ELECTROMAGNETICS AND APPLICATIONS

6.013 Content:

Fundamentals and Applications
Fields, media, and boundaries
Circuits
Motors, generators, and MEMS
Limits to computation speed
Microwave communications and radar
Wireless communications and waves
Optical devices and communications
Acoustics

Prerequisites:
6.003 or 6.02+6.007
[6.002+8.02+18.02]
Appendices B-E

Handouts:
Administration sheet, Equations
Subject outline, lecture notes
Prob. set 1, Objectives & Outcomes

WHAT ARE $\overline{E}$ AND $\overline{H}$?

Lorentz Force Law:

$$\overline{f} = q(\overline{E} + \overline{v} \times \mu_o \overline{H}) \quad \text{[Newtons]}$$

$4\pi \times 10^{-7}$

Velocity [m/s]

Charge [Coulombs]

"EM fields were invented to explain forces"

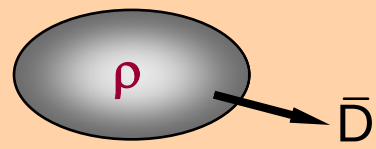
- Find $\overline{E}$ by setting $\overline{v} = 0$ and measuring $\overline{f}$
- Find $\overline{H}$ by setting $\overline{v} = 0$, $\overline{E} = 0$, and measuring $\overline{f}$
  (requires 2 $\overline{f}$ measurements using 2 $\overline{v}$ tests)

INTEGRAL MAXWELL'S EQUATIONS

Graphical Equations:

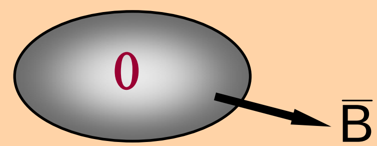
$$\oint\!\!\oint_S \overline{D} \bullet \hat{n} da = \iiint_V \rho \, dv$$

Gauss ⟺



$$\oint\!\!\oint_S \overline{B} \bullet \hat{n} da = 0$$

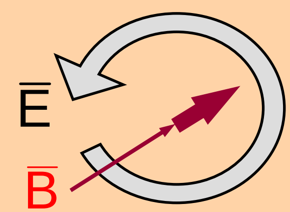
Gauss ⟺



0 (statics)

$$\oint_c \overline{E} \bullet d\overline{s} = -\frac{\partial}{\partial t} \iint_A \overline{B} \bullet \hat{n} da$$

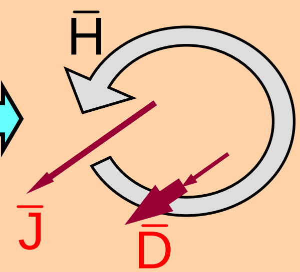
Faraday ⟺



0 (statics)

$$\oint_c \overline{H} \bullet d\overline{s} = \iint_A \overline{J} \bullet \hat{n} da + \frac{\partial}{\partial t} \iint_A \overline{D} \bullet \hat{n} da$$

Ampere ⟺



$$\overline{D} = \varepsilon \overline{E}, \qquad \overline{B} = \mu \overline{H}$$

Constitutive relations

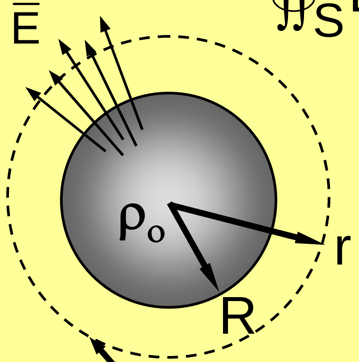
EXAMPLE: SPHERICAL CHARGE

Example I. Sphere of radius R, charge density ρ_o

Spherical symmetry precludes θ and ϕ components for $\overline{E}$

Solution:

$$\oiint_S \overline{D} \bullet \hat{n} da = \iiint_V \rho \, dv = Q \quad \text{(Gauss's Law)}$$

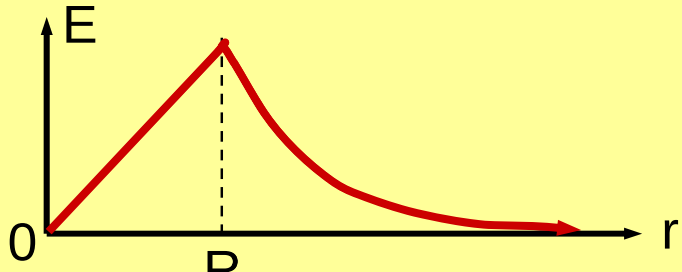


Surface = $4\pi r^2$

$$r > R \Rightarrow \varepsilon E_r (4\pi r^2) = Q \quad \Rightarrow E_r = \frac{Q}{4\pi\varepsilon r^2} \text{ [V/m]}$$

$$r < R \Rightarrow \varepsilon E_r (4\pi r^2) = \rho_o (\frac{4}{3}\pi r^3) \quad \Rightarrow E_r = \frac{\rho_o r}{3\varepsilon} \text{ [V/m]}$$

Equivalently, $r > R \Rightarrow E_r = \dfrac{\rho_o R^3}{3\varepsilon r^2}$



EXAMPLE: CYLINDRICAL CHARGE

Example 2. Cylinder of radius R, charge density ρ_o

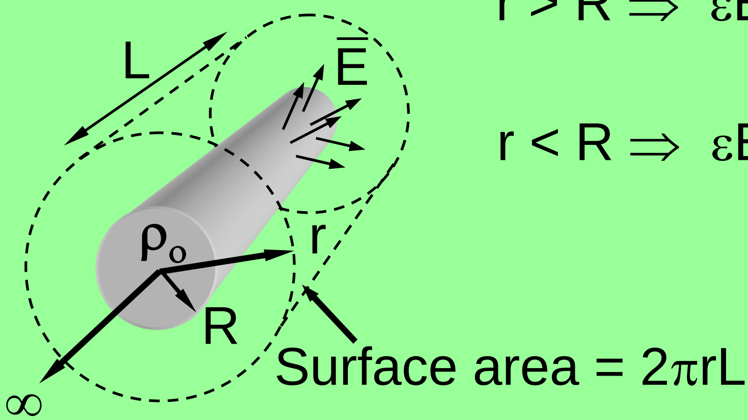
Cylindrical symmetry precludes θ and z components for $\overline{E}$

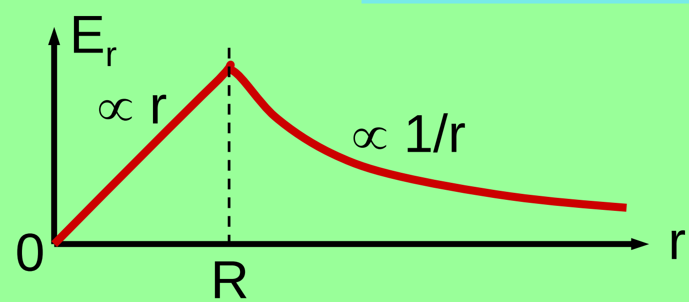
Solution:

$$\oiint_S \overline{D} \bullet \hat{n}\, da = \iiint_V \rho\, dv \qquad \text{(Gauss)}$$

$$r > R \Rightarrow \varepsilon E_r (2\pi r L) = \rho_o (\pi R^2 L) \;\Rightarrow\; E_r = \frac{\rho_o R^2}{2\varepsilon r} \ \text{[V/m]}$$

$$r < R \Rightarrow \varepsilon E_r (2\pi r L) = \rho_o (\pi r^2 L) \;\Rightarrow\; E_r = \frac{\rho_o r}{2\varepsilon} \ \text{[V/m]}$$





HOLLOW SPHERICAL CHARGE

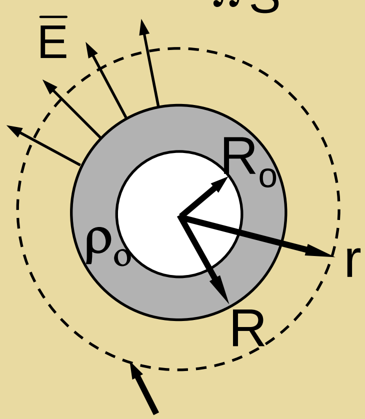
Example 3:

Hollow sphere of inner radius R_o, charge density ρ_o

Spherical symmetry precludes θ and ϕ components for $\overline{E}$

Solution:

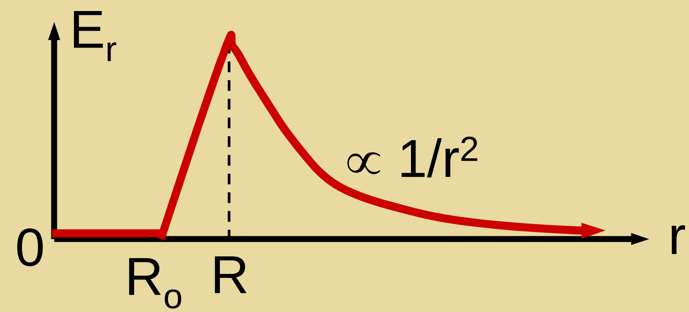
$$\oiint_S \overline{D} \bullet \hat{n} da = \iiint_V \rho \, dv = Q \quad \text{(Gauss's Law)}$$



Surface = $4\pi r^2$

$$r > R \Rightarrow \varepsilon E_r(4\pi r^2) = Q \Rightarrow E_r = \frac{Q}{4\pi\varepsilon r^2} \ \text{[V/m]}$$

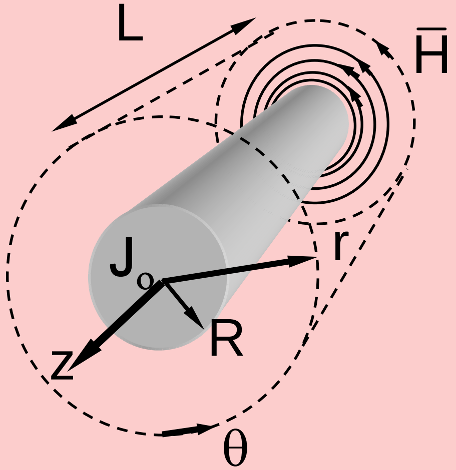
$$r < R_o \Rightarrow \varepsilon E_r(4\pi r^2) = 0 \Rightarrow E_r = 0 \ \text{[V/m]}$$



Hollow cylinders also have zero $\overline{E}$

CYLINDRICAL CURRENT $\bar{J}$[A/m²]

Example 4. Uniform current J_z for $r < R$:

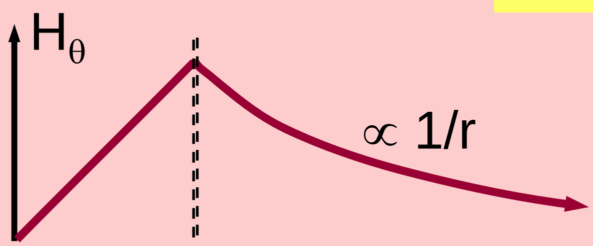


$$\oint_c \bar{H} \bullet d\bar{s} = \iint_A \bar{J} \bullet \hat{n} da + \frac{\partial}{\partial t} \iint_A \bar{D} \bullet \hat{n} da$$

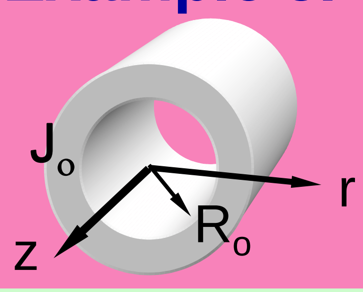
0

$r > R$: $2\pi r H_\theta = I = J_o \pi R^2 \Rightarrow H_\theta = I/2\pi r$ [A/m]

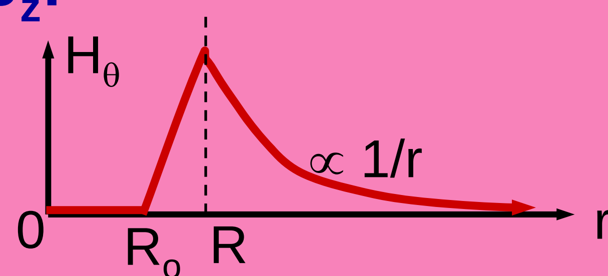
$r < R$: $2\pi r H_\theta = J_o \pi r^2 \Rightarrow H_\theta = J_o r/2$ [A/m]



Example 5. Hollow current J_z:



Hollow cylinders also have zero H for $r < R_o$

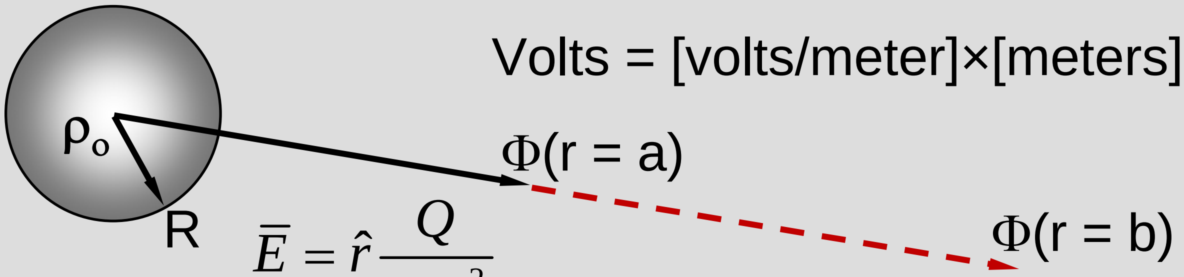


STATIC POTENTIALS Φ(Volts)

Example 6. Spherical charge density ρ_o, radius R:

What is the electric potential $\Phi(r)$ [volts] for r > R?

Volts = [volts/meter]×[meters]



$$\overline{E} = \hat{r}\frac{Q}{4\pi r^2}$$

Solution:

$$\Phi_a - \Phi_b = \int_a^b \overline{E} \bullet d\overline{r} \quad \text{(Volts)}$$

$$\Phi(r) - \Phi_\infty = \int_r^\infty \overline{E} \bullet d\overline{r} = \Phi(r)$$

$$\Phi(r) = \int_r^\infty \frac{Q}{4\pi} r^{-2}\ \hat{r} \bullet d\overline{r} = -\frac{Q}{4\pi} r^{-1}\Big|_r^\infty$$

$$\Phi(r) = \frac{Q}{4\pi r} \text{ Volts,} \quad (r > R)$$

Absolute potentials are defined relative to infinity: $\Phi(\infty) \triangleq 0$

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