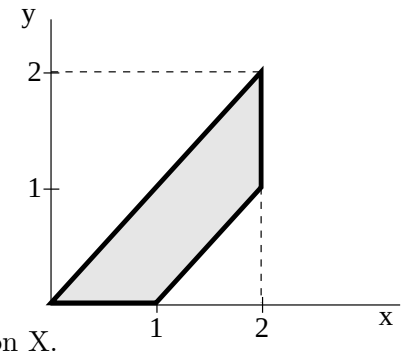
Tutorial 11

1. Continuous random variables X and Y have a joint PDF given by

$$f_{X,Y}(x,y) = \begin{cases} 2/3 & \text{if } (x,y) \text{ belongs to the closed shaded region} \\ 0 & \text{otherwise} \end{cases}$$



We want to estimate Y based on X.

   (a) Find the LMS estimator $g(X)$ of Y.

   (b) Calculate the conditional mean squared error $\mathbf{E}\left[(Y-g(X))^2 \mid X=x\right]$.

   (c) Calculate the mean squared error $\mathbf{E}\left[(Y-g(X))^2\right]$. Is it the same as $\mathbf{E}\left[\text{var}(Y|X)\right]$?

   (d) Derive $L(X)$, the linear LMS estimator of Y based on X.

   (e) How do you expect the mean squared error of $L(X)$ to compare to that of $g(X)$?

   (f) What problem do you expect to encounter, if any, if you try to find the MAP estimator for Y based on observations of X.

2. Consider a noisy channel over which you send messages consisting of 0s and 1s to your friend. It is known that the channel independently flips each bit sent with some fixed probability p; however the value of p is unknown. You decide to conduct some experiments to estimate p and seek your friend's help. Your friend, cheeky as she is, insists that you send her messages consisting of three bits each (which you will both agree upon in advance); for each message, she will only tell you the total number of bits in that message that were flipped. Let X denote the number of bits flipped in a particular three-bit message.

   (a) Find the PMF of X.

   (b) Derive the ML estimator for p based on $X_1, \ldots, X_n$, the numbers of bits flipped in the first n three-bit messages.

   (c) Is the ML estimator unbiased?

   (d) Is the ML estimator consistent?

   (e) You send $n = 100$ three-bit messages and find that the total number of bits flipped is 20. Construct a 95% confidence interval for p. If necessary, you may use a conservative bound on the variance of the number of bits flipped.

   (f) What are some other ways to estimate the variance. How do you expect your confidence interval to change with different estimates of the variance.

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