Tutorial 3: Answers
March 2-3, 2006

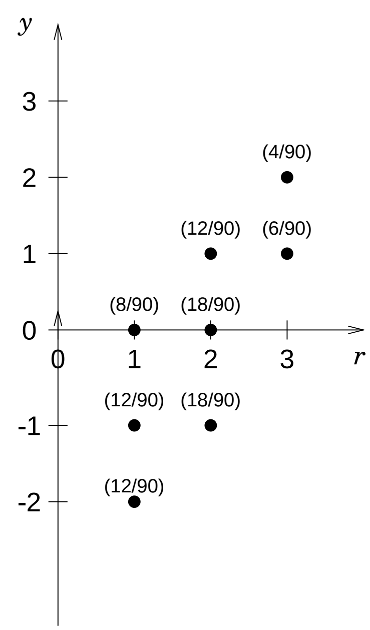
1. (a)

$$p_S(s) = \begin{cases} 24/90, & s = 1; \\ 36/90, & s = 2; \\ 30/90, & s = 3; \\ 0, & \text{otherwise}, \end{cases}$$

$$p_{S|A}(s) = P(S = s \cap A)/P(A) = \begin{cases} 24/60, & s = 1; \\ 36/60, & s = 2; \\ 0, & \text{otherwise}. \end{cases}$$

(The required sketches are omitted.)

(b) The joint PMF for R and Y is:



(c)

$$p_{X|A}(x) \begin{cases} 8/60, & x = 2; \\ 24/60, & x = 3; \\ 22/60, & x = 4; \\ 6/60, & x = 5; \\ 0, & \text{otherwise}. \end{cases}$$

(The required sketch is omitted.)

2. (a)

$$p_{N,K}(n, k) = \begin{cases} 1/4k & \text{if } k = 1, 2, 3, 4 \text{ and } n = 1, \ldots, k \\ 0 & \text{otherwise} \end{cases}$$

(b)

$$p_N(n) = \begin{cases} 1/4 + 1/8 + 1/12 + 1/16 = 25/48 & n = 1 \\ 1/8 + 1/12 + 1/16 = 13/48 & n = 2 \\ 1/12 + 1/16 = 7/48 & n = 3 \\ 1/16 = 3/48 & n = 4 \\ 0 & \text{otherwise} \end{cases}$$

(c) The conditional PMF

$$p_{K|N}(k|2) = \frac{p_{N,K}(2,k)}{p_N(2)} = \begin{cases} 6/13 & k = 2 \\ 4/13 & k = 3 \\ 3/13 & k = 4 \\ 0 & \text{otherwise} \end{cases}$$

(d) Let A be the event that Chuck bought at least 2 but no more than 3 books, $E[K|A] = 3$ $var(K|A) = \frac{3}{5}$

(e) $E[T] = \frac{21}{4}$

3. (a)

$$p_{X,Y,Z}(x,y,z) = \begin{cases} \frac{1}{4}p(1-p)^{x-1}, & \text{if } x \text{ is odd and } (y,z) \in \{(0,0),(0,2),(2,0),(2,2)\} \\ p(1-p)^{x-1}, & \text{if } x \text{ is even and } (y,z) = (0,0) \\ 0, & \text{otherwise.} \end{cases}$$

(b) (i) No.
(ii) Yes.
(iii) No.
(iv) Yes.

(c) The variance is

$$(0-2)^2\frac{1}{4} + (4-2)^2\frac{1}{4} = 2.$$