6.450 Principles of Digital Communication   Wednesday October 21, 2009
MIT, Fall 2009

Quiz 1

- You have 90 minutes to complete the quiz.

- This is an open-book quiz. You may use your book and three pages of note. Calculators are allowed, but probably won't be useful.

- The problems are not necessarily in order of difficulty. Do the problems in whichever order you find most natural.

- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgement of your level of understanding as reflected by what you have written.

- If we can't read it, we can't grade it.

- If you don't understand a problem, please ask.

Problem Q1:

(35 points) True or False. Explain your answer.

(a) A codebook $\mathcal{C}$ is prefix free if and only if $\sum_j (2^{l_j}) \leq 1$, where l_j is the length of codeword c_j.

(b) Suppose $u(t)$ is a measurable function. Then, $f(u(t))$ is also measurable.

(c) Suppose $u(t)$ is $\mathcal{L}_2$ and its Fourier transform, $\hat{u}(f)$, is zero for $|f| > W$. Then, $u(t)$ can be completely described by its values at $u(kT)$, for all integer k and $T = 1/(2W)$.

(d) Suppose $u(t) = \begin{cases} 1 & t \text{ is an integer} \\ 0 & o.w. \end{cases}$. Then, $\forall f$, $\hat{u}(f) = 0$, where $\hat{u}(f)$ is the Fourier transform of $u(t)$.

(e) The Lloyd-Max algorithm finds an optimal solution for quantization regions and points.

(f) Within the class of scalar quantizers, a uniform scalar quantizer followed by a discrete entropy coding is nearly optimal in terms of the mean square distortion.

(g) Performing the Lempel-Ziv algorithm on a sequence may lead to a longer sequence than the input sequence.

Problem Q2:

Consider two discrete sources, $X = \{X_i\}_{i=1}^n$ and $Y = \{Y_i\}_{i=1}^n$.

(i) Suppose X is a memoryless source such that $P(X_i = 0) = 1/3$ and $P(X_i = 1) = 2/3$, for all $1 \le i \le n$. Consider T_ϵ^n as the typical set of source X.

    (a) (10 points) Consider Y as another discrete source such that, at each time i, it is drawn according to,

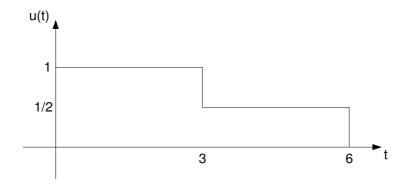$$P_{Y|X}(y_i|x_i) = \begin{cases} \alpha & y_i = x_i \\ 1 - \alpha & y_i \ne x_i \end{cases}.$$

    We perform an asymptotically optimal source coding on source Y, when source X is available at the encoder and the decoder. What is the expected number of bits per source symbol of Y?

    (b) (10 points) Describe an asymptotically error-free scheme to achieve a rate that approaches asymptotically to the rate found in the part (i-a).

(ii) In this part, suppose source X is not memoryless, and we have, $P(X_i = 0|X_{i-1} = 0) = 1/2$ and $P(X_i = 1|X_{i-1} = 1) = 3/4$.

    (c) (10 points) Consider source Y as defined in part (i-b). Note that here X may not be memoryless. Repeat part (i-a).

Problem Q3:

Consider the signal, $u(t)$, shown in the following:

(a) (10 points) Consider the first three terms for each segment of the T-spaced truncated sinusoid expansion of $u(t)$, for $T = 1$, starting at $t = 0$. By using these terms, we form a signal $u_1(t)$. Find $u_1(t)$.

(b) (15 points) Find $u_2(t)$ which is defined as in part (a), except that $T = 2$.

(c) (10 points) Define reconstruction error signals $e_1(t) = u(t) - u_1(t)$ and, $e_2(t) = u(t) - u_2(t)$. Compute and compare their energy.

6.450 Principles of Digital Communication I
Fall 2009