

Recitation 20
Markov Chains: Steady State Behavior
May 09, 2006

1. (Problem 6.9) A professor gives tests that are hard, medium or easy. If she gives a hard test, her next test will be either medium or easy, with equal probability. However, if she gives a medium or easy test, there is a 0.5 probability that her next test will be of the same difficulty, and a 0.25 probability of each of the other two levels of difficulty. Construct an appropriate Markov chain and find the steady-state probabilities.
2. (Problem 6.10) Alvin likes to sail each Saturday to his cottage on a nearby island off the coast. Alvin is an avid fisherman, and enjoys fishing off his boat on the way to and from the island, as long as the weather is good. Unfortunately, the weather is good on the way to or from the island with probability p , independently of what the weather was on any past trip (so the weather could be nice on the way to the island, but poor on the way back). Now, if the weather is nice, Alvin will take one of his n fishing rods for the trip, but if the weather is bad, he will not bring a fishing rod with him. We want to find the probability that on a given leg of the trip to or from the island the weather will be nice, but Alvin will not fish because all his fishing rods are at his other home.
 - (a) Formulate an appropriate Markov chain model with $n + 1$ states and find the steady state probabilities.
 - (b) What is the steady-state probability that on a given trip, Alvin sails with nice weather but without a fishing rod?
3. (Problem 6.13) **Ehrenfest model of diffusion.** We have a total of n balls, some of them black, some white. At each time step, we either do nothing, which happens with probability ϵ , where $0 < \epsilon < 1$, or, we select a ball at random, so that each ball has probability $(1 - \epsilon)/n > 0$ of being selected. In the latter case, we change the color of the selected ball (if white it becomes black, and vice versa), and the process is repeated indefinitely. What is the steady-state distribution of the number of white balls?