6.012 - Microelectronic Devices and Circuits

Lecture 24 - Intrin. Freq. Limits - Outline

- **Announcements**

  Final Exam - Tuesday, Dec 15, 9:00 am - 12 noon

- **Review - Shunt feedback capacitances:** C_μ and C_{gd}

  Miller effect: any C bridging a gain stage looks bigger at the input

  Marvelous cascode: CE/S-CB/G (E/SF-CB/G work, too **- see µA741**)

  large bandwidth, large output resistance

  used in gain stages and in current sources

  Using the Miller effect to advantage: Stabilizing OP Amps - the µA741

- **Intrinsic high frequency limitations of transistors**

  General approach

  MOSFETs: f_T

  biasing for speed

  impact of velocity saturation

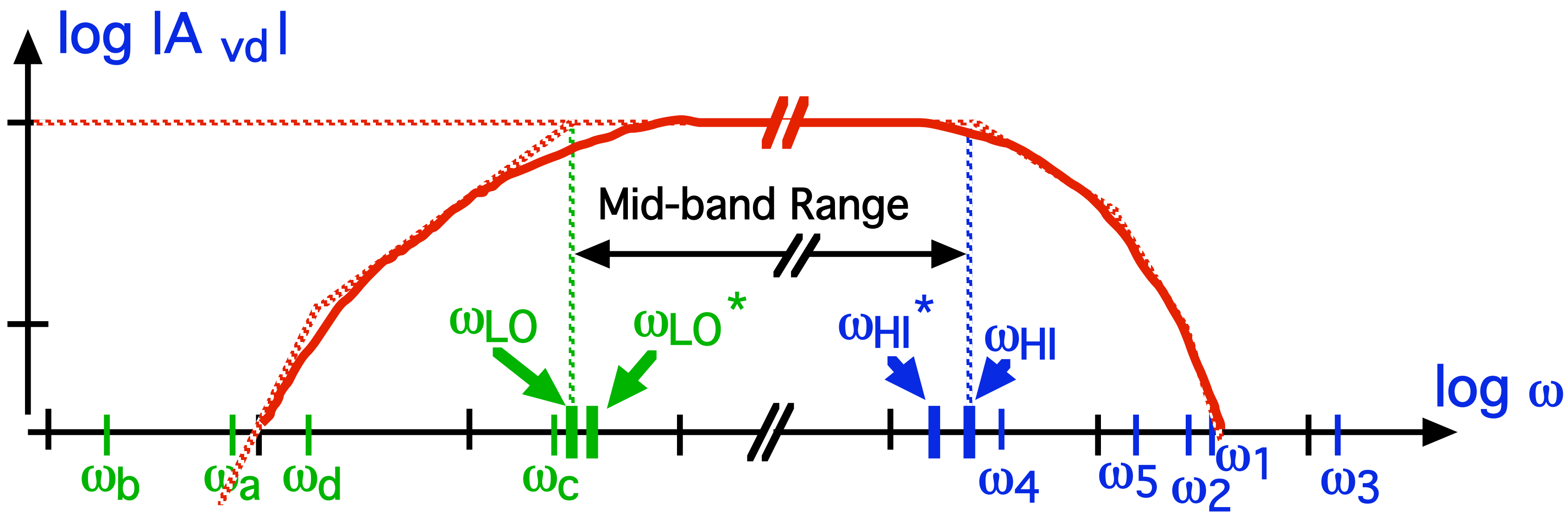
  design lessons

  BJTs: f_β, f_T, f_α

  biasing for speed

  design lessons

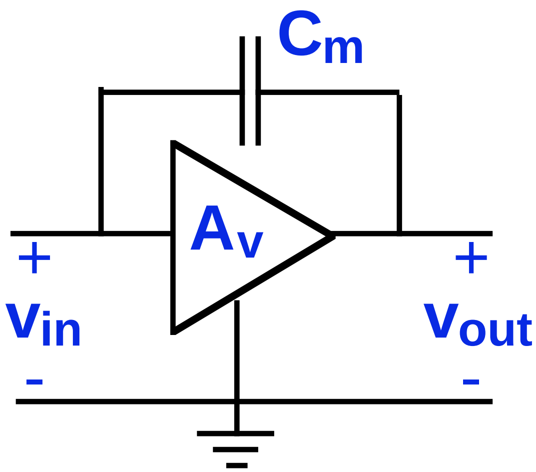
Summary of OCTC and SCTC results

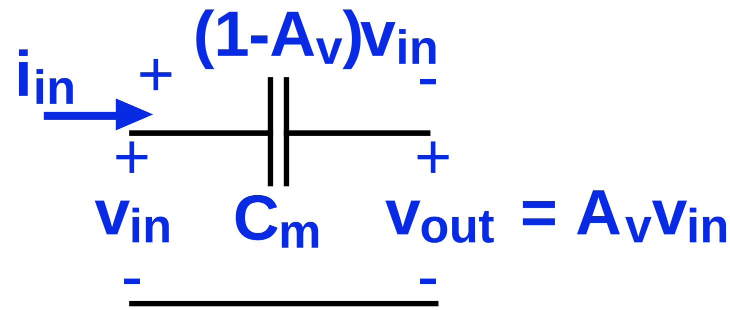


- **OCTC: an estimate for ω_{HI}**
  1. **ω_{HI}* is a weighted sum of ω's associated with device capacitances: (add RC's and invert)**
  2. **Smallest ω (largest RC) dominates ω_{HI}***
  3. **Provides a lower bound on ω_{HI}**
- **SCTC: an estimate for ω_{LO}**
  1. **ω_{LO}* is a weighted sum of w's associated with bias capacitors: (add ω's directly)**
  2. **Largest ω (smallest RC) dominates ω_{LO}***
  3. **Provides a upper bound on ω_{LO}**

The Miller effect (general)

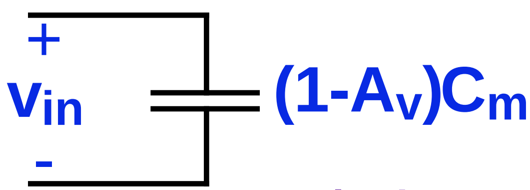
Consider an amplifier shunted by a capacitor, and consider how the capacitor looks at the input and output terminals:



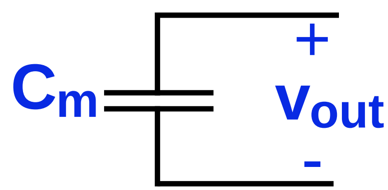


$$i_{in} = C_m \frac{d\left[(1-A_v)v_{in}\right]}{dt} = (1-A_v)C_m \frac{dv_{in}}{dt}$$

Note: A_v is negative



C_{in} looks much bigger than C_m



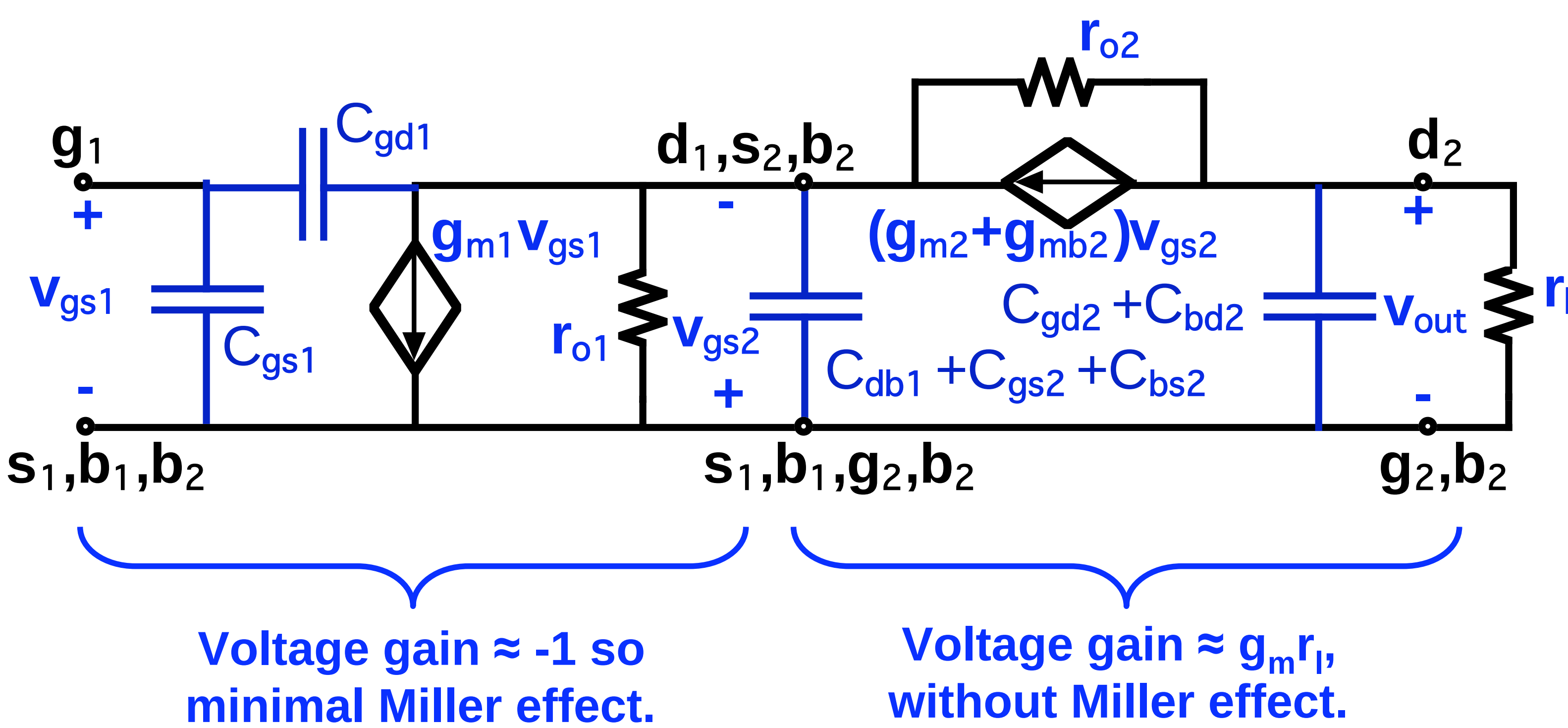
$$C_m \frac{(1-A_v)}{A_v} \approx C_m$$

C_{out} looks like C_m

The cascode when the substrate is grounded:

High frequency issues:

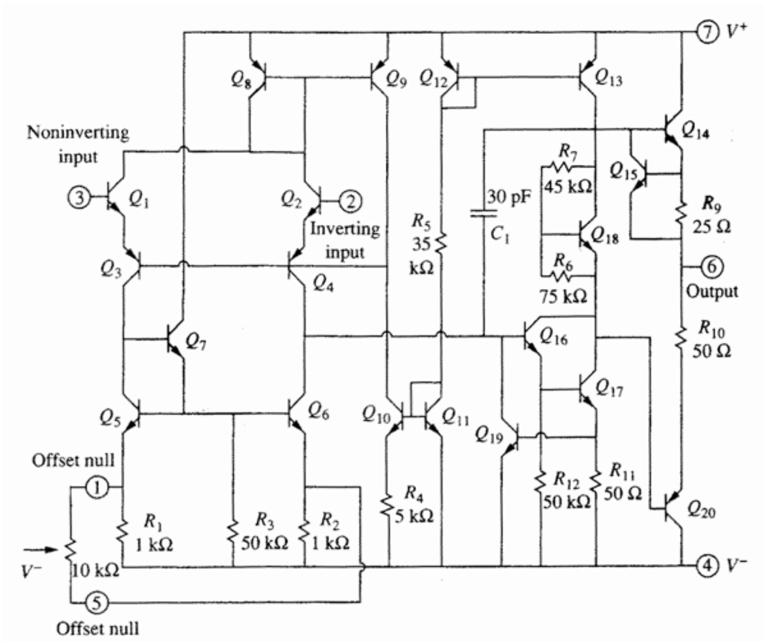
L.E.C. of cascode: can't use equivalent transistor idea here because it didn't address the issue of the C's!



Common-source gain without the Miller effect penalty!

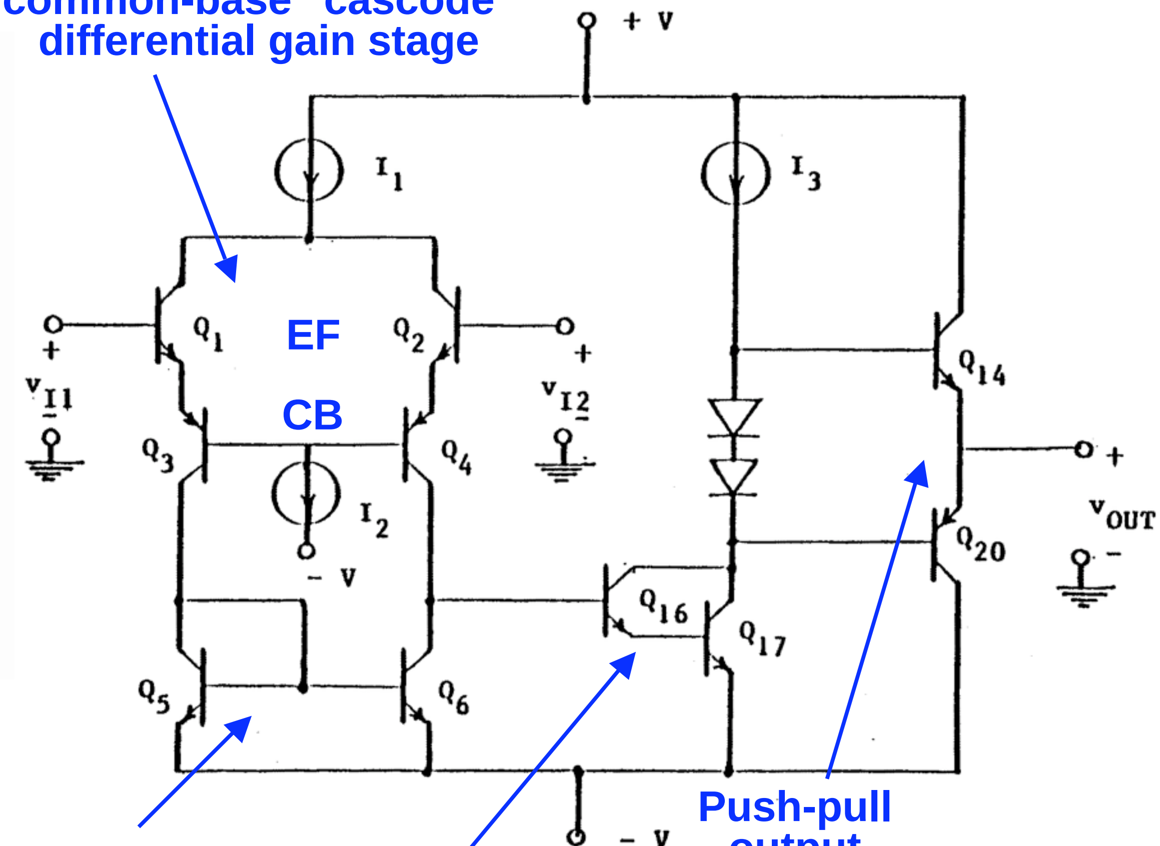
Multi-stage amplifier analysis and design: The µA741

Figuring the circuit out:



The full schematic

Emitter-follower/ common-base "cascode" differential gain stage



Current mirror load

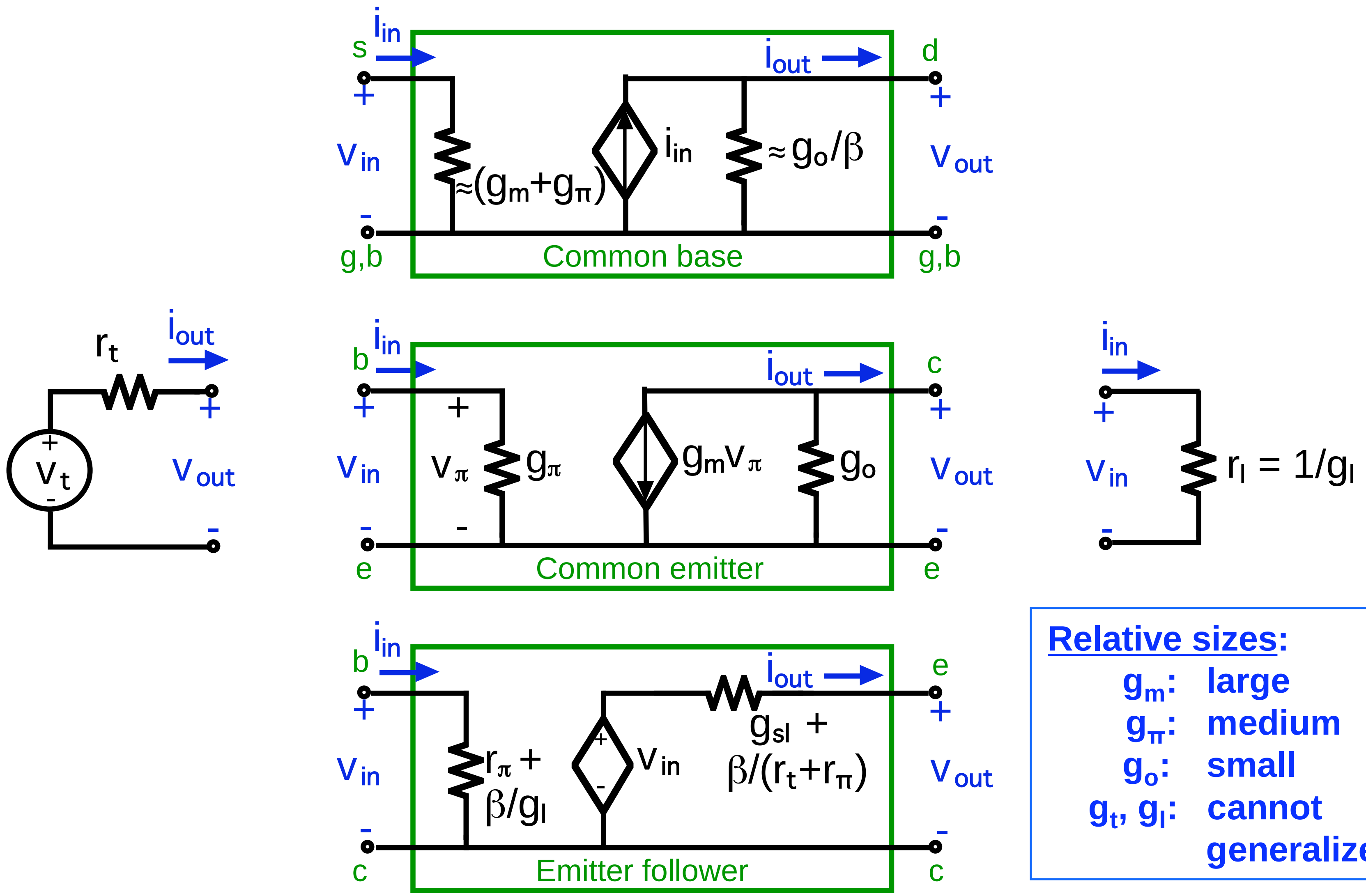
Push-pull output

Simplified schematic

Darlington common-emitter gain stage

Multi-stage amplifier analysis and design: Understanding the μA741 input "cascode"

Begin with the BJT building-block stages:

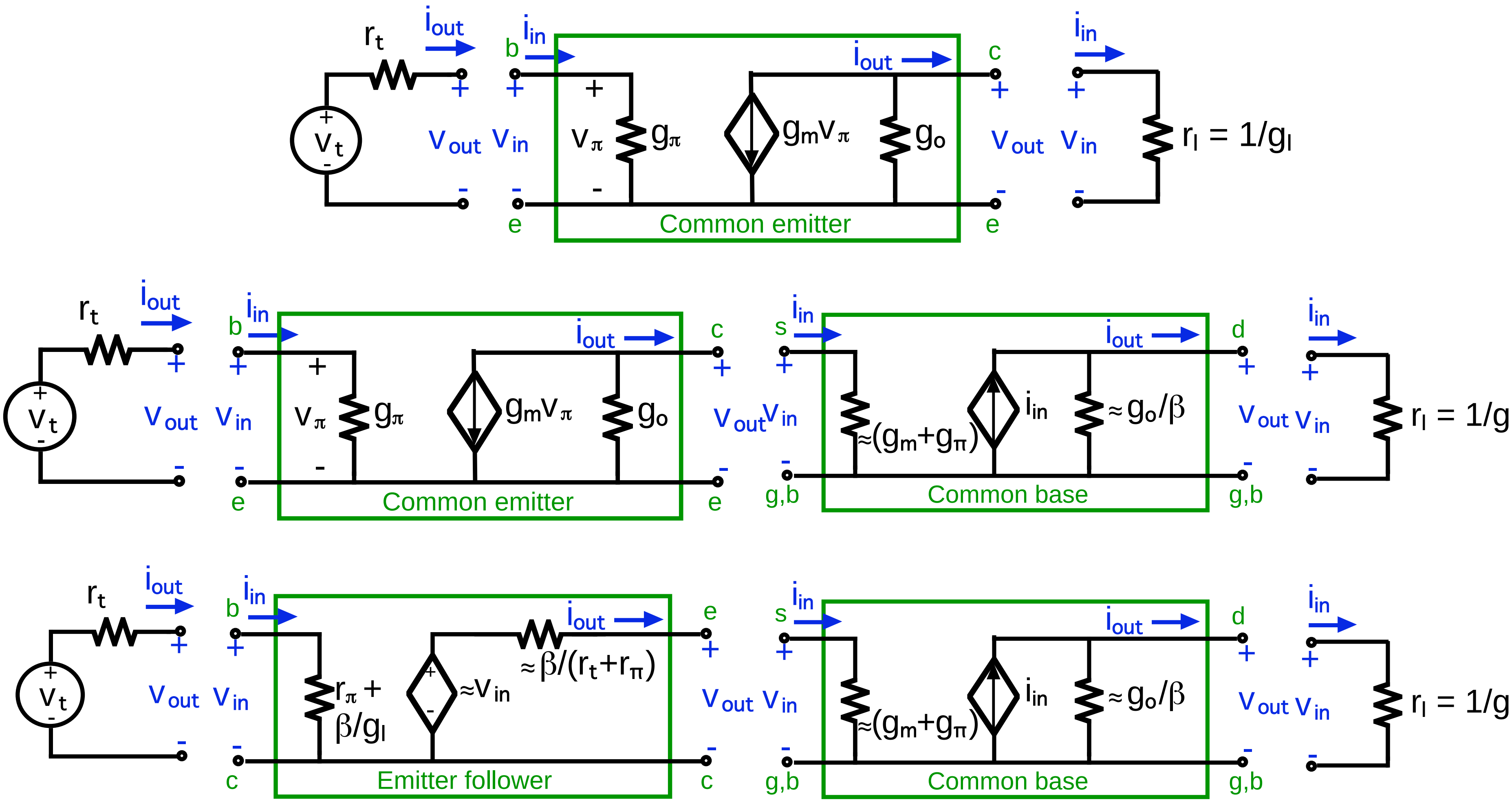


Relative sizes:

g_m:	large
g_π:	medium
g_o:	small
g_t, g_l:	cannot generalize

Multi-stage amplifier analysis and design: Two-port models

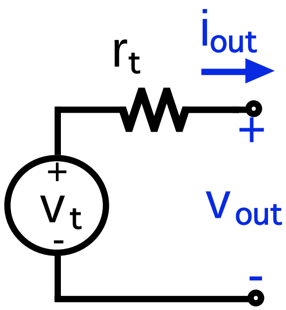
Two different "cascode" configurations, this time bipolar:

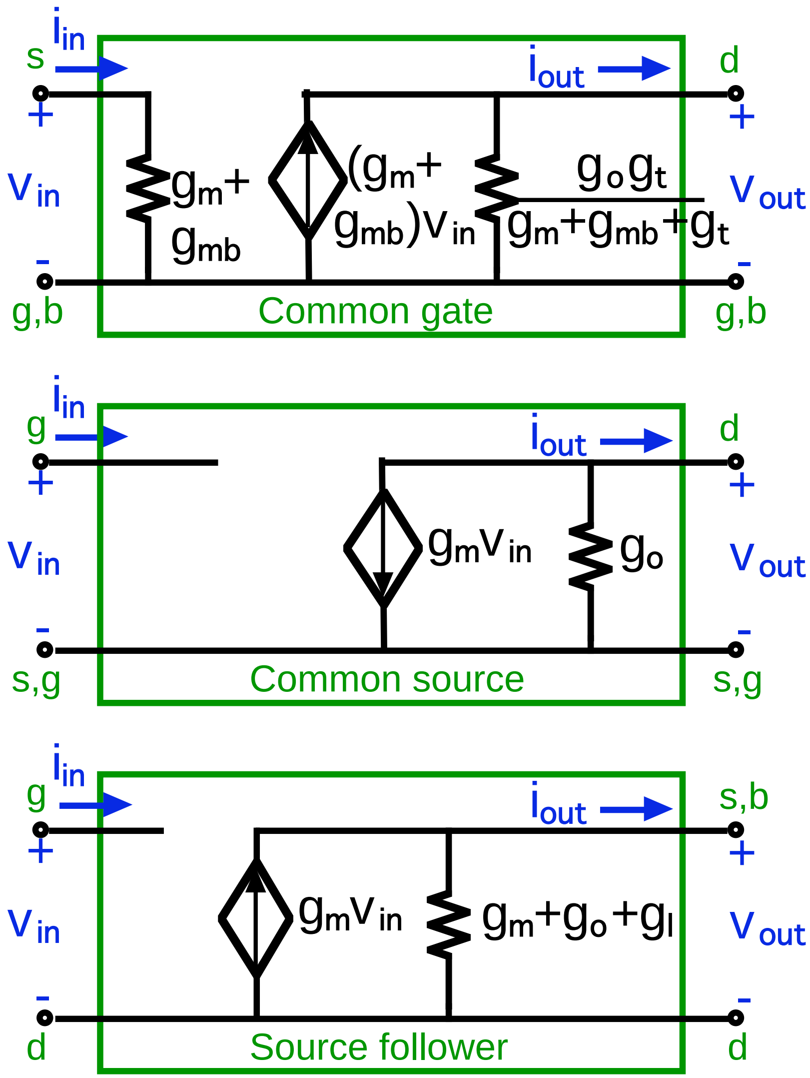


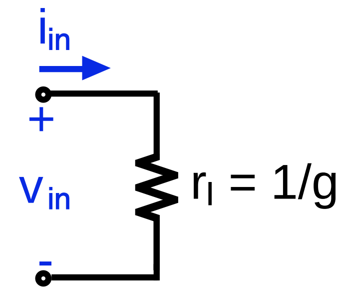
In a bipolar cascode, starting with an emitter follower still reduces the gain, but it also gives twice the input resistance, which is helpful.

Multi-stage amplifier analysis and design: MOSFET 2-port models

Reviewing our building-block stages:







Relative sizes:

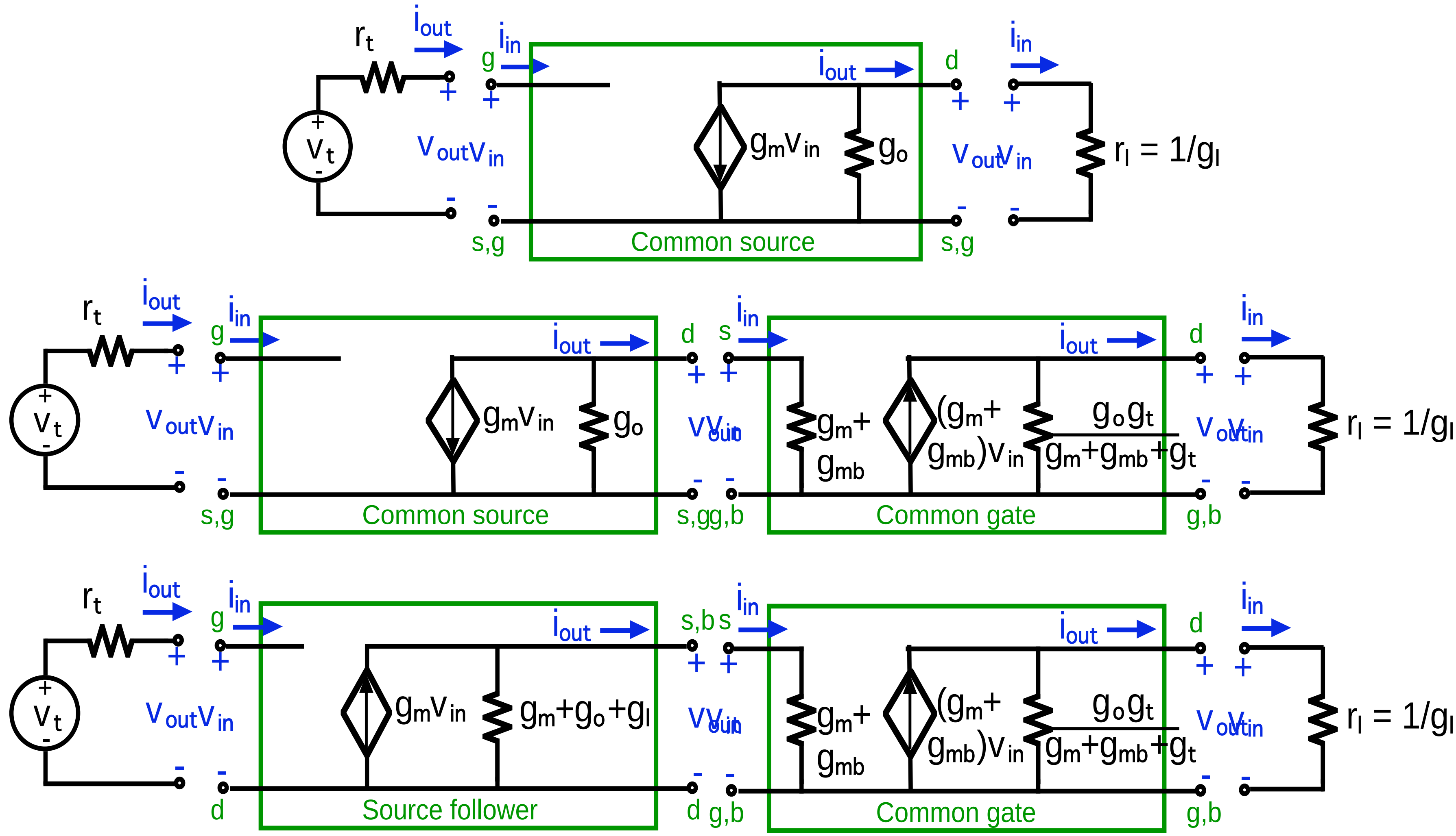
g_m, g_{mb}: **large**

g_o: **small**

g_t, g_l: **cannot generalize**

Multi-stage amplifier analysis and design: Two-port models

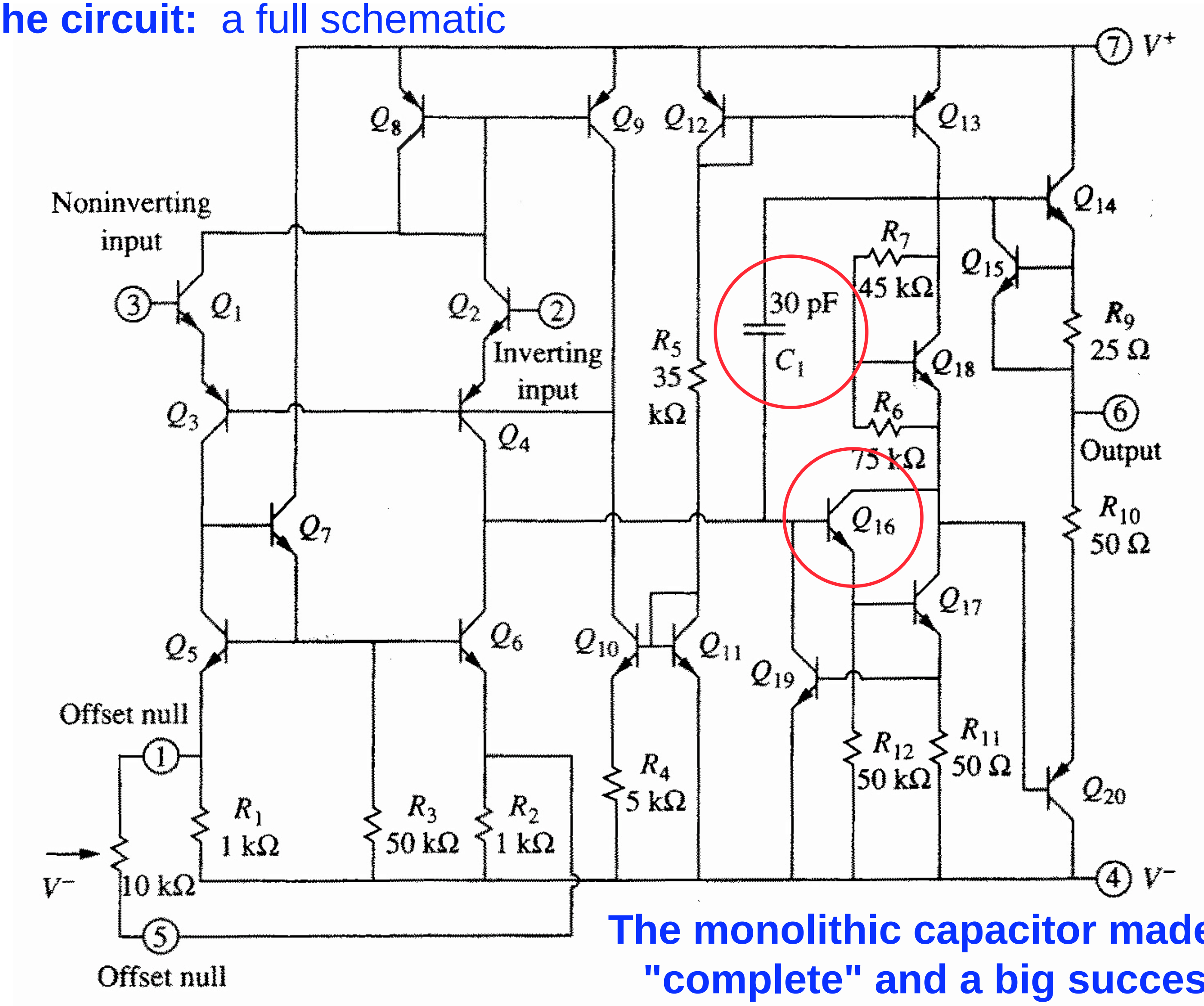
Two different "cascode" configurations:



With MOSFETs, starting a cascode with a source follower costs a factor of two in gain because r_{out} for an SF is small, so it isn't very attractive.

Multi-stage amplifier analysis and design: The μA741

The circuit: a full schematic



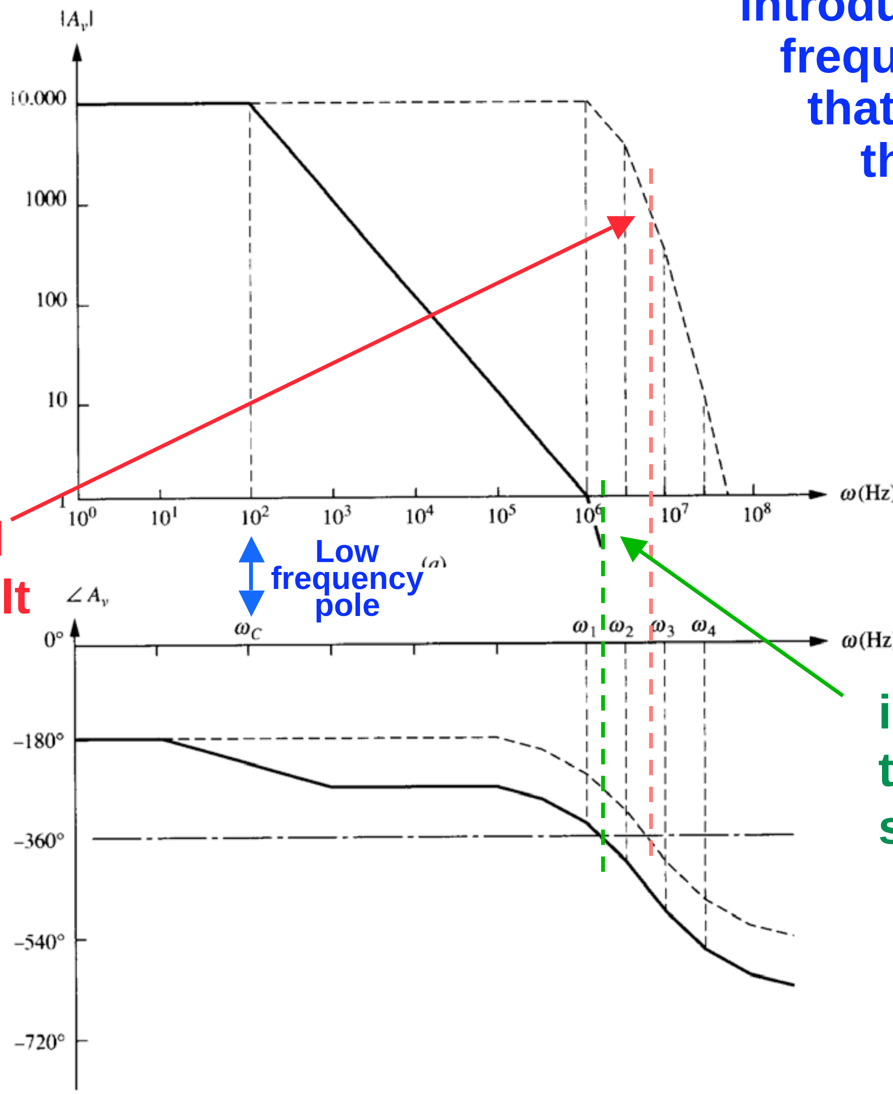
C_1 is in a Miller position across Q_{16}

The monolithic capacitor made the μA741 "complete" and a big success. Why is it needed? What does it do?

Multi-stage amplifier analysis and design: The µA741

Why is there a capacitor in the circuit?: the added capacitor introduces a low frequency pole that stabilizes the circuit.

Without it the gain is still greater than 1 when the phase shift exceeds 180° (dashed curve). This can result in positive feedback and instability.



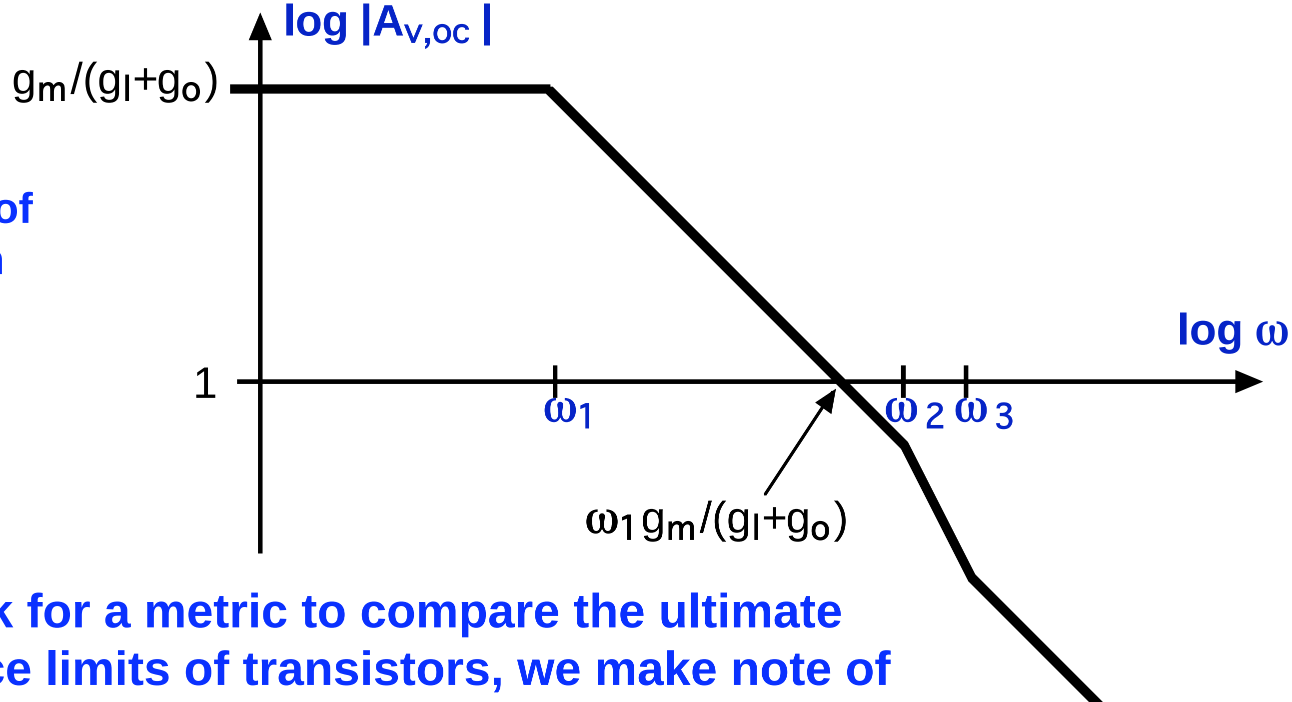
With it the gain is less than 1 by the time the phase shift exceeds 180° (solid curve).

Intrinsic performance - **the best we can do**

We've focused on ω_{HI}, the upper limit of mid-band, but even when $\omega > \omega_{HI}$ the $|A_v| > 1$, and the circuit is useful. For example, for the common source stage we had

$$A_v(j\omega) = \frac{-g_t\left(g_m - j\omega C_{gd}\right)}{\left\{(j\omega)^2 C_{gs}C_{gd} + j\omega\left[(g_l+g_o)C_{gs} + (g_l+g_o+g_t+g_m)C_{gd}\right] + (g_l+g_o)g_t\right\}}$$

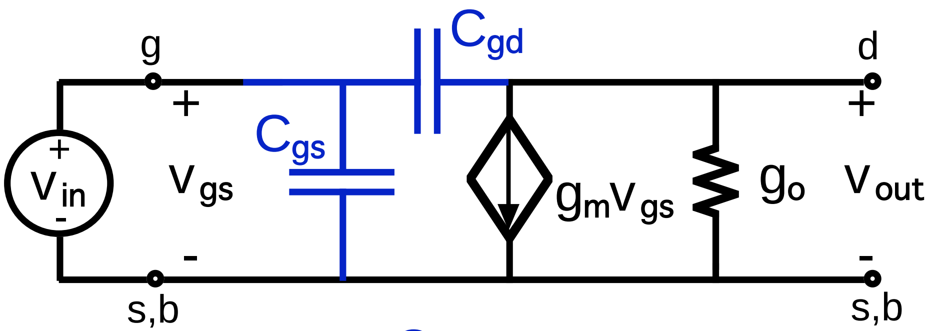
A Bode plot of A_v is shown to the right:



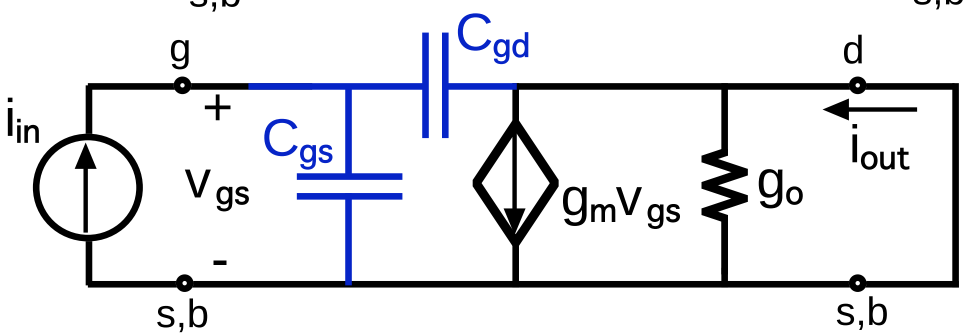
When we look for a metric to compare the ultimate performance limits of transistors, we make note of this and ask how high can a device in isolation have provide voltage or current gain?

Intrinsic performance - the best we can do, cont.

Consider the two possibilities shown below, one for a voltage input and output where the metric would be the open circuit voltage gain, $A_{v,oc}$, and the other for a current input and output with the metric being the short circuit current gain, $A_{i,sc}$ (commonly written β_{sc}):



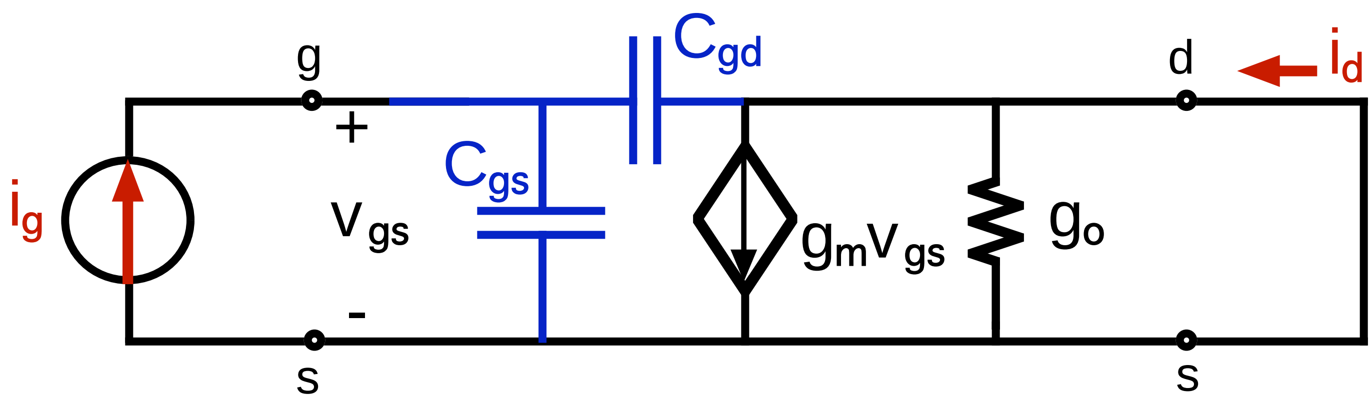
$$A_{v,oc}(s) \equiv \frac{v_{out}(j\omega)}{v_{in}(j\omega)} = -\frac{g_m - j\omega C_{gd}}{g_o - j\omega C_{gd}}$$



$$\beta_{sc}(j\omega) \equiv \frac{i_d(j\omega)}{i_g(j\omega)} = \frac{g_m - j\omega C_{gd}}{j\omega\left(C_{gs} + C_{gd}\right)}$$

Of these two alternatives, β_{sc} is the more useful. $A_{v,oc}$ is derived with a voltage source driving a capacitor, something that doesn't give a meaningful result and leads to ever increasing input power. It also does not involve g_m and C_{gs}. Consequently, short circuit current gain is used as the intrinsic high frequency performance metric for transistors.

Intrinsic ω_{HI}'s for MOSFETs - short-circuit current gain



The common-source short-circuit current gain is:

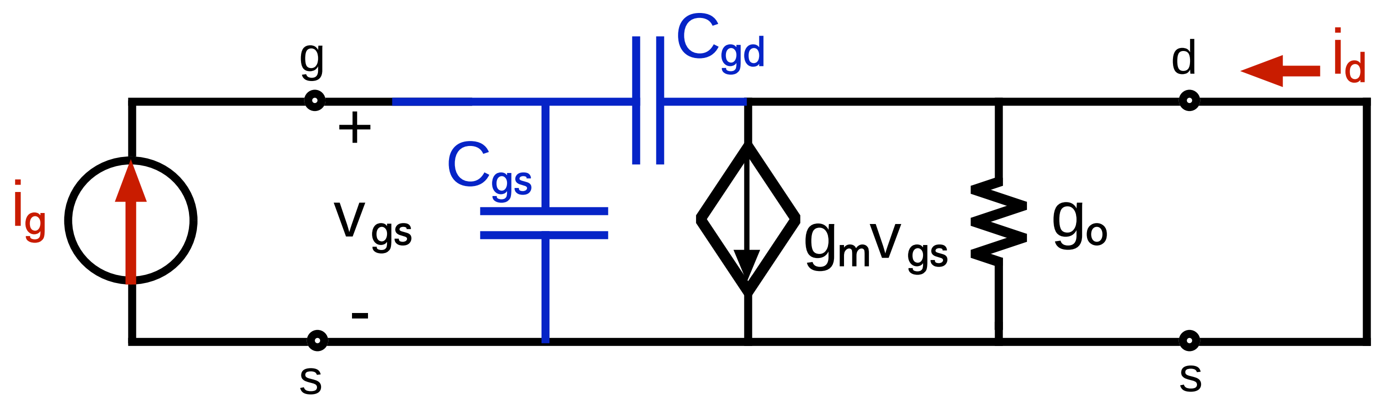
$$\beta_{sc}(j\omega) \equiv \frac{i_d(j\omega)}{i_g(j\omega)} = \frac{g_m - j\omega C_{gd}}{j\omega\left(C_{gs} + C_{gd}\right)}$$

there is one pole at $\omega = 0$, and one zero, ω_z:

$$\omega_z = \frac{g_m}{C_{gd}}$$

The short circuit current gain, β_{sc}, is infinite at DC ($\omega = 0$) , and its magnitude decreases linearly with increasing frequency.

Intrinsic ω_{HI}'s for MOSFETs - short-circuit current gain, cont.



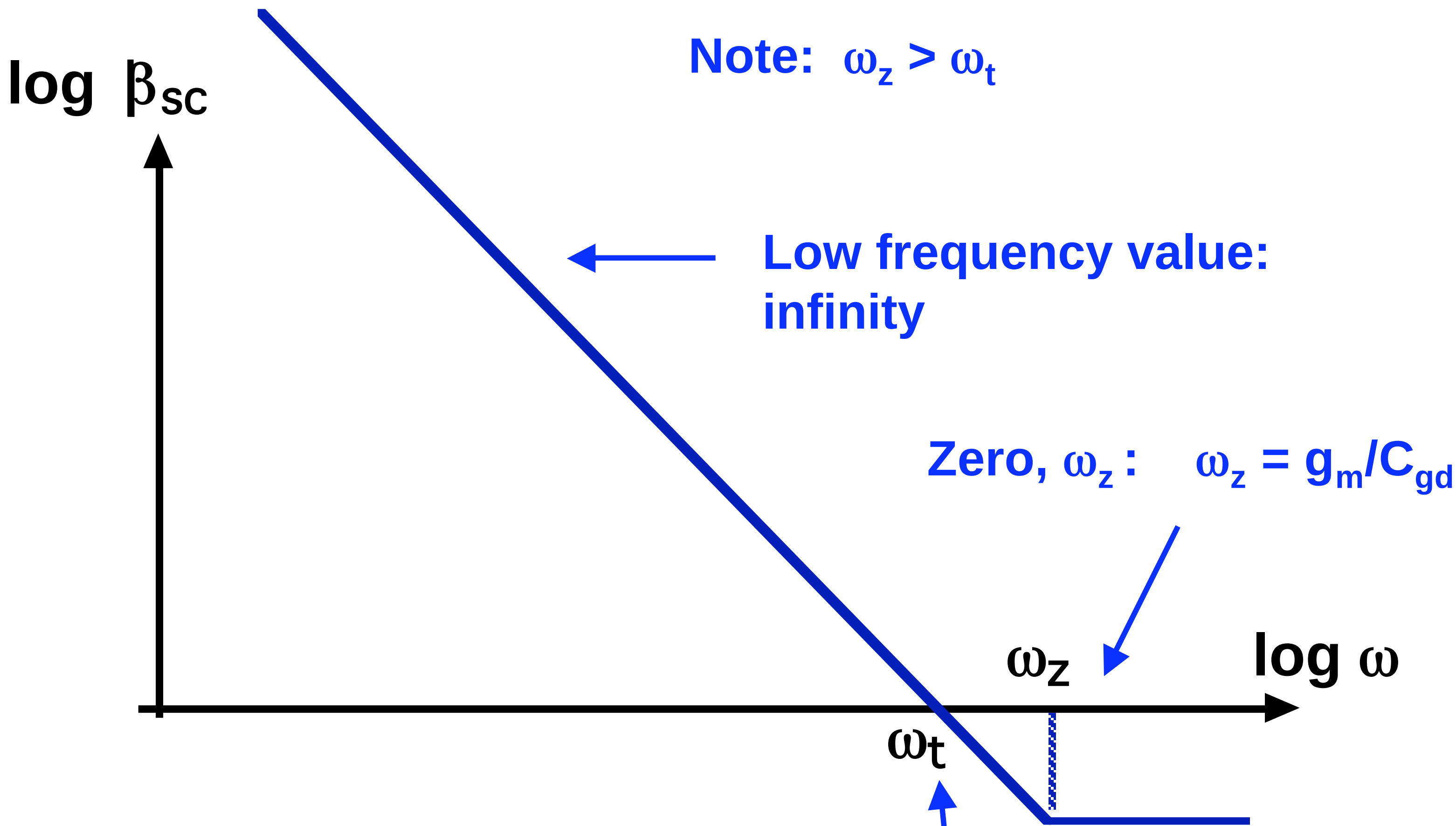
The magnitude of β_{sc} decreases with ω, but it is still greater than one for a wide range of frequencies.

$$|\beta_{sc}(j\omega)| = \sqrt{\frac{g_m^2 + \omega^2 C_{gd}^2}{\omega^2 \left(C_{gs} + C_{gd}\right)^2}}$$

The transistor is useful until $|\beta_{sc}|$ is less than one. The frequency at which this occurs is called ω_t. Setting = 1 and solving for ω_t yields:

$$\omega_t = \sqrt{\frac{g_m^2}{\left[\left(C_{gs} + C_{gd}\right)^2 - C_{gd}^2\right]}} \approx \frac{g_m}{\left(C_{gs} + C_{gd}\right)}$$

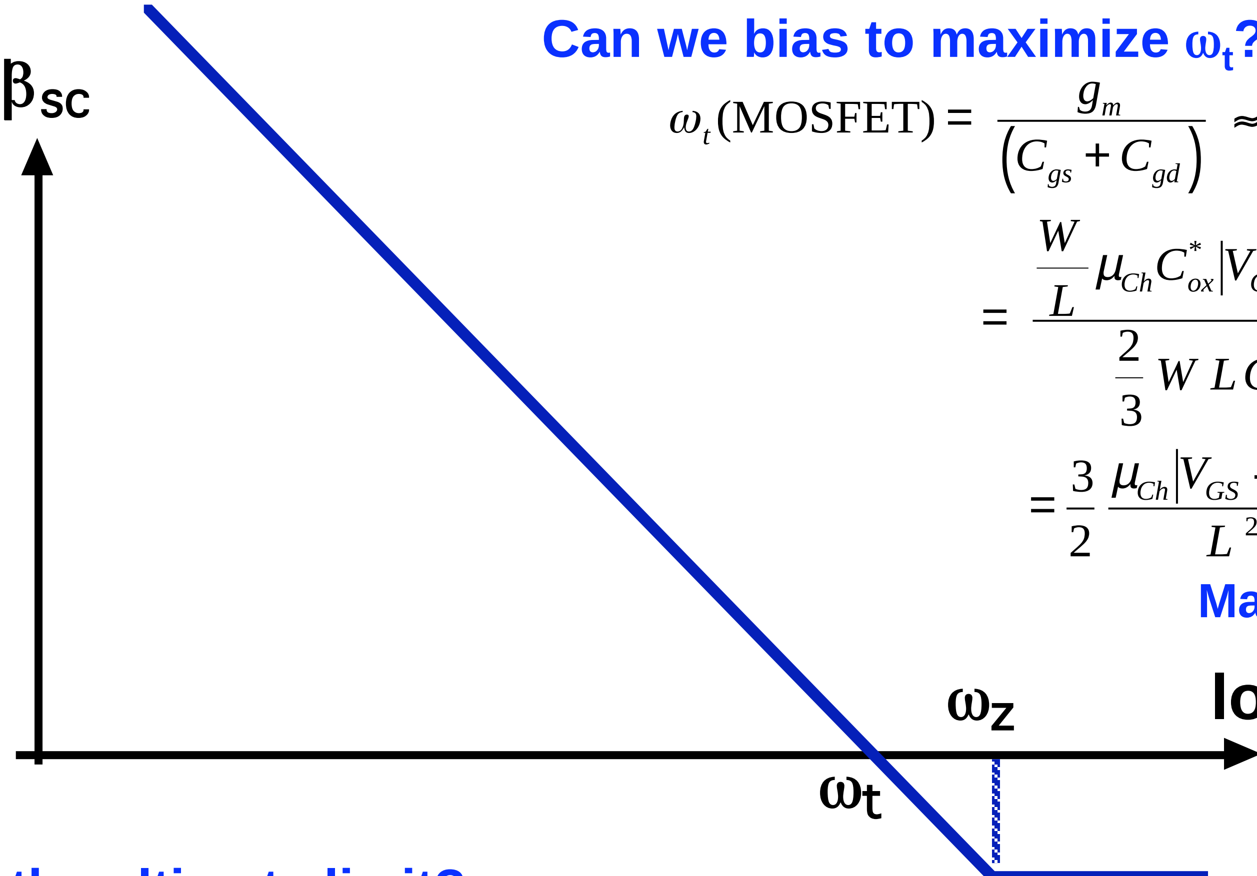
MOSFET short-circuit current gain, $\beta_{sc}(j\omega)$, cont.



No 3dB point, ω_b.

Unity gain point, ω_t : ω_t @ $g_m/(C_{gs}+C_{gd})$

MOSFET short-circuit current gain, $\beta_{sc}(j\omega)$, cont.



Can we bias to maximize ω_t?

$$\omega_t(\text{MOSFET}) = \frac{g_m}{\left(C_{gs} + C_{gd}\right)} \approx \frac{g_m}{C_{gs}}$$

$$= \frac{\frac{W}{L}\mu_{Ch} C_{ox}^* \left|V_{GS} - V_T\right|}{\frac{2}{3} W\, L\, C_{ox}^*}$$

$$= \frac{3}{2}\frac{\mu_{Ch}\left|V_{GS} - V_T\right|}{L^2}$$

Maximize V_{GS}.

What is the ultimate limit?

$$\omega_t(\text{MOSFET}) = \frac{3}{2}\frac{\mu_{Ch}\left|V_{GS} - V_T\right|}{L^2} = \frac{3}{2L}\mu_{Ch}\frac{\left|V_{DS}\right|}{L} = \frac{3}{2L}\mu_{Ch}\overline{E_{Ch}} = \frac{3}{2}\frac{\overline{s_{Ch}}}{L} = \frac{1}{\tau_{Ch}}$$

Channel transit time!

Lessons: Bias at well above V_T; make L small, use n-channel.

An aside: looking back at CMOS gate delays

CMOS: switching speed; minimum cycle time (from Lec. 15)

Gate delay/minimum cycle time:

For MOSFETs operating in strong inversion, no velocity saturation:

$$\tau_{Min\,Cycle} = \frac{12 n L_{\min}^2 V_{DD}}{\mu_e \left[V_{DD} - V_{Tn}\right]^2}$$

Comparing this to the channel transit time:

$$\tau_{Ch\,Transit} = \frac{L_{\min}}{\bar{s}_{e,Ch}} = \frac{L_{\min}}{\mu_e \mathrm{E}_{Ch}} = \frac{L_{\min}}{\mu_e \left(V_{DD} - V_{Tn}\right) / L_{\min}}$$

We see that the cycle time is a multiple of the transit time:

$$\tau_{Min\,Cycle} = \frac{12 n V_{DD}}{\left(V_{DD} - V_{Tn}\right)} \tau_{Channel\,Transit} = n' \, \tau_{Channel\,Transit}$$

When velocity saturation dominated, we found the same thing:

$$\tau_{Min.Cycle} \propto \frac{L_{\min} V_{DD}}{s_{sat}\left[V_{DD} - V_{Tn}\right]} = n' \tau_{ChanTransit} \quad \textbf{where} \quad \tau_{ChanTransit} = \frac{L}{s_{sat}}$$

Intrinsic ω_{HI}'s for MOSFETs - $\beta_{sc}(j\omega)$ and ω_t w. velocity saturation

What about the intrinsic ω_{HI} of a MOSFET operating with full velocity saturation?

The basic result is unchanged; we still have:

$$\omega_t = \sqrt{\frac{g_m^2}{\left[\left(C_{gs} + C_{gd}\right)^2 - C_{gd}^2\right]}} \approx \frac{g_m}{\left(C_{gs} + C_{gd}\right)} \approx \frac{g_m}{C_{gs}}$$

However, now g_m is different:

$$g_m = W\, s_{sat}\, C_{ox}^*$$

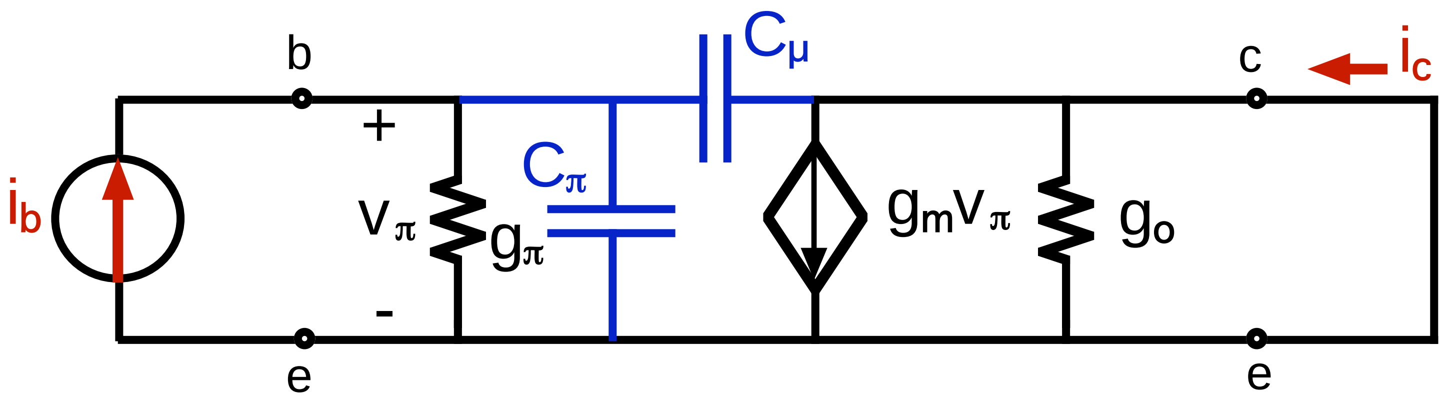
With this we have:

$$\omega_t \approx \frac{g_m}{C_{gs}} = \frac{W\, s_{sat}\, C_{ox}^*}{W\, L\, C_{ox}^*} = \frac{s_{sat}}{L} = \frac{1}{\tau_{Ch}}$$

In the case where <u>velocity saturation</u> dominates, we once again find that it is the channel transit time that is the ultimate limit.

Do you care to speculate on the intrinsic ω_{HI} of a BJT?

Intrinsic ω_{HI}'s for BJTs - short-circuit current gain



The common-emitter short-circuit current gain is:

$$\beta_{sc}(j\omega) \equiv \frac{i_c(j\omega)}{i_b(j\omega)} = \frac{g_m - j\omega C_\mu}{\left[g_\pi + j\omega\left(C_\pi + C_\mu\right)\right]}$$

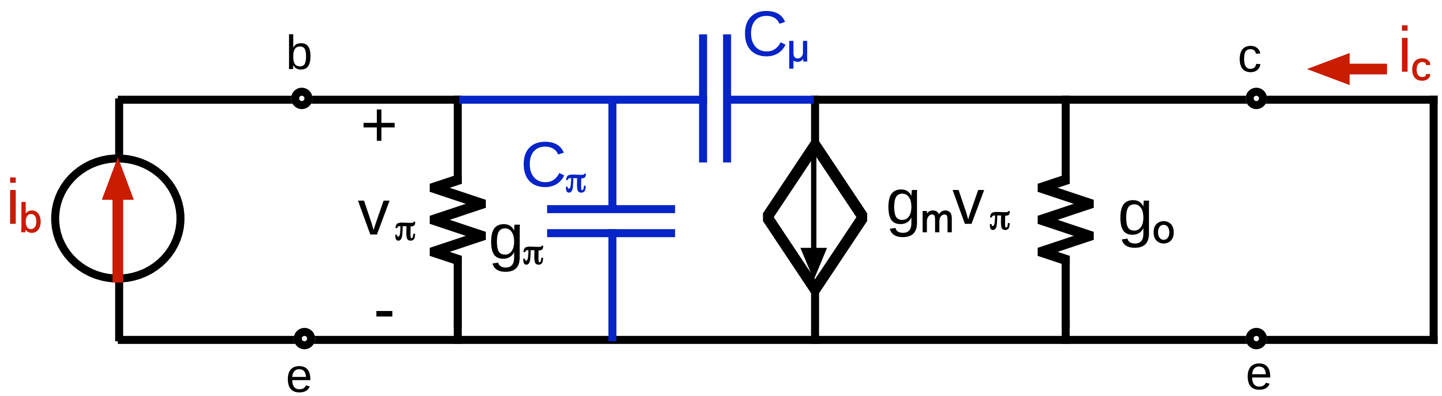
there is one pole, call it ω_p, and one zero, ω_z:

$$\omega_p = \frac{g_\pi}{\left(C_\pi + C_\mu\right)}, \qquad \omega_z = \frac{g_m}{C_\mu}$$

Of these two, ω_p is much smaller and this is the 3dB point of the common-emitter short-circuit current gain. We give it the name ω_β:

$$\omega_\beta = \frac{g_\pi}{\left(C_\pi + C_\mu\right)}$$

Intrinsic ω_{HI}'s for BJTs - short-circuit current gain, cont.



The magnitude of β_{sc} decreases above ω_b, but it is still greater than one initially:

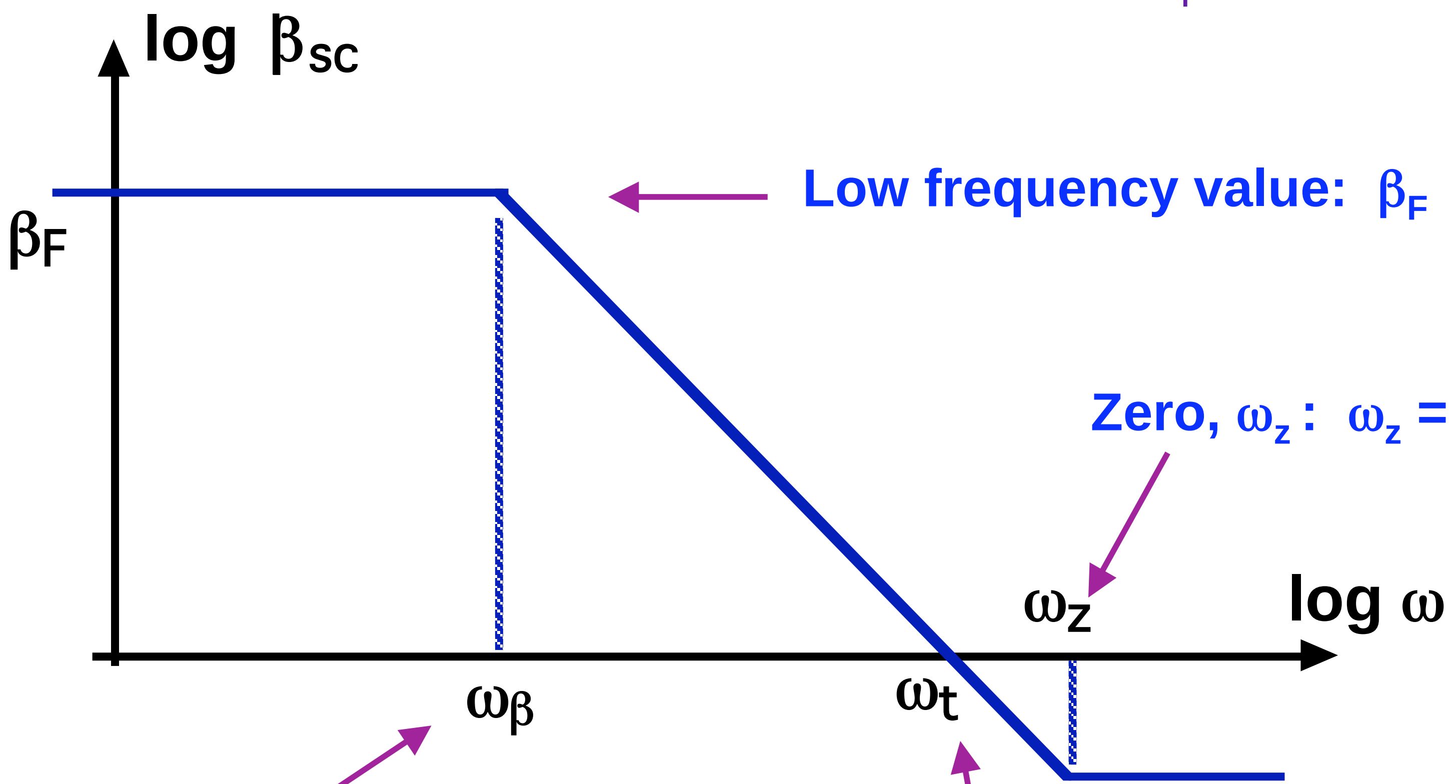
$$\left|\beta_{sc}(j\omega)\right| = \sqrt{\frac{g_m^2 + \omega^2 C_\mu^2}{\left[g_\pi^2 + \omega^2 (C_\pi + C_\mu)^2\right]}}$$

The transistor is useful until $|\beta_{sc}|$ is less than one. The frequency at which this occurs is called ω_t. Setting = 1 and solving for ω_t yields:

$$\omega_t = \sqrt{\frac{(g_\pi^2 + g_m^2)}{\left[(C_\pi + C_\mu)^2 - C_\mu^2\right]}} \approx \frac{g_m}{(C_\pi + C_\mu)}$$

BJT short-circuit current gain, $\beta_{sc}(j\omega)$, cont.

Note: $\omega_z > \omega_t >> \omega_\beta$ $(= \omega_t / \beta_F)$



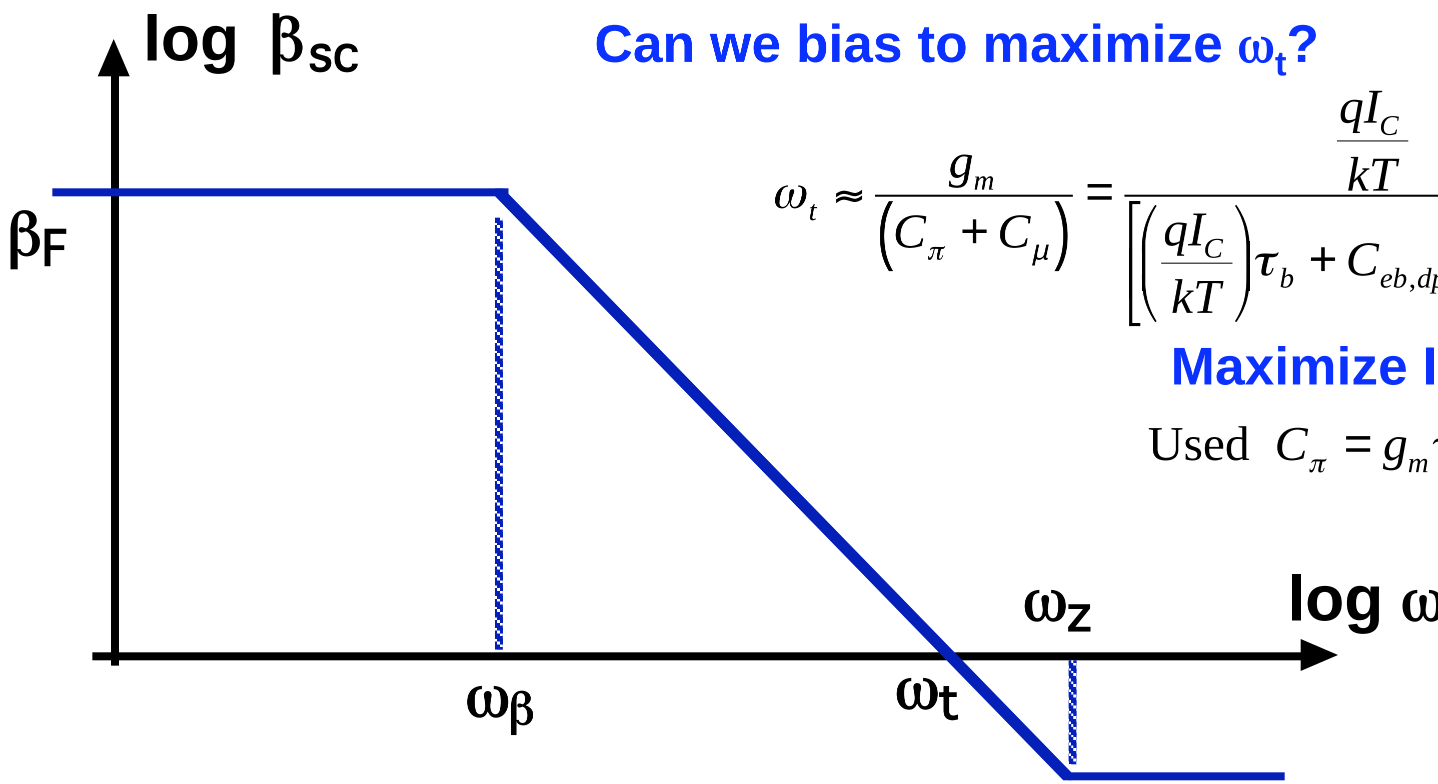
Low frequency value: β_F

Zero, ω_z **:** $\omega_z = g_m/C_\mu$

3dB point, ω_b**:** $\omega_b = g_\pi/(C_\pi+C_\mu)$

Unity gain point, ω_t **:** ω_t @ $g_m/(C_\pi+C_\mu)$

BJT short-circuit current gain, $\beta_{sc}(j\omega)$, cont.



Can we bias to maximize ω_t?

$$\omega_t \approx \frac{g_m}{\left(C_\pi + C_\mu\right)} = \frac{\dfrac{qI_C}{kT}}{\left[\left(\dfrac{qI_C}{kT}\right)\tau_b + C_{eb,dp} + C_{cb,dp}\right]}$$

Maximize I_C.

Used $C_\pi = g_m \tau_b + C_{eb,dp}$

Base transit time

In the limit of large I_C: $\lim_{I_C \to \infty} \omega_t \approx \dfrac{1}{\tau_b} = \dfrac{2D_{\min,B}}{w_B^2} = \dfrac{2\mu_{\min,B} V_{thermal}}{w_B^2}$

Base transit time

Lessons: Bias at large I_C; make w_B small, use npn.

Lecture 24 - Intrinsic Limits of Transistor Speed - Summary

- **Intrinsic high frequency limits for transistors**

  General approach: short-circuit current gains

- **Limits for MOSFETs:**

  Metric - CS short-circuit current unity gain pt: $\omega_T = g_m/[(C_{gs}+C_{gd})^2 - C_{gd}^2]^{1/2}$

  ω_T is approximately $g_m/C_{gs} = 3\mu_e(V_{GS}-V_T)/2L^2$

  $g_m = (W/L)\mu_e C_{ox}^*(V_{GS}-V_T)$ and $C_{gs} = (2/3)WLC_{ox}^*$

  so $\omega_T \approx 3\mu_e(V_{GS}-V_T)/2L^2 = 1/\tau_{ch}$

  Design lessons: bias at large I_D

  minimize L **(win as L^2; as L in velocity saturation)**

  use n-channel rather than p-channel **($\mu_e >> \mu_h$)**

- **Limits for BJTs:**

  Metrics - CE short-circuit current gain 3B pt: $\omega_b = g_p/(C_\pi + C_\mu)$

  CE short-circuit current gain unit gain pt: $\omega_T = g_m/(C_\pi + C_\mu)$

  ω_T approaches $1/\tau_b$ as I_c increases and $\tau_b = w_B^2/2D_{min,B}$

  so $\omega_T \approx 2D_{min,B}/w_B^2 = 2\mu_e V_t/w_B^2 = 1/\tau_b$

  CB short-circuit current gain unit gain pt: $\omega_\alpha = g_m/C_\pi$

  Design lessons: bias at high collector current

  minimize w_B **(win as w_B^2)**

  use npn rather than pnp **($\mu_e >> \mu_h$)**