Problem Wk.6.2.1: Control System Analysis

Part 1: System 1

You are given two difference equations, one that describes a "controller" and one that describes a "plant". For example:

Controller: $v[n] = K(y[n] - x[n])$

Plant: $y[n] = y[n-1] - Tv[n-1]$

where K and T are constants.

The first step in the analysis is to combine the two equations to produce a single equation relating x and y. We can do this by converting to polynomials in R, doing algebra and converting back.

1. Manipulate the two difference equations above to produce one equation by eliminating $v[n]$.

   A difference equation is in the form:

   $y[n] = c_0 y[n-1] + c_1 y[n-2] + \ldots + c_{k-1} y[n-k] + d_0 x[n] + d_1 x[n-1] + \ldots + d_j x[n-j]$

   Specify the `dCoeffs`: $d_0 \ldots d_j$ and the `cCoeffs`: $c_0 \ldots c_{k-1}$ for each of the difference equations below. For each question, enter a sequence of numbers representing the coefficients.

   If one set of coefficients is empty, enter `none`, otherwise enter a sequence of numbers separated by spaces (no commas, parens, brackets, etc).

   Enter values of the cofficients assuming that $K = 1$ and $T = 0.2$:

   Difference equation:
   dCoeffs: ______ cCoeffs: ______

2. What is the largest value of K that produces a stable system (assuming $T = 0.2$)?
   ______

Part 2: System 2

Now, we'll repeat the analysis using a different (more realistic) controller equation:

Controller: $v[n] = K(y[n-1] - x[n-1])$

Everything else remains the same.

1. Manipulate the two difference equations above to produce one equation by eliminating $v[n]$.

   A difference equation is in the form:

$$y[n] = c_0 y[n-1] + c_1 y[n-2] + \ldots + c_{k-1} y[n-k] + d_0 x[n] + d_1 x[n-1] + \ldots + d_j x[n-j]$$

Specify the `dCoeffs`: $d_0 \ldots d_j$ and the `cCoeffs`: $c_0 \ldots c_{k-1}$ for each of the difference equations below. For each question, enter a sequence of numbers representing the coefficients.

If one set of coefficients is empty, enter none, otherwise enter a sequence of numbers separated by spaces (no commas, parens, brackets, etc).

Enter values of the cofficients assuming that $K = 1$ and $T = 0.2$:

Difference equation:
dCoeffs: [ ] cCoeffs: [ ]

2. What are the poles of this system? You can use the software from lab to get these.
The larger pole is: [ ]
The smaller pole is: [ ]

Note that $(1 - p_1 R)(1 - p_2 R)$ - where p_1, p_2 are the poles - should be equal to the denominator polynomial of the system function for this difference equation.