MASSACHUSETTS INSTITUTE of TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.161 Modern Optics Project Laboratory

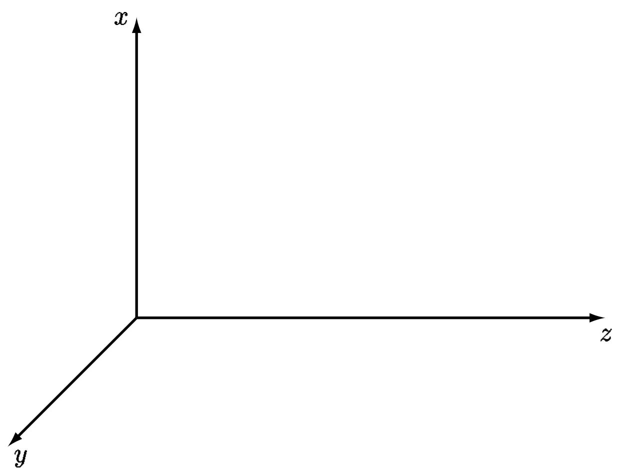
Problem Set No. 4	**Diffraction**	Issued Tues. 10/4/2005
Spring Term, 2005	**-**	Due Thurs. 10/13/2005

Reading recommendation: Class Notes, Chapter 4. Be neat in your work!

Problem 4.1

In the coordinate system below, the following definitions hold:

$$\begin{aligned} \overline{\rho} &= x\hat{x} + y\hat{y} \\ \overline{r} &= x\hat{x} + y\hat{y} + z\hat{z} \\ \overline{k} &= k_x\hat{x} + k_y\hat{y} + k_z\hat{z} \end{aligned}$$



Show the direction of propagation, and algebraically derive, sketch and describe the shape of the wavefront associated with the following elementary unit amplitude waves:

(a) $\underline{U}(\overline{r}) = e^{j\overline{k}\cdot\overline{r}}$

(b) $\underline{U}(\overline{r}) = e^{jkz}e^{-j\frac{k\rho^2}{2F}}$

(c) $\underline{U}(\overline{r}) = e^{j\frac{k}{2z}(x^2+y^2)}$

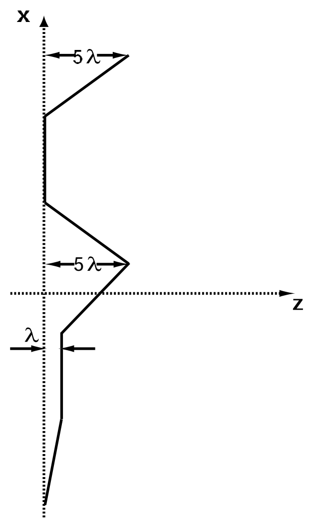
(d) $\underline{U}(\overline{r}) = e^{j\frac{k}{2z}[(x-x_0)^2+(y-y_0)^2]}$

(e) $\underline{U}(\overline{r}) = e^{jk(z^2+x^2+y^2)^{1/2}}$

(f) $\underline{U}(\overline{r}) = e^{jk[z^2+(x-x_0)^2+(y-y_0)^2]^{1/2}}$

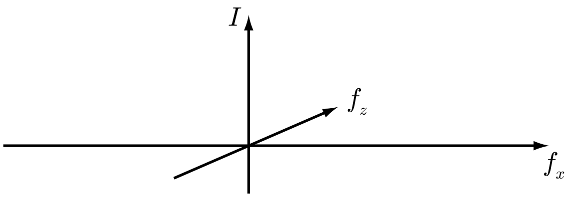
Problem 4.2

The figure below is a 1-D crosssectional plot of a 2-D piecewise-continuous approximation to an actual wavefront. The actual 2-D wavefront (not shown) is smooth (no sharp corners). For simplicity, the normal to the wavefront segments all lie in the $x - z$ plane, and the wavefront, of wavelength λ, is travelling nominally in the $+z$-direction. Assume all segments of the 2-D piecewise continuous wavefront are of the same area (10λ in the x-z plane, 10λ in y) and they all carry the same power density of I Watt/m^2.



(a) Write a frequency-domain expression, $\underline{U}(f_x, f_z)$, that describes the primary directions of power flow in this wavefront. (ignore any effective aperturing and, therefore, diffraction effects).

(b) Sketch the spatial-frequency content of this wavefront on a graph with the co-ordinate system shown below.



(c) Assume the above wavefront exists in the back focal plane of a lens of focal length F and is traveling nominally toward the lens. Sketch the intensity pattern that would be seen on a screen placed in the front focal plane of the lens, and label the positions and the sizes of any critical features that will be present on the screen.

Problem 4.3

A 100 mW CO_2 laser of wavelength $\lambda = 10.6\ \mu m$ aboard an unmanned aerial vehicle (UAV) is pointed at a passenger plane flying at a distance of 2 km from the UAV. The laser beam is 4 mm in diameter immediately after exiting the laser. Assume there is no atmospheric turbulence.

(a) What is the spot size of the laser beam on the passenger plane?

(b) Assuming that all the light from the laser is distributed evenly within the spot on the airplane, what is the power density of the beam on the passenger plane?

(c) Can a passenger looking out of the window of the airplane at the UAV see the laser light? Why?

(d) Can you devise a simple, practical means (that is implementable aboard the UAV) that will increase the optical power density on the passenger plane?

Problem 4.4

A plane-wave of amplitude A and wavelength λ is incident at normal incidence in the $\hat{z}$-direction on the transmission object $\underline{U}_a(x, y)$ described below.

$$\underline{U}_a(x,y) = \begin{cases} e^{j\frac{\pi}{2}} & 0 < x < a \\ e^{-j\frac{\pi}{2}} & -a < x < 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Draw a sketch of this object in the x-y plane

(b) Give an example of how you would fabricate such an object.

(c) Compute analytically the intensity of the Fraunhofer diffraction field owing to this object.

(d) Plot the Fraunhofer diffraction field of part (c) using your favorite software package.