will be able to model and understand all of the important semiconductor devices, including diodes, bipolar transistors, and MOSFETs.

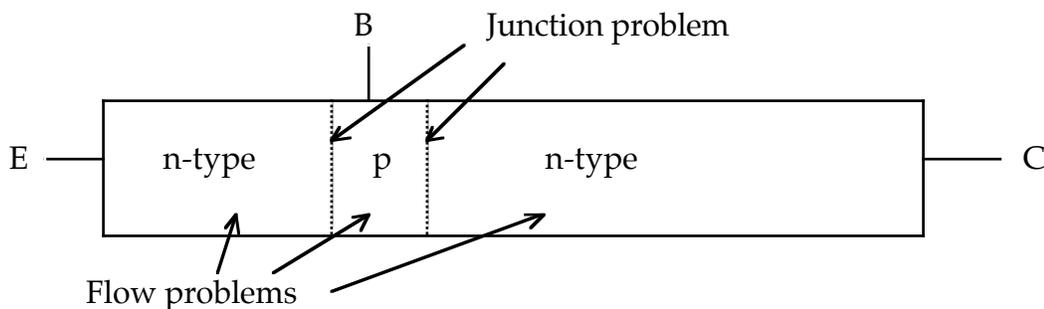<div align="center">(See page 3 for an illustration of this point)</div>

Why we care: Understanding Flow Problems, the Depletion Approximation, and Drift, we can under-stand all of the basic devices we see in 6.012:

<u>Diodes</u> involve flow problems in two regions and the depletion approximation about one junction:

```
        ┌──────────────┊──────────────┐
────────┤   p-type     ┊    n-type    ├────────
        └──────────────┊──────────────┘
```
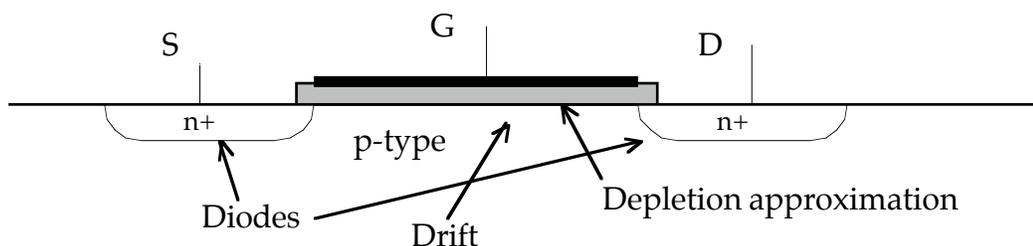Flow problem   Junctiion problem   Flow problem

Note: This is true not only for simple electronic diodes, but also for light emitting and laser diodes, and for photodiodes and solar cells.

<u>Bipolar transistors</u> involve flow problems in three regions and the depletion approximation about two junctions:

B   Junction problem

```
        ┌──────────┊──┊──────────────────┐
E ──────┤ n-type   ┊p ┊    n-type        ├────── C
        └──────────┊──┊──────────────────┘
```
Flow problems

<u>MOSFETs</u> involve two diodes, the depletion approxi-mation in the gate region, and drift in the channel:

S   G   D

```
                 ┌─────────────────┐
─────────(  n+  )┤     p-type      (  n+  )──────
                 └─────────────────┘
```
Diodes   Drift   Depletion approximation

<u>Special Case</u>: **UNIFORM DOPING, THERMAL EQUIL.**

Uniform material, thermal equilibrium: $\partial/\partial x = 0$, $\partial/\partial t \approx 0$, and $g_L = 0$. In this case n and p are constant and denoted as n_o and p_o. Equations 3 and 4 give: $n_o p_o = n_i^2$, and Equation 5 gives: $p_o - n_o + N_d - N_a = 0$. Combining and solving for n_o and p_o yields:

when $N_d - N_a \gg n_i$ (i.e., n-type), $n_o \approx N_d - N_a$, and $p_o = n_i^2/n_o$

and

when $N_a - N_d \gg n_i$ (i.e., p-type), $p_o \approx N_a - N_d$, and $n_o = n_i^2/p_o$

<u>Special Case</u>: **DRIFT**

Uniform material; uniform, slowly varying (quasi-static) externally applied electric field: $\partial/\partial x = 0$, $\partial/\partial t \approx 0$, and $g_L = 0$.

Equations (3) and (4) yield: $np = n_i^2$, and Equation (5) yields: $p - n + N_d - N_a = 0$. Combining these we see p and n are the same as p_o and n_o in thermal equilibrium.

Using this with Equations (1) and (2) yields:

$$J_e(t) = q\,\mu_e\,n_o\,E(t), \quad J_h(t) = q\,\mu_h\,p_o\,E(t)$$

$$J_{TOT}(t) = J_e(t) + J_h(t) = q\,(\mu_e\,n_o + \mu_h\,p_o)\,E(t)$$

We see that under these conditions we have a current proportional to any externally applied electric field. This is Ohm's law on a microscopic scale.

<u>Special Case</u>: **UNIFORM OPTICAL EXCITATION**
             (Photoconductivity)
             (Minority Carrier Lifetime)

Uniform material; uniform optical generation, $g_L(t)$; uniform applied electric field; low level injection:

$$\partial/\partial x = 0$$

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We first define excess carrier populations, n' and p', as:

$$n(t) = n_o + n'(t), \quad \text{or} \quad n'(t) = n(t) - n_o$$
$$p(t) = p_o + p'(t), \quad \text{or} \quad p'(t) = p(t) - p_o$$

Symmetry tells us we must have zero internal electric field, and since any externally applied electric field must be uniform, there is no gradient in the electric field. Using this in Equation (5), yields

$$n'(t) = p'(t)$$

Using all of this in Equations (1) and (2) shows us that we only have drift currents, but they are larger than in thermal equilibrium. This is photoconductivity:

$$J_e(x,t) = q\,\mu_e\,[n_o + n'(t)]\,E, \quad J_h(x,t) = q\,\mu_h\,[p_o + p'(t)]\,E$$

$$J_{TOT}(t) = [q\,(\mu_e\,n_o + \mu_h\,p_o) + q\,(\mu_e + \mu_h)\,n'(t)]\,E$$

"thermal equilibrium conductivity"   "photoconductivity"

Returning now to p'(t) and n'(t), which are still unknowns, we know they are equal, and to determine what they are we use either Equation (3) or Equation (4):

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$$\frac{dn(t)}{dt} = \frac{dn'(t)}{dt} = g_L(t) - [\{n_o + n'(t)\}\{p_o + p'(t)\} - n_i^2] \, r$$

yielding

$$\frac{dn'(t)}{dt} = g_L(t) - [\{n_o + p_o + n'(t)\} \, n'(t)\}] \, r$$

To linearize this first order differential equation we assume <u>low level injection</u>, i.e. $n'(t) \ll n_o + p_o$. This sum is essentially the majority carrier population, and we will focus on the excess minority carrier population. Assuming for sake of discussion that we have a p-type sample, we normally would write this as $n'(t) \ll p_o$. In this case we have

$$\{n_o + p_o + n'(t)\} \approx p_o$$

and thus

$$\frac{dn'(t)}{dt} \approx g_L(t) - n'(t) \, p_o \, r$$

We define the <u>minority carrier lifetime</u>, τ_e, as $1/p_o \, r$, so:

$$\frac{dn'(t)}{dt} + \frac{n'(t)}{\tau_e} \approx g_L(t)$$

This is a first order, linear differential equation well known to us from RC circuits. The homogeneous solutions of this equation are of the form:

$$n'(t) = A \, e^{-t/\tau_e}$$

where A is chosen so that the total solution (homogeneous plus particular) satisfies the initial condition.

<u>Special Case</u>: **DOPING PROFILES and JUNCTIONS**

Arbitrary doping profile; thermal equilibrium:

$$\partial/\partial t = 0, \ g_L = 0; \text{arbitrary } N_d(x), N_a(x)$$

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In thermal equilibrium the currents must be zero so from Equations (3) and (4) we find that even with an arbitrary doping profile,

$$n_o(x) \ p_o(x) \ = \ n_i^2,$$

and Equations (1) and (2) tell us that

$$0 \ = \ q \, \mu_e \, n_o(x) \, E(x) + q \, D_e \frac{dn_o(x)}{dx}$$

and

$$0 = \ q \, \mu_h \, p_o(x) \, E(x) - q \, D_h \frac{dp_o(x)}{dx}$$

We can rewrite these equations using $E(x) = -d\Phi(x)/dx$ as:

$$\frac{1}{n_o(x)} \frac{dn_o(x)}{dx} \ = \ - \frac{\mu_e}{D_e} \frac{d\Phi(x)}{dx}$$

and

$$\frac{1}{p_o(x)} \frac{dp_o(x)}{dx} \ = \ \frac{\mu_h}{D_h} \frac{d\Phi(x)}{dx}$$

Integrating both sides with respect to x, and adapting the convention that $\Phi(x)$ is zero in intrinsic material, i.e., where $n_o(x) = p_o(x) = \ n_i$, we have

$$n_o(x) \ = \ n_i \, e^{\frac{\mu_e}{D_e} \Phi(x)}$$

and

$$p_o(x) \,=\, n_i\, e^{-\frac{\mu_h}{D_h}\Phi(x)}$$

We see immediately that the requirement that $n_o(x)\, p_o(x) \,=\, n_i^2$, tells us that we must have:

$$\frac{\mu_e}{D_e} \,=\, \frac{\mu_h}{D_h}$$

In fact, the ratio of μ to D is equal to a very specific quantity, q/kT:

$$\frac{\mu_e}{D_e} \,=\, \frac{\mu_h}{D_h} \,=\, \frac{q}{kT}$$

This equality is known as the Einstein relation; it is easy to remember because it rhymes as written, and it also rhymes inverted. We use it frequently.

We thus have:

$$n_o(x) \,=\, n_i\, e^{\,q\Phi(x)/kT}$$

and

$$p_o(x) \,=\, n_i\, e^{\,-\,q\Phi(x)/kT}$$

Alternatively, we can write in terms of $n_o(x)$ and/or $p_o(x)$:

$$\Phi(x) \,=\, \frac{kT}{q}\, \ln\frac{n_o(x)}{n_i} \,=\, -\frac{kT}{q}\, \ln\frac{p_o(x)}{n_i}$$

Finally, we use these results in Equation (5) (which also relates $\Phi(x)$, $p_o(x)$, and $n_o(x)$) to write:

$$\varepsilon\, \frac{\partial^2\Phi(x)}{\partial x^2} \,=\, q\,[n_i\, e^{\,-\,q\Phi(x)/kT} - n_i\, e^{\,q\Phi(x)/kT} + N_d(x) - N_a(x)]$$

Once again we have reduced our five equations to one equation in one unknown ($\Phi(x)$ in this case). However, the solution of this equation is, in general, impossible to obtain analytically, but it can easily be solved iteratively by computer, and in many cases a perfectly adequate solution can be found by hand using the "depletion approximation".

An important observation is that the electrostatic potential, $\Phi(x)$, depends only logarithmically on the equilibrium carrier concentrations, $p_o(x)$ and $n_o(x)$. This means that large changes in carrier concentration result in only relatively small changes in the electrostatic potential. Said the other way around, a small change in electrostatic potential corresponds to a relatively large change in the carrier concentrations. A useful rule of thumb to keep in mind is that an order of magnitude change in carrier concentration, corresponds to a 60 mV change in electrostatic potential. This is called "The 60 mV rule."

Special Profiles

Gradual spatial variation: If $N_d(x)$ and/or $N_a(x)$ vary only slowly with position (where "slowly" can be defined using the concept of Debye length), the equilibrium carrier concentrations track the doping profile, i.e. in an n-type sample where, $N_d(x) - N_a(x) > 0$

$$n_o(x) \approx N_d(x) - N_a(x) \text{ and } p_o(x) = n_i^2/n_o(x)$$

Abrupt doping changes; depletion approximation: If the doping changes abruptly, for example from p-type to n-type at a p-n junction, the majority carrier concentration will fall so quickly at the electrostatic potential begins to

change that a "depletion region", i.e., a region effectively void of mobile carriers will be created. In that region there will be a net charge density approximately equal to that of the net donor or acceptor concentration.

For example, if the depletion region on the n-side extends from 0 to x_n, the depletion approximation says that net charge density, $\rho(x)$, can be approximated as:

$$\rho(x) \approx q[N_d(x) - N_a(x)] \text{ for } 0 < x < x_n$$

The <u>depletion approximation model</u> gives an estimate for the net charge density profile. Having this estimate, we can integrate it once to get an estimate for the electric field profile, $E(x)$. Integrating again gives us an estimate for the electrostatic potential profile, $\Phi(x)$. Having this we can calculate $n_o(x)$ and $p_o(x)$, recalculate $\rho(x)$, etc., and continue interating until we are satisfied. Usually we stop after one time through, i.e., after calculating $\Phi(x)$ the first time.

<u>Special Case</u>: **FLOW PROBLEMS**

Uniform material; quasi-static (slowly varying), low level optical excitation of arbitrary distribution, i.e. $g_L(x,t)$; no externally applied electric field (may have internal field):

$$\partial/\partial t \approx 0; \; n', \, p' << n_o + p_o$$

Assume p-type for purposes of discussion.

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We already know n_o and p_o in this situation, so the problem is one of finding the excess carrier populations, the currents, and the electric field.

A fundamental assumption we make is that the material remains essential charge neutral. We call this condition <u>quasineutrality,</u> and specify it by saying that we can assume

$$p'(x,t) \approx n'(x,t), \text{ and } \partial n'(x,t)/\partial x \approx \partial p'(x,t)/\partial x$$

We will not justify this assumption rigorously in 6.012, but one can show that it is well satisfied in semiconductors. You can refer to Appendix D in the course text for more.

We proceed with Equation (3), which yields:

$$-\frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} \approx g_L(x,t) - \frac{n'(x,t)}{\tau_e}$$

We next turn to Equations (1) and (2), and we argue that any minority carrier drift must be negligible under <u>low level injection</u> conditions. The minority carrier drift

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current is always much less than the majority carrier drift current, so the only way there will be a non-negligible minority carrier current is if it is a <u>minority carrier diffusion current</u>. Thus, in our present case of a p-type sample, Equation (1) becomes:

$$J_e(x,t) \approx q\,D_e\frac{\partial n'(x,t)}{\partial x}$$

Combining the last two equations yields a single second order differential equation in the minority carrier concentration:

$$\frac{\partial^2 n'(x,t)}{\partial x^2} - \frac{n'(x,t)}{D_e\tau_e} \approx -\frac{g_L(x,t)}{D_e}$$

The homogeneous solutions of this equation are of the form:

$$n'(x,t) = A\,e^{-x/L_e} + B\,e^{+x/L_e}$$

where we have defined the <u>minority carrier diffusion length</u>, L_e, as

$$L_e = \sqrt{D_e\tau_e}$$

The constants A and B are chosen so that the total solution, consisting of the sum of the homogeneous solution and the particular solutiion, satisfy the boundary conditions.

Now we can continue to calculate the rest of the quantities of interest, i.e., $p'(x,t)$, $J_e(x,t)$, $J_h(x,t)$, and $E(x,t)$. We begin by noticing that once we know $n'(x,t)$, we can calculate $J_e(x,t)$ using the equation above.

Then we calculate $J_h(x,t)$ using $J_{Tot}(t) = J_e(x,t) + J_h(x,t)$. We will in general know what $J_{Tot}(t)$ is from the problem statement, or from some other piece of information. To see that $J_{Tot}(t)$ is not a function of position in quasistatic situations (i.e., $\partial/\partial t \approx 0$), subtract Equations (3) and (4) to get:

$$\frac{\partial J_e(x,t)}{\partial x} + \frac{\partial J_h(x,t)}{\partial x} = \frac{\partial[J_e(x,t) + J_h(x,t)]}{\partial x} = \frac{\partial J_{Tot}(t)}{\partial x} = 0$$

Knowing $J_h(x,t)$, we calculate $E(x)$ using Equation (2), along with our assumptions of low level injection (i.e., $p(x,t) \approx p_o$) and quasineutrality (i.e., $\partial n'(x,t)/\partial x \approx \partial p'(x,t)/\partial x$).

Once we have $E(x)$, we calculate $p'(x)$ using Equation (5).

Finally, we look at all of our answers and confirm that all of our assumptions were valid. If they are, we are done. If they are not, we start over, this time not making the invalid assumptions.

Boundary Conditions

An important aspect of solving flow problems is knowing the boundary conditions. Once you know the boundary conditions, you can often sketch the answer (usually the excess minority carrier distribution). The key boundary conditions we encounter in 6.012 can be summarized as follows:

ohmic contacts - excesses are identically zero

reflecting surfaces - net current in or out is zero (i.e. slope of minority carrier profile is zero) unless there is surface generation, in which case the flux away from the surface equals the generation rate

internal boundaries - excess concentration profiles are never discontinuous; the current can only be discontinuous if there is generation/recombination in the boundary plane, in which case the net flux out/in equals the generation/recombination rate

injecting surface/boundaries - in devices we will have boundaries which establish the value of (a) the excess minority carrier population, or (b) the excess minority carrier flux (current density)

Infinite Lifetime Approximation

In cases where the minority carrier diffusion length, L_e, is much larger than the dimensions of the sample, we can often ignore the n'/L_e^2 term in the diffusion equation, so that our equation becomes simply

$$\frac{\partial^2 n'(x,t)}{\partial x^2} \approx -\frac{g_L(x,t)}{D_e}$$

Important points to make about this result are:
1. We can find n' for an arbitrary g_L simply by integrating this equation twice, and then using the boundary conditions to get the two constants of integration.
2. When g_L is zero, the profiles are straight lines.
3. This is called the infinite lifetime approximation because it is equivalent to neglecting recombination.

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