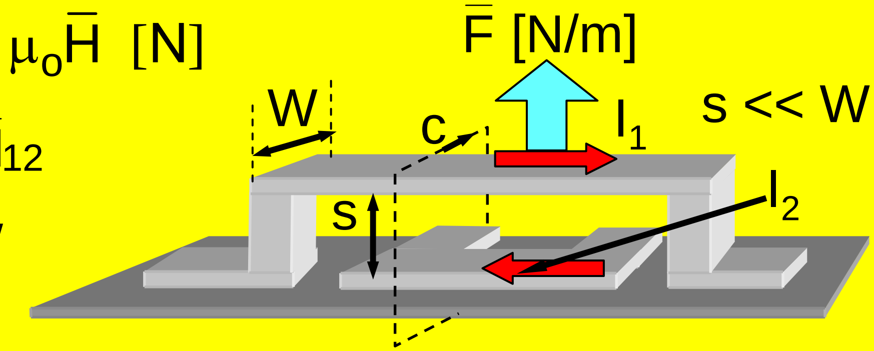
MAGNETIC FORCES ON FLAT SURFACES

Lorentz Force Law: $\bar{f}_m = q\bar{v} \times \mu_o \bar{H}$ [N]

$\bar{F}$ [N/m] $= Nq\bar{v}_1 \times \mu_o \bar{H}_{12} = \bar{I}_1 \times \mu_o \bar{H}_{12}$

$\oint_C \bar{H} \bullet d\bar{s} = \iint_A \bar{J} \bullet d\bar{a} = I_2 \cong 2H_{12}W$

$\bar{F} = \hat{z}\mu_o I_1 I_2 / 2W$ [N/m]



Currents exert no net force on themselves

Total H method:

$$\oint_C \bar{H} \bullet d\bar{s} = \iint_A \bar{J} \bullet d\bar{a} = \begin{cases} 0 \text{ for } C_1 \\ I \text{ for } C_2 \end{cases}$$

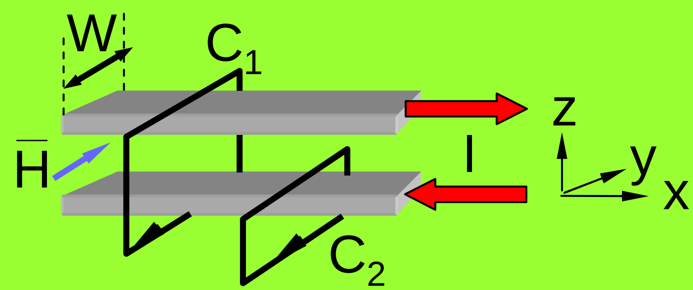
Let $I_1 = I_2 = I = H_o W$

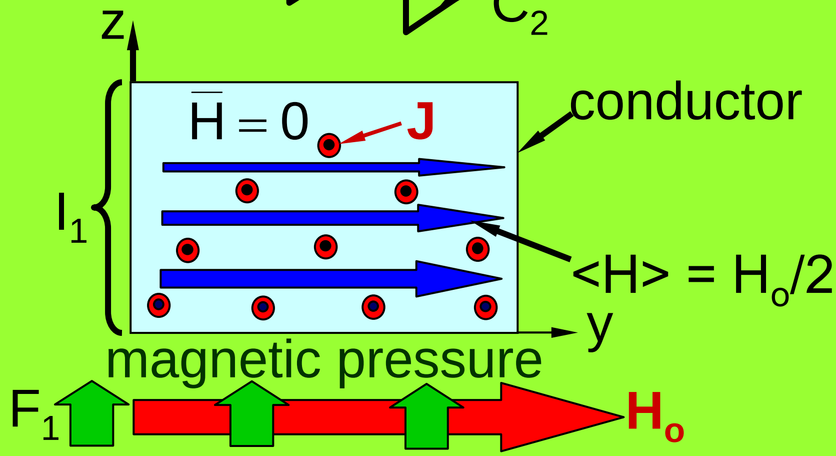
$\bar{F}_1$ [N/m] $= \bar{I}_1 \times \mu_o \langle \bar{H} \rangle$

$= \hat{z}\, I_1 \mu_o \dfrac{H_o}{2} = \hat{z}\mu_o I_1 I_2 / 2W$ [N/m]

$\bar{P} = \bar{F}/W = \hat{z}\,\dfrac{1}{2}\mu_o H_o^2$ [N/m^2]

"Magnetic pressure"





ROTARY WIRE MOTOR

Single wire loop spinning in uniform $\overline{H}$:

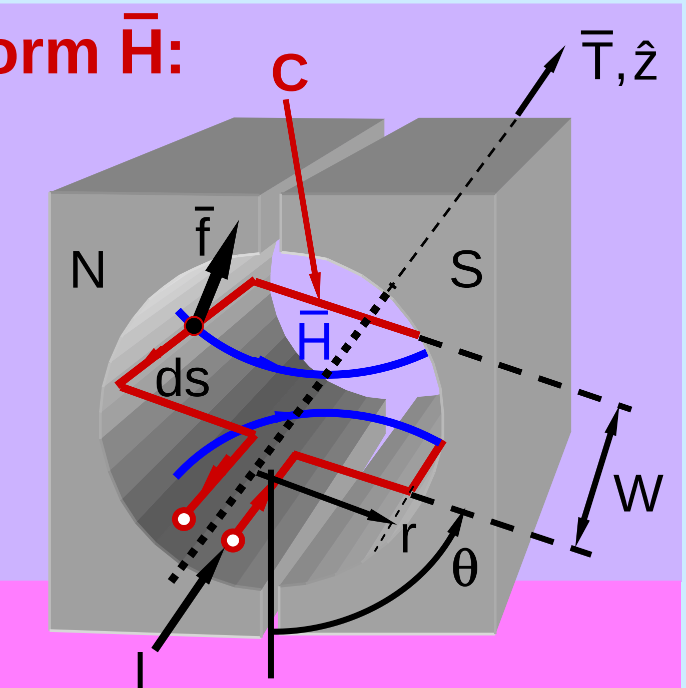
$\overline{F}\ [\mathrm{Nm}^{-1}] = \overline{I} \times \mu_o \overline{H}$

$\overline{T} = \oint_c \overline{r} \times \overline{F}\, ds = r2WI\mu_o H\hat{z}\ \ [\mathrm{Nm}]$

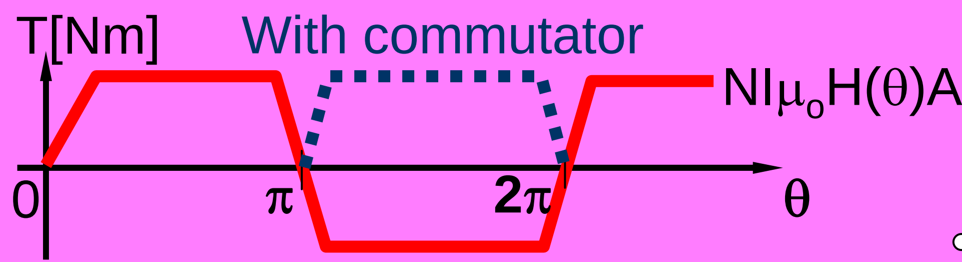
$\overline{T} = IA\mu_o H\hat{z}$ (A is loop area, N = 1 turn)

$\overline{T} = NIA\mu_o H\hat{z}$ for N-turn coil

Axial forces from wires at ends cancel

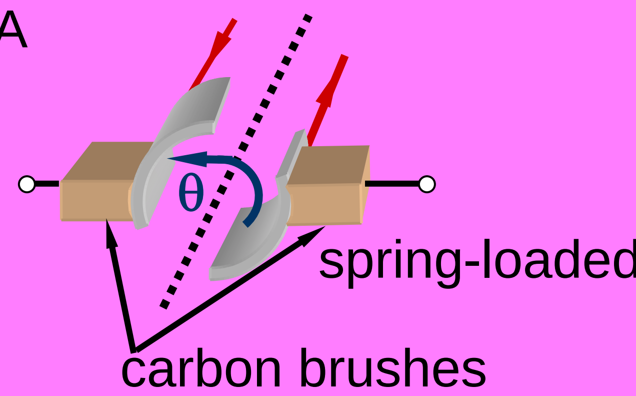


Torque = f(θ):



Commutators:

Switch currents to maximize torque
Can have N coils and 2N brushes



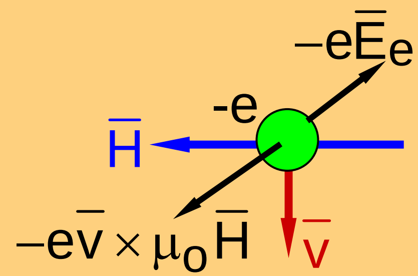
MOTOR BACK VOLTAGE

Force $\bar{f}_e$ on electron inside moving wire:

Open-circuit wire: $\bar{f}_e = -e\left(\bar{E}_e + \bar{v} \times \mu_o \bar{H}\right) = 0$

$\Rightarrow \bar{E}_e = -\bar{v} \times \mu_o \bar{H}$ inside

Open-circuit voltage: $\Phi = E_e W = v \mu_o H W$ [V]



Force balance

Mechanical power output, N turns:

$P_m = \omega T = \omega N I A \mu_o H$ [W]

$I = (V - \Phi)/R$

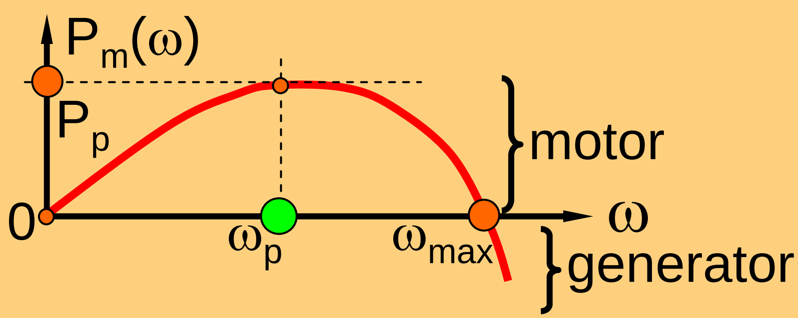
$\Phi = 2 N v \mu_o H W = 2 N \omega r \mu_o H W = N A \mu_o H \omega$

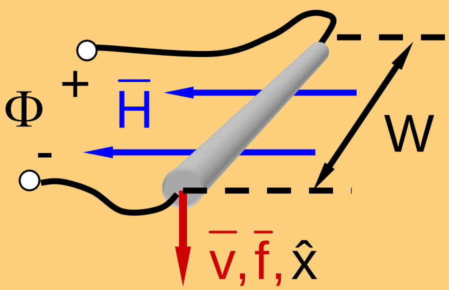
$P_m = \omega N(V - N A \mu_o H \omega) A \mu_o H / R = \omega K_1 - \omega^2 K_2$

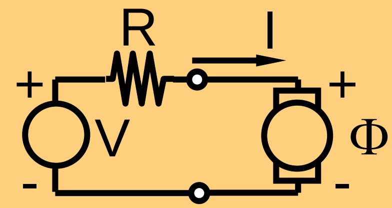
$\omega_{max} = V / N A \mu_o H$

$= 2\omega_p$

$\Phi_p = V/2$







RELUCTANCE MOTOR FIELDS

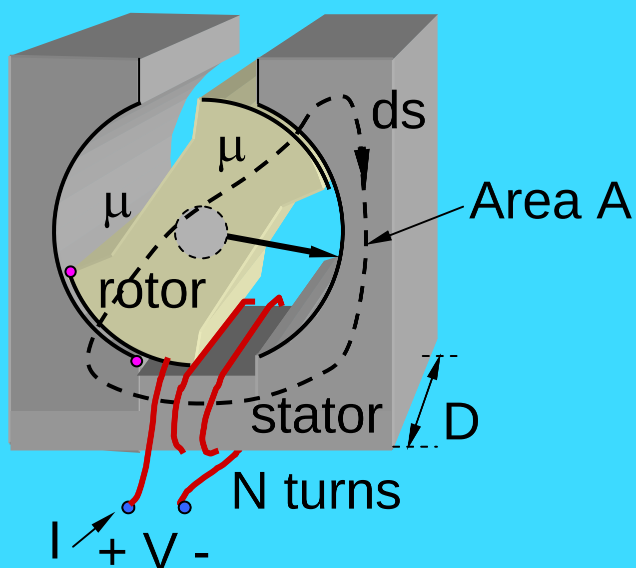
2-Pole Reluctance Motor:

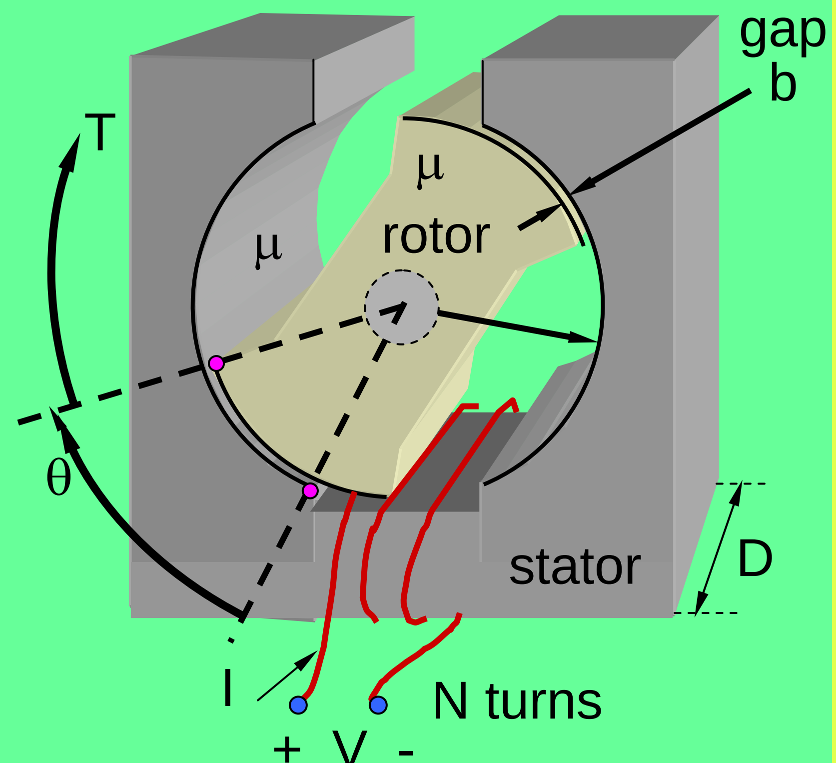
$$\nabla \bullet \overline{B} = 0 \;\Rightarrow\; \overline{B}_\mu \cong \overline{B}_{gap} \;\Rightarrow\; \overline{H}_\mu \cong \frac{\mu_o}{\mu}\overline{H}_{gap} << \overline{H}_{gap} \quad [\frac{\mu}{\mu_o} > \sim 10^4]$$

$$\int_A \overline{J}\bullet d\overline{a} = NI = \oint_c \overline{H}\bullet d\overline{s} = \oint_c \left(\overline{H}_{gap} + \overline{H}_\mu\right)\bullet d\overline{s} \cong 2bH_{gap}$$

(if $H_\mu \oint_c ds << 2bH_{gap}$)

Therefore $\boxed{H_{gap} \cong \frac{NI}{2b} \quad [A/m]}$





RELUCTANCE MOTOR TORQUE

Fields:

$$H_{gap} = \frac{NI}{2b}$$

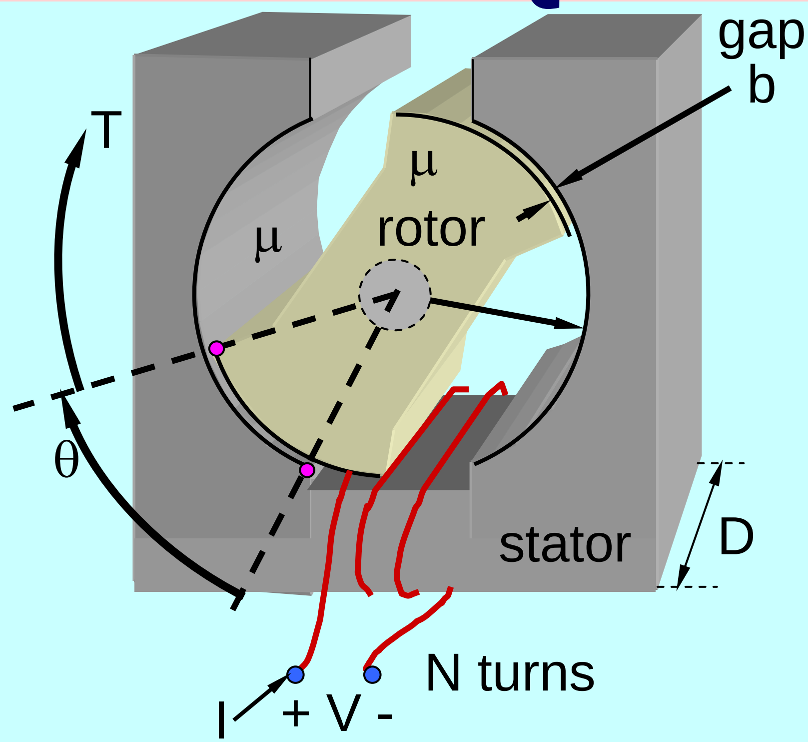
Magnetic Flux Linkage Λ:

$$\Lambda = N\iint_A \bar{B} \bullet da = N\mu_o H_{gap} A_{gap}$$

$$= N^2 \mu_o I A_{gap}/2b \quad (A_{gap} = RD\theta)$$

$$\Lambda = LI \quad \Rightarrow \quad L = N^2 \mu_o RD\theta/2b$$

$$\boxed{w_m = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\Lambda^2}{L}}$$



Set V = dΛ/dt = 0:

$$\boxed{T = -\frac{dw_m}{d\theta}} = -\frac{\Lambda^2}{2}\frac{dL^{-1}}{d\theta} = \frac{\Lambda^2}{2}\frac{2b}{N^2\mu_o RD\theta^2} \quad [\propto I^2],\ \Lambda = N\mu_o H_{gap} R\theta D$$

$$= \boxed{\frac{1}{2}\mu_o H_{gap}^{\ 2}} 2bD\,R = \boxed{W_{gap}\frac{dV_{olume}}{d\theta} \quad [\text{Nm}] \ \text{Torque}}$$

We power coil until overlap is maximum, then coast until it is zero

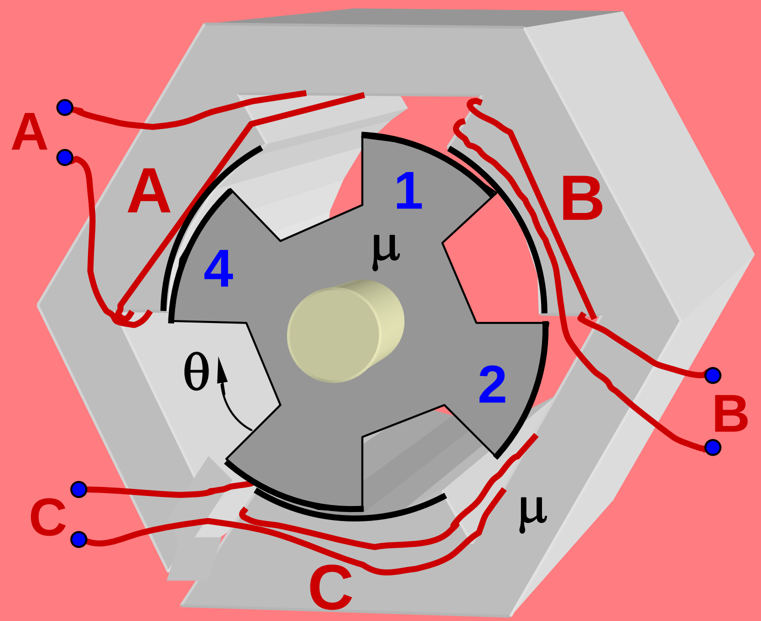
Magnetic pressure = Energy density [$J/m^3 = N/m^2$]

¾-POLE RELUCTANCE MOTOR

Winding Excitation Plan:

First excite windings A and B,
pulling pole 1 into pole B.
Pole area A = constant, temporarily.

When $\Delta\theta = \pi/3$, excite B and C.
When $\Delta\theta = 2\pi/3$, excite C and A.
Repeating this cycle results in
nearly constant clockwise torque.



To go counter-clockwise, excite BC, then AB, then CA.

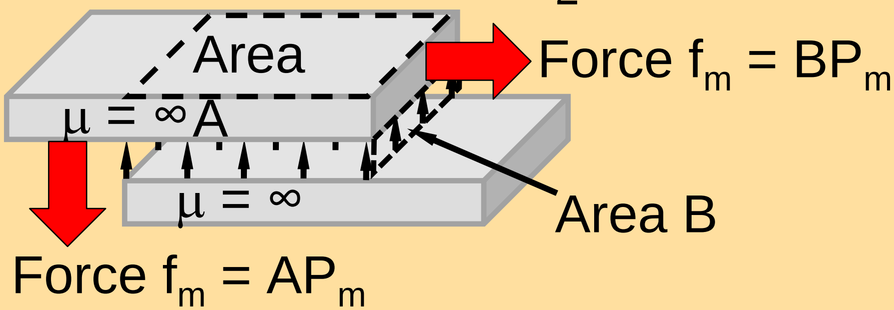
Torque:

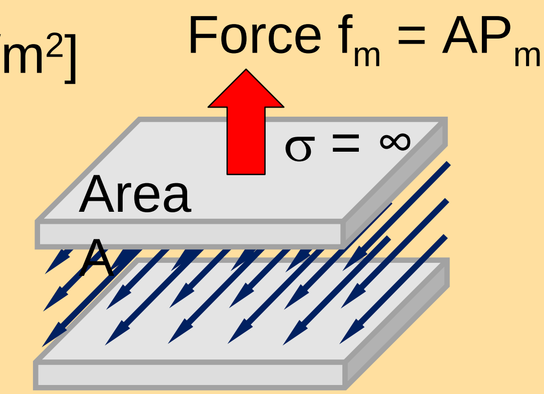
Only one pole is being pulled in here; the other excited winding has either one rotor pole fully in, or one entering and one leaving that cancel. Many pole combinations are used (more poles, more torque).

ELECTRIC AND MAGNETIC PRESSURE

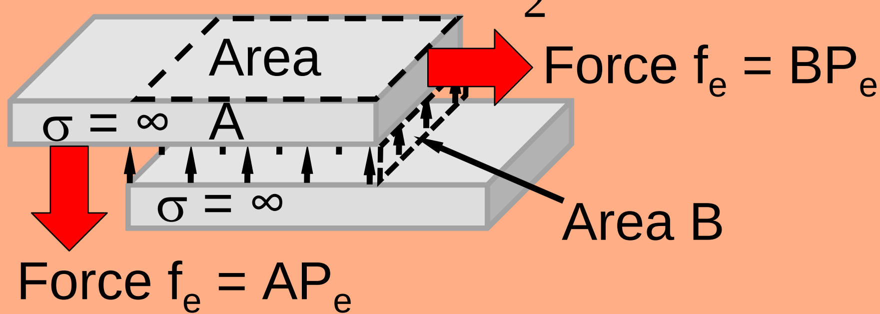
Electric and magnetic pressures equal the field energy densities, J/m^3
Both field types only pull along their length, and only push laterally
The net pressure is the difference between two sides of any boundary

Magnetic pressure $P_m = \frac{1}{2}\mu_o |\overline{H}|^2$ [N/m^2] or [J/m^2]





Electric pressure $P_e = \frac{1}{2}\varepsilon_o |\overline{E}|^2$ [N/m^2] or [J/m^2]



FORCES ON NEUTRAL MATTER

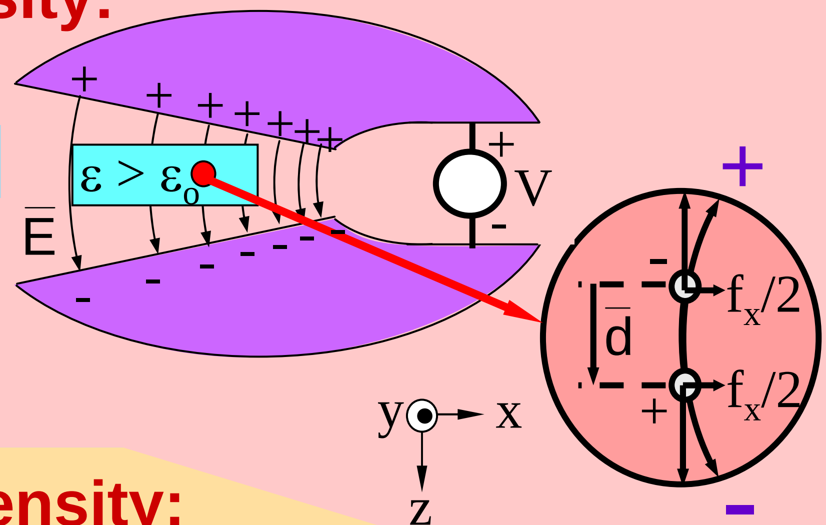
Kelvin polarization force density:

If $\nabla \times \overline{E} = 0 = \nabla \bullet \overline{E}$, then:

Field gradiants $\perp \overline{E} \Rightarrow \overline{E}$ is curved

Curved $\overline{E}$ pulls electric dipoles

into stronger field regions for $\varepsilon > \varepsilon_o$



Kelvin magnetization force density:

If $\nabla \times \overline{H} = 0 = \nabla \bullet \overline{B}$, then:

Field gradiants $\perp \overline{H} \Rightarrow \overline{H}$ is curved

Curved $\overline{H}$ pulls current loops into

stronger field regions for $\mu > \mu_o$