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6.013/ESD.013J Electromagnetics and Applications, Fall 2005

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Markus Zahn, Erich Ippen, and David Staelin, *6.013/ESD.013J Electromagnetics and Applications, Fall 2005*. (Massachusetts Institute of Technology: MIT OpenCourseWare). <http://ocw.mit.edu> (accessed MM DD, YYYY). License: Creative Commons Attribution-Noncommercial-Share Alike.

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Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.013 FORMULA SHEET

1. DIFFERENTIAL OPERATORS IN CYLINDRICAL AND SPHERICAL COORDINATES

If r , ϕ , and z are circular cylindrical coordinates and \hat{i}_r , \hat{i}_ϕ , and \hat{i}_z are unit vectors in the directions of increasing values of the corresponding coordinates,

$$\nabla U = \text{grad} U = \hat{i}_r \frac{\partial U}{\partial r} + \hat{i}_\phi \frac{1}{r} \frac{\partial U}{\partial \phi} + \hat{i}_z \frac{\partial U}{\partial z}$$

$$\nabla \cdot \vec{A} = \text{div} \vec{A} = \frac{1}{r} \frac{\partial (rA_r)}{\partial r} + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \text{curl} \vec{A} = \hat{i}_r \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{i}_\phi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{i}_z \left(\frac{1}{r} \frac{\partial (rA_\phi)}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \phi} \right)$$

$$\nabla^2 U = \text{div grad} U = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 U}{\partial \phi^2} + \frac{\partial^2 U}{\partial z^2}$$

If r , θ , and ϕ are spherical coordinates and \hat{i}_r , \hat{i}_θ , and \hat{i}_ϕ are unit vectors in the directions of increasing values of the corresponding coordinates,

$$\nabla U = \text{grad} U = \hat{i}_r \frac{\partial U}{\partial r} + \hat{i}_\theta \frac{1}{r} \frac{\partial U}{\partial \theta} + \hat{i}_\phi \frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \text{div} \vec{A} = \frac{1}{r^2} \frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (A_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \text{curl} \vec{A} = \hat{i}_r \left(\frac{1}{r \sin \theta} \frac{\partial (A_\phi \sin \theta)}{\partial \theta} - \frac{1}{r \sin \theta} \frac{\partial A_\theta}{\partial \phi} \right) + \hat{i}_\theta \left(\frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial (rA_\phi)}{\partial r} \right) + \hat{i}_\phi \left(\frac{1}{r} \frac{\partial (rA_\theta)}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \theta} \right)$$

$$\nabla^2 U = \text{div grad} U = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial U}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial U}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 U}{\partial \phi^2}$$

2. SOLUTIONS OF LAPLACE'S EQUATIONS

A. Rectangular coordinates, two dimensions (independent of z):

$$\Phi = e^{kx} (A_1 \sin ky + A_2 \cos ky) + e^{-kx} (B_1 \sin ky + B_2 \cos ky)$$

(or replace e^{kx} and e^{-kx} by $\sinh kx$ and $\cosh kx$).

$$\Phi = Axy + Bx + Cy + D; (k = 0)$$

B. Cylindrical coordinates, two dimensions (independent of z):

$$\Phi = r^n (A_1 \sin n\phi + A_2 \cos n\phi) + r^{-n} (B_1 \sin n\phi + B_2 \cos n\phi)$$

$$\Phi = \ln \frac{R}{r} (A_1 \phi + A_2) + B_1 \phi + B_2; (n = 0)$$

C. Spherical coordinates, two dimensions (independent of ϕ):

$$\Phi = Ar \cos \theta + \frac{B}{r^2} \cos \theta + \frac{C}{r} + D$$