Recitation 8
October 5, 2010

1. Let Z be a continuous random variable with probability density function

$$f_z(z) = \begin{cases} \gamma(1+z^2), & \text{if } -2 < z < 1, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) For what value of γ is this possible?

   (b) Find the cumulative distribution function of Z.

2. Problem 3.9, pages 186–187 in the text.

   The taxi stand and the bus stop near Al's home are in the same location. Al goes there at a given time and if a taxi is waiting, (this happens with probability 2/3) he boards it. Otherwise he waits for a taxi or a bus to come, whichever comes first. The next taxi will arrive in a time that is uniformly distributed between 0 and 10 minutes, while the next bus will arrive in exactly 5 minutes. Find the CDF and the expected value of Al's waiting time.

3. Let λ be a positive number. The continuous random variable X is called **exponential** with parameter λ when its probability density function is

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the cumulative distribution function (CDF) of X.

   (b) Find the mean of X.

   (c) Find the variance of X.

   (d) Suppose X_1, X_2, and X_3 are independent exponential random variables, each with parameter λ. Find the PDF of $Z = \max\{X_1, X_2, X_3\}$.

   (e) Find the PDF of $W = \min\{X_1, X_2\}$.

6.041SC Probabilistic Systems Analysis and Applied Probability

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