6.301 Solid State Circuits

Recitation 11: Analog Computation
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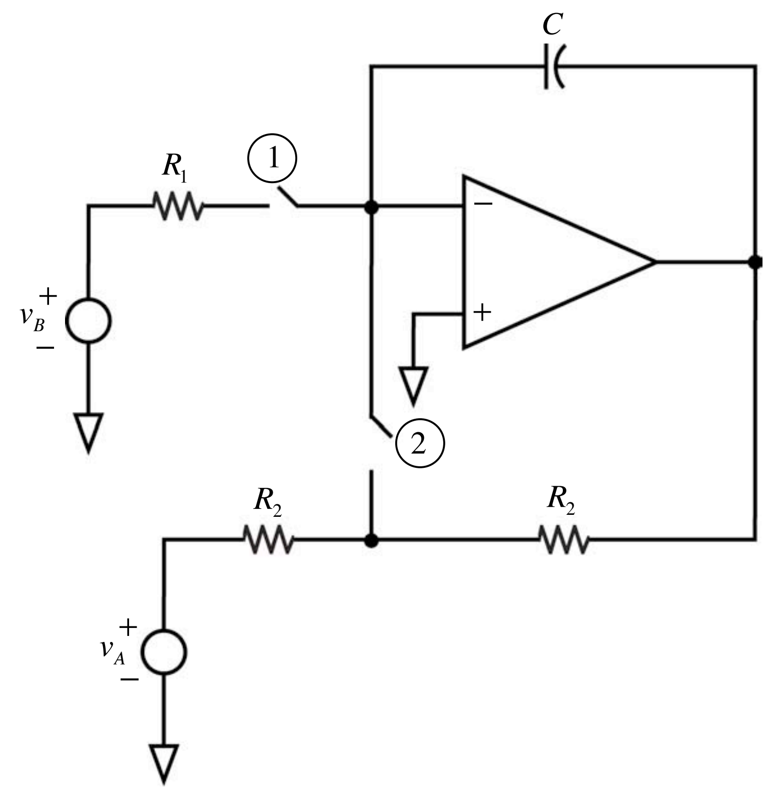
At one time, analog computers were the only machines that we had for performing simulations of real world systems. Although almost never used now, analog computation did stimulate a great deal of innovation in the design of low-offset, high DC gain amplification. Many of the techniques that were developed are still in use today.

For our class exercise, we're going to look at one building block that is very useful in analog computation: the three-mode integrator.

CLASS EXERCISE



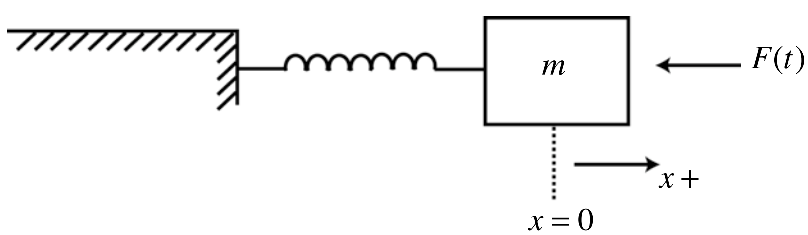
Describe the operation of this circuit in each of the following three modes: (A) switch (1) open and switch (2) closed; (B) switch (1) closed and switch (2) open; (C) switch (1) and switch (2) open.

(Workspace)

Analog computers were frequently used for simulating systems described by differential equations. Recall that an n^{th} order differential equation requires n initial conditions to be specified. Mode (A) of the above circuit would be used for setting the initial condition on one of the integrators.

Now, let's see how we might go about simulating some common physical systems.

<u>Mass-Spring System with Damping</u>



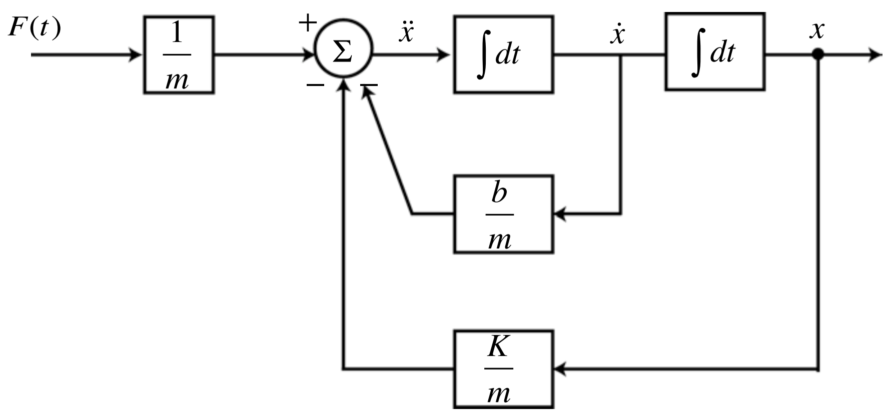
Write equation of motion:

$$m\ddot{x} = \underbrace{F(t)}_{\text{Forcing function}} - \underbrace{kx}_{\text{spring}} - \underbrace{b\dot{x}}_{\text{Viscous damping}}$$

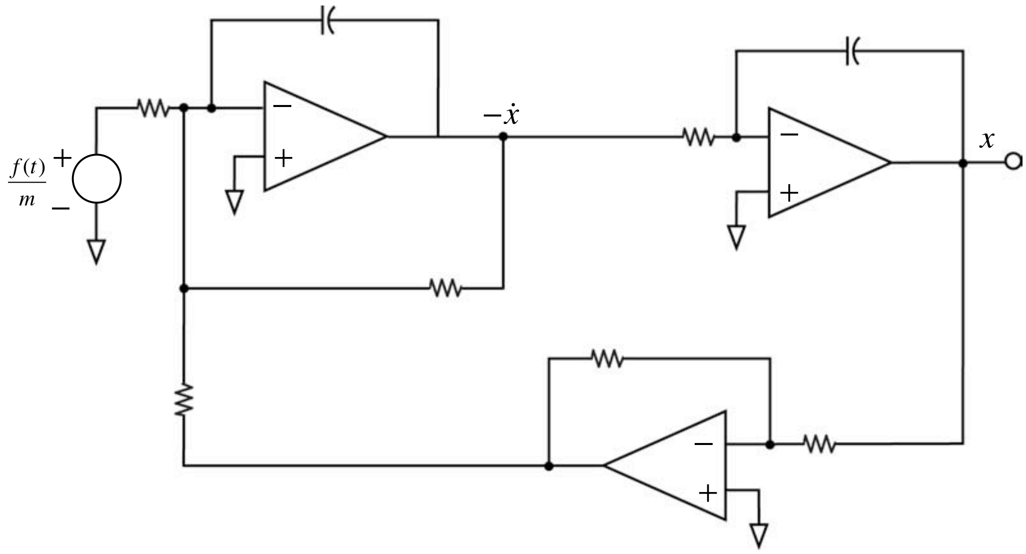
Dividing through by the mass m, we get:

$$\ddot{x} = \tfrac{1}{m}F(t) - \tfrac{b}{m}\dot{x} - \tfrac{K}{m}x$$

How could we set up an electronic <u>analog</u> to this physical system? Consider:



Note that the second-order equation required two integrators to realize. The initial conditions, namely $x(0)$ and $\dot{x}(0)$, could be set with the aid of the 3-mode integrators. Using op-amps, we might wind up with a realization like the following:



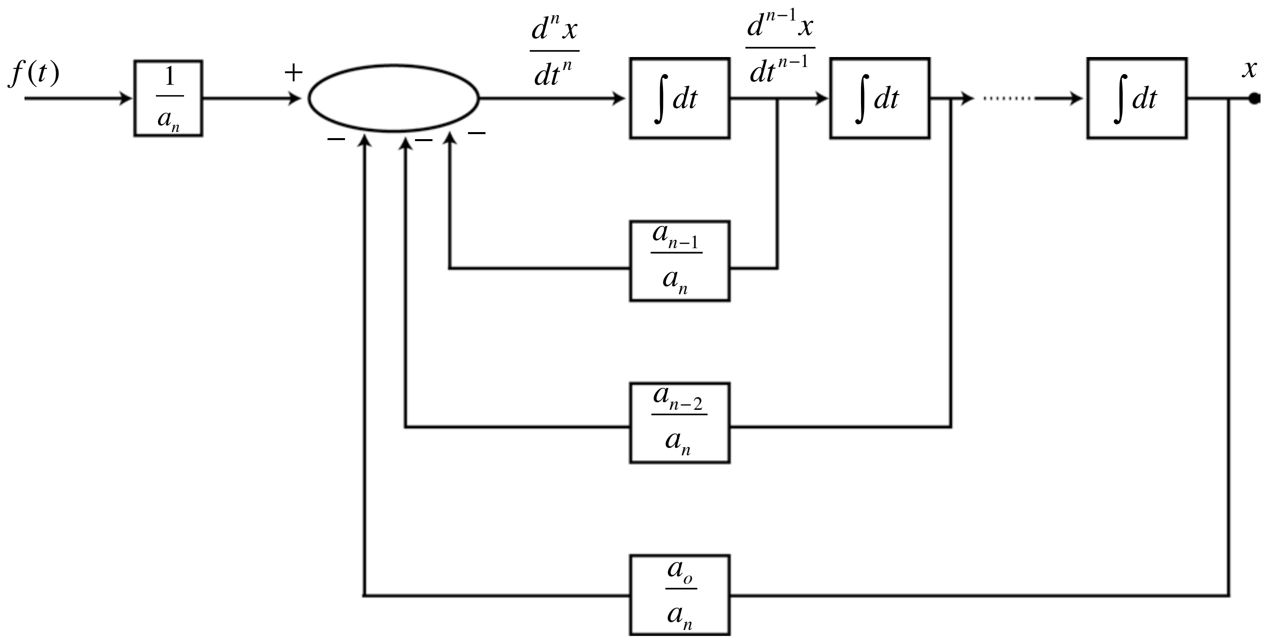
It turns out that there is a general synthesis procedure for linear differential equations that take us from the equation to an equivalent block diagram. Suppose that we start with

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \ldots + a_1 \frac{d_x}{dt} + a_0 x = f(t)$$

Our first step is to solve for the highest order derivative:

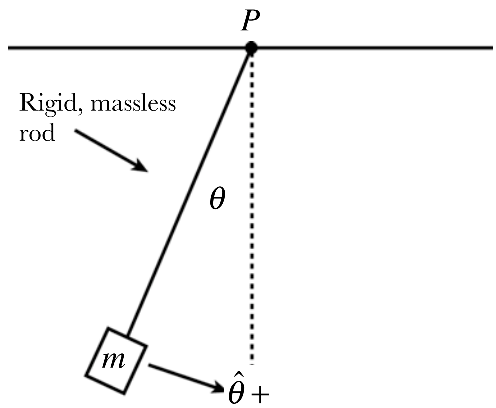
$$\frac{d^n x}{dt^n} = -\frac{a_{n-1}}{a_n} \frac{d^{n-1} x}{dt^{n-1}} - \ldots - \frac{a_1}{a_n} \frac{dx}{dt} - \frac{a_0}{a_n} x + \frac{1}{a_n} f(t)$$

In the resulting block diagram, we make $\frac{d^n x}{dt^n}$ the output of a large summation junction, and follow that with a string of integrators to generate the lower order derivatives:



As a final note, bear in mind that the independent variable of the system that we're studying needn't always be time. In some systems, the independent variable may be position, or temperature, or any number of things. In these cases, the proper interpretation of "time" in our analog simulation is simply applied to the simulation result.

Our methods are not confined merely to linear differential equations. Consider modeling the common pendulum system



Writing out the torque about the point P:

$$I\ddot{\theta} = -mgl\sin\theta$$

The moment of inertia, I, for a mass at the end of a (massless) rod is ml^2:

$$ml^2\ddot{\theta} = -mgl\sin\theta$$

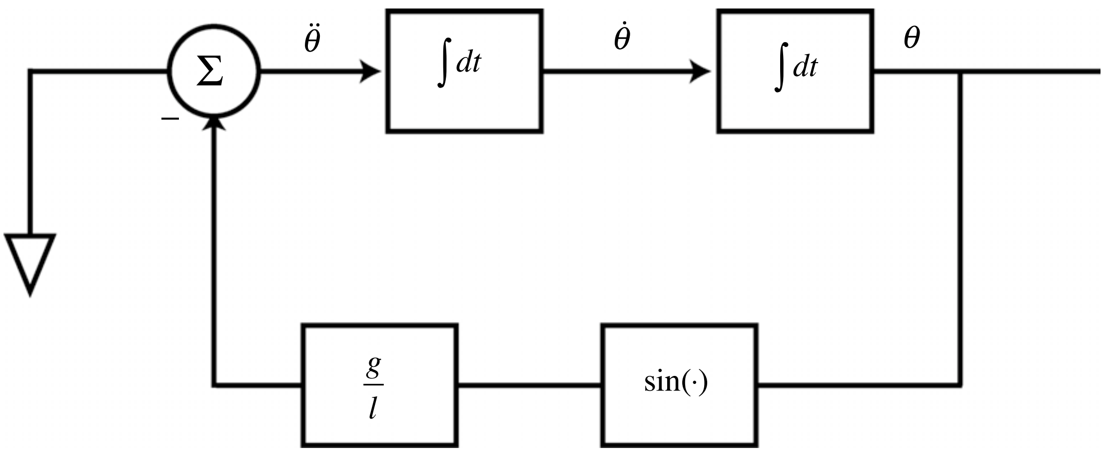
$$\ddot{\theta} = -\frac{g}{l}\sin\theta$$

Normally at this point we invoke "small" displacements in and write the linearized equation

$$\ddot{\theta} = -\frac{g}{l}\theta$$

This system has a natural frequency of $\omega = \sqrt{\frac{g}{l}}$. But what if we wanted to contend with the original nonlinear system? Same as before:

$$\ddot{\theta} = -\frac{g}{l}\sin\theta$$



This would allow us to simulate the real system.

Of course, we have the practical problem of realizing a $\sin(\cdot)$ analog block. Barrie Gilbert of Analog Devices, a legendary circuit designer, evidently came up with an *IC* that performed exactly this function!