Massachusetts Institute of Technology

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6.243j (Fall 2003): DYNAMICS OF NONLINEAR SYSTEMS

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Problem Set 4[1]

Problem 4.1

Find a function $V: \mathbf{R}^3 \mapsto \mathbf{R}_+$ which has a unique minimum at $\bar{x} = 0$, and is strictly monotonically decreasing along all non-equilibrium trajectories of system

$$\begin{aligned}
\dot{x}_1(t) &= -x_1(t) + x_2(t)^2, \\
\dot{x}_2(t) &= -x_2(t)^3 + x_3(t)^4, \\
\dot{x}_3(t) &= -x_3(t)^5.
\end{aligned}$$

Problem 4.2

System Δ takes arbitrary continuous input signals $v: [0,\infty) \mapsto \mathbf{R}$ and produces continuous outputs $w: [0,\infty) \mapsto \mathbf{R}$ in such a way that the series connection of Δ and the LTI system with transfer function $G_0(s) = 1/(s+1)$, described by equations

$$\dot{x}_0(t) = -x_0(t) + w(t), \quad w(\cdot) = \Delta(v(\cdot)),$$

has a non-negative storage function with supply rate

$$\sigma_0(\bar{x}_0, \bar{v}, \bar{w}) = (\bar{w} - 0.9\bar{x}_0)(\bar{v} - \bar{w}).$$

(a) Find at least one *nonlinear* system Δ which fits the description.

(b) Derive constraints to be imposed on the values $G(j\omega)$ of a transfer function

$$G(s) = C(sI - A)^{-1}B$$

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with a Hurwitz matrix A, which guarantee that $x(t) \to 0$ as $t \to \infty$ for every solution of

$$\dot{x}(t) = Ax(t) + Bw(t),\ v(t) = Cx(t),\ w(\cdot) = \Delta(v(\cdot)).$$

Make sure that your conditions are satisfied at least for one non-zero transfer function $G = G(s)$.

Problem 4.3

For the pendulum equation

$$\ddot{y}(t) + \dot{y} + sin(y) = 0,$$

find a single continuously differentiable Lyapunov function $V = V(y, \dot{y})$ that yields the maximal region of attraction of the equilibrium $y = \dot{y} = 0$. (In other words, the level set

$$\{\bar{x} \in \mathbf{R}^2 :\ V(\bar{x}) < 1\}$$

schould be a union of disjoint open sets, one of which is the attractor Ω of the zero equilibrium, and $V(y(t), \dot{y}(t))$ schould have negative derivative at all points of Ω except the origin.)