PROFESSOR: We've just started talking about wireless communication. We spent a lot of time talking about how do you communicate, essentially on wire lines, where essentially the only problem is white Gaussian noise. Namely, you transmit a signal, noise gets added to it, you receive the sum of the transmitted signal plus noise and all of the going from base band to pass band. All of that stuff is all messy analytically, but essentially all that's happening is that you're going from -- you take a signal, you move up to pass band, you add noise, you move back to base band, in which case you get the original transmitted signal plus the noise moved back to -- move back -- moved to base band. So what we wound up with there was a relatively simple situation.

Wireless communication is not so simple. As you all know, if you use a cell phone, and I'm sure most of you do, because of something called fading -- what we want to understand is where this fading comes from, how it arises and some of the things you might do about it. One of the things you should recognize right at the beginning -- and we won't spend much time talking about it -- is if you happen to be trying to use a cellular phone and there's a big wall, which is perfectly reflecting right in front of you and the base station you're trying to communicate with is on the other side of that wall, you're not going to get through. In other words, there are some situations where no matter how you design a cellular phone, you just can't get any communication. It's part of life. What we want to do, however, is to make sure that when you can get communication, you will get it as well as possible. You will prolong the period, but you're still communicating while the channel's getting worse and worse -- and part of the way to do that is to understand what it is in the physical mechanisms that's making the problem difficult.

OK, so to start out with this, we'll start out kind of easy. We'll assume an input, which is just a cosine of 2 pi ft at some ficed antenna -- and this is radiating outwards and the electric field -- anywhere in free space, if you go a long distance away. If you go a very short distance from the antenna, if you study electromagnetism, you know that all sorts of crazy things are going on. When you get very far away, there's something called the far field and essentially what happens is that all of these disturbances close in just sort of disappear and very far away what's happening is that the field strength -- and this is true for the magnetic field also, which is just in -- if the electric field is this way, the magnetic field is that way. They're both propogating outwards and they're going down as 1 over r, just like when we deal with linear systems, we never talk about voltage and current anymore. This is why we use a square root of minus 1 at this point -- because after you learn about voltage and current, you don't have to talk about them anymore. You just deal with one of them for the function that exists someplace and the other one just follows along from the impedance. Same thing happens for electric fields and magnetic fields. You don't have to bother about both of them, unless you're really trying to solve Maxwell's field equations, which is a very challenging endeavor. I admire any of you who can do this. I used to be able to do it many, many years ago and I've given up on it because I decided that's for younger people than me.

This far field has a very simple kind of behavior, because what happens is it has to go down as 1 over r. How do you know it has to go down as 1 over r? Namely, the distance away from this radiating antenna. If you look at a sphere, which is very, very far away from the radiating antenna, all you find is this field that's radiating outwards and you look at how much energy is radiating outwards. The energy which is radiating outwards, there's no place to lose it because we're transmitting just an open space now -- or at least that's what we're imagining at this point.

So we're transmitting out in open space. Energy doesn't get beaten up any place so it just keeps going out until it disappears out of the outer edges of the known universe. It travels out there with the speed of light. That's what Maxwell's laws say if you solve them. Since this sphere has an area which is proportional to r squared, the only thing that can be happening in this wave, which is propogating outward spherically, is it has to be going down as 1 over r, because that's the only way for the power to balance out. You can't be creating power out in this free space and you can't be losing power. Whatever you're sending is just radiating outwards, so we have to have this 1 over r dependence any time you're dealing with free space.

The other thing that's happening is we have an antenna pattern. The antenna pattern is a function of two angles. We'll think of theta as being the angle around this way and psi as being the angle this way. If you like to think of angles in some other way, be my guest. The only thing is, when we're radiating outwards through this sphere, if the antenna is directional, it's going to be radiating more in some directions than it is in other directions and this simply takes that into account. I'm not going to pay any attention to this at all. It just exists and it's there. For people who design antennas, that's an important issue: How do you make antennas which are directional, which will shine their energy in one direction rather than another? We're not going to go into that at all. We're just representing the fact that it's there.

This is just a factor which says, how much loss do you have in the antenna and how much of the power that you're radiating goes in each one of these directions and how does it depend on the frequency that you're transmitting at? Antennas are sometimes designed to be rather frequency dependent, so they're tuned to a certain frequency. They work very well at that frequency and on other frequencies, they just go to pot.

The other part of this equation is that if I send a signal, as it radiates outward, it's going to be radiating outward at the speed of light. Therefore, whatever I receive is going to be delayed by the distance that I am away divided by the speed of light. So this equation is something you don't really have to know any electromagnetics to derive. If you want to find out what this term is, yes, you'd need some electromagnetics. All this is saying is the power has to be going down as 1 over r and you have a propogation delay, which has to be going as r divided by the speed of light.

Now, if we look at what happens when we put a receiving antenna out at some distance r away from the transmitting antenna and in this direction theta and psi, that receiving antenna is going to distort this electromagnetic wave locally around the receiving antenna, but it's not going to distort the whole thing. In other words, its power is radiating outwards. It doesn't know anything about this little receiving antenna until it gets close to it, then the electromagnetic wave gets distorted somewhat. The only thing that's happening because of this receiving antenna is that there's some added antenna pattern due to the receiving antenna -- namely, some added attenuation which multiplies by the attentuation in the source antenna. You have both the source antenna pattern, the receiving antenna pattern. We put the two together. We call that alpha and we don't bother about it anymore except recognizing that it might change with frequency also.

Then we have this propagation delay between transmitting antenna and receiving antenna If you're transmitting in free space from one antenna to another antenna, this is what happens, with some arbitrary pattern here for the two antennas, which depends on whether they radiate spherically or whether they radiate in some sort of direction. That's the received wave form.

That said, if you look at it, at the received field -- this is supposed to be valid for any -- oh, sorry. Yes, that would help. This equation is supposed to be valid for any frequency we want to transmit at, at any time, and if we're transmitting two signals, both together, the response is going to be the sum of the response to one signal and a response to the other signal. In other words, Maxwell's laws are linear and therefore the response that you get when you solve Maxwell's laws -- I've never solved Maxwell's laws for anything this complicated and probably none of you had either. You might have, but anyway, they are linear and therefore, in fact, what's going on is that this gives you a system function which says what the response is to any given input that you might want to transmit. It says the response -- at frequency f to a sinusoidal input, which is what we were assuming before. We were assuming that we transmitted cosine 2 pi ft. The system function is just this antenna pattern times e to the minus 2 pi i times f times the distance away divided by c divided by r. Namely, this takes into account the propogation delay. The only thing it doesn't take into account is what the input is. The received field is then the real part of the system function times what we already assume that we were going to be transmitting. Namely, the real part -- e to the 2 pi i ft.

So at this point, what we're doing is taking into account the fact that the solution to Maxwell's equations is going to be linear and therefore we can just add up what happens for each frequency of input. What you notice from looking at that is that when we have a fixed transmitting antenna, a fixed receiving antenna and free space between them, we are right back to the problem that we started with. Namely, white Gaussian noise on a channel, because nothing is varying with time. There isn't any fading. Nothing interesting is going on. This is sort of like the case of microwave towers. Microwave towers are set up and they have nice directional antennas, nice horns which are blowing at each other. Nothing changes except every once in awhile is a rainstorm or something and the communication goes to pot. Usually, you're just transmitting as if it was white Gaussian noise and you usually view microwave towers sending microwave as being almost equivalent to wire line communication. So there's nothing very interesting there.

Now if the receiving antenna starts to move -- at this point, we are transmitting from a fixed sending antenna. We have a receiving antenna, which for example, is in a car where somebody's running along, driving a car with their feet and talking on two cellular phones at the same time -- and the car's driving along at 100 miles an hour and something's going to happen soon, but it hasn't happened yet. At this point what we're interested in is not the response of some fixed place r, but what we're interested in is the electromagnetic field in the absence of receiver at this point, which is moving. We're interested in the electromagnetic field at a point r 0 plus v2, where v is the velocity of this vehicle. We'll assume for the time being that the vehicle is going directly away from the transmitting antenna. If it's going at some angle, it just changes these equations a little bit. And the electric field there, before we put the car in the car changes all of the field patterns, but just changes it in a local way again. Just like when we put the receiving end antenna in a fixed location, it changed all the local field equation, but it didn't change anything globally. Again, what we have when we put in the receiving antenna -- is now the electric field at a point r, which is varying with time. There's a time dependence with this now. 1 over r 0 plus v.t. times the real part of this antenna pattern -- which we'll assume remains fixed -- times e to the 2 pi if times t minus r 0 plus vt over c. This is for a velocity away from the antenna.

That's just what this same equation says, but we can now interpret this nicely if we take the vt over c and combine it with the ft here and then we get something which looks like this antenna pattern again -- e to the 2 pi i f times 1 minus v over c times t. This v over c here is just this term here. It's coming down to there. Nothing mysterious has happened here, but what you see is this well known phenomenon called Doppler shift. If you throw some screaming person over a cliff, what you'll hear coming back to you is a scream in velocity much smaller than the actual scream that the person is actually transmitting to you. You're all familiar with this. You're familiar with having planes fly overhead and when you hear them coming towards you, you hear a higher pitched sound. When it passes by you -- this is a nicer example than throwing somebody over a cliff, obviously -- and you then hear a lower frequency sound as the plane starts moving away from you. This Doppler shift is a well known phenomena as far as sound is concerned. The same phenomena exists with electromagnetic radiation. And there's nothing more to it than just this -- it's just that as you are transmitting from here to a point which is moving away, it keeps taking longer for the electromagnetic wave to get out to there than it takes to get here, so if you look at the wave fronts going along, the peak of the wave as it travels along, the peak of the wave takes a little longer to get out here than it took to get here, which means that from the viewpoint of the receiver, it looks like the receive frequency is much smaller than it was when it was actually being transmitted.

We get this thing called the Doppler shift. Just to get some idea of the magnitude of this Doppler shift, what you're interested in is the speed of the vehicle divided by the speed of light. That's the relative change in the frequency that you observe. The situation here is quite different than it is in sound. Sound travels rather slowly. Light travels pretty fast and therefore, you need a really rapidly speeding vehicle to make this be any appreciable fraction of one. So it looks like this is a very small effect. The trouble is, what you are multiplying this effect by is the carrier frequency, which can be up in the gigahertz range. To look at it another way, we are looking at situations where the wavelength is small fractions of the meter. What this equation says is that any time this receiving vehicle moves by one wavelength, namely a small fraction of a meter, the crest of this wave goes from maximum down to minimum back up to maximum again. In other words, in a quarter wavelength, it will go from maximum down to zero. What you are observing at this carrier frequency is very, very different and it keeps changing rather rapidly.

All these other terms are just junk, of course. This f times r 0 over c -- all that is is just a fixed phase difference, so we don't care about that. This is just some fixed term. This quantity here is changing with t also. If we're thinking of a distance away, it's several kilometers and we're thinking of the amount of time for this to become an appreciable fraction of one. For this to becoming an appreciable fraction of this, that's a pretty long time. The amount of time for this to change appreciably is seconds or minutes. The amount of time for this to go through a wavelength change is milliseconds. So despite the fact that you see this sitting in an important place down there, this is not important. Everything that goes on as far as fading is concerned is tied up with that term there. This is important.

So we now have a system which is linear. We still have the linear field equation, but it's not time invariant anymore. It's changing with time. The response is changing with time. You send an exponential, what you received. If you have a linear time invariant system, when you send an exponential of frequency f, you receive an exponential with frequency f. The only thing that a linear time invariant system can do is change the phase of that signal and change the amplitude of it. Can't do anything more complicated than that. That's why we love to study it; because it's so simple.

Now we have something more. We have a system that can also change the frequency of what's getting received. This small change down here -- and of course, if obstacles get in the way or something then there's this huge shadowing difference and all those important things, but you can't do anything about that. You can do something about this, which is why we're focusing on that.

Let's go to the next example. Incidentally, that example is no problem at all for communication. I'll show you why in a little bit. You can get around that problem very, very easily and I'll show you why. This is a problem you can't get around so easily. Here we have a vehicle, which is travelling, say, at 60 kilometers an hour. Person's talking on his two cell phones, has his eyes closed because something surprising is happening, there's this big reflecting wall right in front of him and he doesn't see it at all, so he's speeding into this wall. We're going to analyze this problem right before he hits the wall. We have two paths here. We have one path which is the path from here out to the vehicle, which has a length, r of t -- this is the distance away from the sending antenna to the receiving antenna. We have another path which has a length d, then it gets reflected and this distances is d minus r of t. The total length -- and you're adding up this length with this length -- is 2d minus r of t. The reason is -- do I have an extra picture there? No, I didn't make my extra pretty picture. The reason is that one way to deal with electromagnetic radiation is when you see a wall, the thing that happens is that you get a reflection which is coming back this way. The reflection has a strength which is equal to the radiation that you would get if there weren't any wall -- uhhuh except, of course, that you've changed directions. In other words, this wall has generated a new plane wave which is going backwards, which is just enough to make the electric field strength on this wall equal to zero, because we're assuming a perfectly reflecting wall.

You can satisfy Maxwell's equations by having this incoming electromagnetic wave. You would like to have an outgoing electromagnetic wave, but you can't do that because there's no way for the wave to get through it. The only way you can do it is to generate a new wave, which is moving backwards, which cancels out the incoming wave right at this point. We really have a path here of length 2d minus r of t and as a result of that, the electric wave has two components. One is the component we were dealing with before where there's this Doppler shift because this is moving away from the sending antenna. The other term -- in fact, we are moving closer to the wall and the distance in this path is getting shorter and shorter as doom approaches. Here we have a positive Doppler shift. Here we have a negative Doppler shift and we have these junk terms in both places. One is fr 0 over c. One is 2d minus rc over c. Here we have r0 plus vt. Here we have 2d minus r0 zero minus v.t.. As we said before, this term and this term are not changing very rapidly. It makes it a little easier to analyze this if we say, let's suppose that this is equal to this. In other words, we'd like to look at this right before the car strikes the wall. That also is where this approximation is best because for those of you who have studied electromagnetism, you know that if a plane wave impinges on a wall, funny things happen.

If you look very far away from the wall, you will find this electromagnetic wave, which looks like a plane wave if the wall is distant from the source. So this electromagnetic wave coming in, there's this wall in here and what happens to the electromagnetic radiation is outside of the wall that's going to go out past the wall -- and because of Maxwell's equations, it just sort of gathers together beyond the wall and it sort of comes together. What you find is a disturbance, which is just around the wall. Far away from the wall, you get the same electromagnetic radiation that you had before and close to the wall you have this disturbance. If you look at the situation -- here it is. If you look at what happens here and the wall is not big enough, if the wall is very small, this reflection is really going to look like what happens when you have an electromagnetic wave hitting the wall and the wall then re-radiates an electromagnetic wave, which very far away, this wall just looks like a point source. What you have is instead of a 1 over r, 1 over 2r minus 1 over 2d minus r attenuation, you have a 1 over d attenuation multiplied by a 1 over d minus r attenuation.

If you didn't get all of that, fine. Doesn't make any difference. The point that I'm trying to make is that this analysis is really limited to the case where there the wall is rather small, where the wall is very large, because otherwise you won't have just this plane wave radiation effect. What we wind up with these two terms instead of one term. If I throw away all of the phase terms and I assume that the denominators are equal -- I'm going through this for some sort of reason -- I wind up with two sinusoids: e to the blah and e to blah plus. When I take the real part of the sum of two sinusoids, and I look in all of the high school books I can think of about elementary geometry and playing around with sine waves, what I find is that this collapses into 2 alpha times the sine of 2 pi ftv over c times the sine of 2 pi ft.

In other words, it collapses into a sinusoidal term, which is the major part of this term, ft, and the major part of this term. In other words, I can cancel out the terms here that are the same as the terms here. When I cancel out those same terms, that's the term that comes out. When I look at the other terms, I get an e to the minus vc over t and an e to the plus vc over to t. When we look at e to the minus vc over to t and e to the plus vc over t, even I know how to deal with that. It looks like either a cosine term or sine term, depending on whether the sines in the same or the sines are different.

What we have at that point is this sinusoid is really a sinusoid at the carrier that we're transmitting at. This term here is really something which expands and contracts slowly. So it's a beat, which says that if you're transmitting from this source at this receiver, what you're hearing is something which contracts, expands, then contracts, then expands again. There's nothing you can do about that problem either. There's just no energy there part of the time.

This sine term here is running along, sort of changing from maximum to minimum at a few milliseconds time period. If you have this vehicle traveling at 60 kilometers per hour -- you just work out the numbers there with the velocity of light and all of that stuff. You find that you really can't communicate over your cellphone in that situation, because of these peak frequencies. It's too fast. It's too slow to ignore and ride over it and it's too fast to be able to get all of your data transmitted before it happens. It sort of is a catastrophe.

So that's what happens because of Doppler shift. You get a response which is periodically fading at the Doppler frequency. This is called multipath fading or fast fading. It's called fast fading because it happens so fast. It's called multipath fading because it happens because of multiple paths, which each have lengths which are changing relative to each other.

Let's go back and look at the thing we had before, where we just had a moving antenna. Here you have a Doppler shift also. Why doesn't it bother you? Here you have this Doppler shift and you're transmitting, let's say, a gigahertz and what the receiver is getting is the gighertz minus -- perhaps a kilohertz or something. So why isn't that a problem? If I demodulate at the carrier frequency, I'm sort of in bad luck because I have a signal then which is changing very rapidly. I have something which looks like a time varying system. But what's going to happen? If I use the same kind of frequency recovery system that we talked about earlier, that frequency recovery system has all the time in the world to track that frequency which is one gigahertz minus a kilohertz. It can track it perfectly, which says so what happens is we start out with a signal. We move it up in frequency by one gigahertz. The Doppler shift moves it down by a kilohertz. We track that frequency, then we move it down again by a gigahertz minus a kilohertz and everything works fine and nobody even knows that there's any Doppler shift there.

So the problem is not Doppler shift. The problem is multiple Doppler shifts which are at different speeds relative to each other. That's an important point and we will come back to it as we move along. If you put all of those phases that we neglected in the analysis that we just went through, this is the equation that arises. I write this down not because it's important, but because the notes -- this is in lecture 20 -- have an error. In the sine term, it fails to put an i in, which should be there and this is the correct term and that is off like ninety degrees. If you write this down, you will then see what's going on. Equation 7 in the notes has an e to the 2 pi f.t. minus f.d. over c, which is not the right thing.

As we said, the fading is due to Doppler spread between different paths. The single Doppler shift does not bother us at all. If you have a vehicle which is traveling away from the sending antenna and you have some kind of reflector, which is not something you're running into, but which is a reflector above, a reflector below or something like that, the thing which is going to happen is that both of those paths then are going to have roughly the same Doppler shift in them and if they both have roughly the same Doppler shift, it's not going to be this kind of beat cancellation that we have here, which is something you really can't get rid of. The other thing that this points out again is that this variation is going to be in terms of minutes or seconds and anything you're doing to track the signal is going to be adequate for that until you get to the point where there just isn't enough energy anymore and then of course you have to move to a different base station or something else.

Want go through one more example of electromagnetism because it's so surprising -- at least, it was surprising to me when I found this out. If you have a sending antenna -- think of this as a base station, which is high up at about maybe 15 meters or something. It's sending to some receive antenna, which is at some height above the ground -- usually quite a bit smaller. Suppose there is some more or less partly reflecting plane, like a road. Here we have a vehicle which is travelling along a road and we have ascending antenna, which is also close to the road, which is sending the signal so we have two paths, one which is the direct path from sending antenna to receive antenna. The other is a reflecting path which goes down here and comes back up again. The rather surprising thing here, the thing that I couldn't believe when I saw it, because it contradicted all of my intuition, is that when r gets quite big, the difference between these two path lengths goes to 0. Is that surprising to anybody else? Anybody awake enough to be surprised?

This difference in path length here really goes down as 1 over r. That's an easy geometric problem to solve. You just write down what this length is and the sum of these two lengths and you will find when you do it that the difference is proportional to 1 over r. What happens then is that as r gets big, these two path lengths get closer and closer together. As they get closer and closer together, eventually they're much closer than one wavelength to each other. When they're much closer than one wavelength to each other, the thing that happens is that we get a reflection in the electric field here, so. This field and this field are going to be canceling each other. They'll be canceling each other, except for this phase difference which is proportional to 1 over r, and the phase difference which is proportional to 1 over r is the only thing that gives us any power here at all. As we move further and further away, that phase difference goes down as 1 over r, which means what's happening is the overall electric field that you receive here, instead of going down as 1 over r, which it would in free space, is going down as 1 over r squared. Since it's going down as 1 over r squared, it means that the power that we're receiving is proportional to 1 over r 4th instead of 1 over r squared.

The analysis of this is not particularly important. Why it is that surfaces such as macadam reflects so well is not particularly important to us either. The point is that when you look at all the problems of electromagnetic radiation, in actual situations, you find some situations which behave like this, you find some situations which behave like this nice plane wave and free space. You find other things where in fact the radiation goes down as 1 over r 6th, rather than 1 over r squared, so that we wind up with each path has its own particular kind of attenuation in with path lengths and it's kind of hard to figure out what all of those are.

Then you go on and say that things are even worse than this because sometimes you're communicating through a wall which is only partly absorbing and if you're transmitting through a wall which is partly absorbing, then the attenuation is exponential in the width of the wall. You have a lot of different paths, all of which are very messy electromagnetic radiation problems and all of which have attenuations which range from 1 over r squared to 1 over r 6th, sometimes with an exponential thrown in for good measure, which says that if you really try to find the electromagnetic field at a wireless cell phone, you're in very deep trouble. It's certainly not something that your cellular phone is going to solve for you and it's certainly not something that you're going to have solved ahead of time and program into your cellular phone or into the base station or anything else.

The question is, what do we do about this? One thing is, we're not going to study these electromagnetic phenomena any further because it's a losing game. If the thing that you're interested in is finding out where to place base stations, all of this kind of analysis is useful and you can go much further with it and you should go much further with it. If your interest is in, how do you build third or fourth or fifth or 20th generation wireless systems? All the electromagnetics that you study is only going to give you gross ideas of what kind of phenomena you have to deal with. We already have some idea of the kind of phenomena we have to deal with. We have to deal with paths which have different attenuations on them, which have different propogation delays on them, and all of these multiple paths are things that we have to somehow deal with without analyzing them in detail.

The question is, how do we do this? Everything that we've done so far is called rate tracing. In fact, even with all the complexity that we're dealing with now, we have highly oversimplified it. Each of these paths that we have is going to give rise to an attenuation. We'll now call the attenuation beta of j of t and a propogation delay, which we'll call passive j of t and the propogation delay is just what we get by assuming that a plane wave is going from source to destination and we add up the distance from source to reflector to destination and that gives us this propogation delay.

These are going to vary with time, but in our rate tracing approximation, we've assumed that they're independent of frequency. I originally assumed that the antenna pattern was a function of frequency. We didn't want to say anything about that. So we have a total of capital J paths. we put in an input of cosine 2 pi ft, what's going to come out is an electromagnetic radiation, which is a sum of these different attenuation factors times e to the 2 pi ft minus this propogation delay.

Everything you can do with ray tracing is included in this formula. What you might be able to do in a wireless system is you might, by looking at the received wave form and knowing things about the transmitted wave form, you might be able to figure out what these attenuation factors are and what these propogation delay factors are. Just like when we tried to do frequency recovery, we can find out what the transmitted frequency was. We can play the same sorts of games here, but they're harder and we'll talk about that later. If you ever hear the term rake receiver, a rake receiver is a receiver that in fact measures all this stuff and responds to it and we'll talk about that probably next Monday.

If we want to look at that the reflecting wall just as an example of what this formula means: beta 1 of t, namely for the direct path. We have an attenuation, which is the magnitude of the antenna patterns divided by r0 plus vt. For the attenuation on the return path from the wall, it's the same alpha. Why is it the same alpha? Because we assumed it was the same alpha to make things simple for ourselves -- divided by 2d minus r0 minus vt.

If you look at these propogation delay terms, the propogation delay terms are r0 plus vt divided by c and this gives us the Doppler shift that we're interested in here. We're also going to have an extra term here, which is really caused by the phase change at the transmitting antenna and the phase change at the receiving antenna and a phase change at a reflector, if there's any there. We have the same sort of term at both of these places. The reason that I talk about that is if you look at the electromagnetic wave that you received for this reflecting wall problem that we've talked about a good deal, the second term is there with a negative sign rather than a plus sign. Here, everything is put in with plus signs. You can create negative signs by phase changes of pi. So the assumption is, we put in a phase change of pi as part of this term here. So all of these terms can be expressed in this general forum here.

As we said before, you only have two choices with a cellular system. You cannot solve the electromagnetic field problem at the cell phone. The person using the cell phone is not going to do it. The cell phone is not going to do it. The base station is not going to do it and you're not going to store all those changes because these radiations change remarkably within a period of just small fractions of one meter. You have a coverage here, which in fact, is at least area coverage of one kilometer times one kilometer. Then the reflectors are going to be moving also, so you can't deal with them very easily either. It's a hopeless problem, to try to solve the electromagnetic problem and store it some place.

Electromagnetism helps us to limit the range and likelihood of choices, but it doesn't help in actual detection. So we're now going to deal with the kind of thing we just talked about, which is this sort of general expression for electric field in terms of attenuation factors and phase changes, as opposed to anything which is wh much more detailed.

We're going to define a channel system function as just this sum of these attenuation terms times phase change terms. The reason that we're doing this is that if we put in an input -- e to the 2 pi ift, then what we get is this system function here -- times x of t, which is e to the 2 pi ift, so we get this quantity here. Here's the minus 2 pi if tau -- that's that term coming down there and here is the e to the 2 pi ift coming down here. So all of this term and this term are both included in this system response term. This is linear also, so we know what the response is to an exponential that we put in. I'm cheating you a little bit here by going from real to complex. And the notes do that a little more carefully, but we ought to be used to that now. If I put in an input, x hat of f e to the 2 pi ift df and integrate it. In other words, if I put in an arbitrary input, which I represent in terms of its Fourier transform, I now know what the response is to every x-hat of f. It's given by this response here. So I just integrate over that and I find that the response y of t to an arbitrary input now is the integral of x-hat of f, h-hat of ft, e to the 2 pi ift df -- namely, the same game that we always play. Namely, that's just the system analysis way of looking at arbitrary systems in terms of their Fourier transforms. So the output y of t is just this integral here.

Important point: When you look at this, this says this is the same as any old linear time invariant system. This is not a linear time invariant system. If you try to take the Fourier transform of this to get y-hat of f, you're not going to get x-hat of f times h-hat of f. Namely, this is not equal to that. Why isn't it equal to it?

First reason is, try to take the Fourier transform of this and see what you get over here and when you deal with the fact that there's a t in here, you will find that there's nothing you can do. You'll find you're stuck. So you can't derive this equation here.

Next, argument is this quantity here is not a function of t. This quantity here is a function of t. You can't have a quality between something which is not a function of t and something else which is a function of t. It just can't happen.

The final argument is, if you look at what's happening here, when you put in a single frequency -- when you put in x of t equals e to the 2 pi ift, what comes out? In terms of this reflecting wall example, the thing that came out was not one sinusoid, but two sinusoids. One sinusoid a little bit above the carrier, the other sinusoid a little bit below the carrier. In other words, because of Doppler shifts, when you put in a sinusoid, what comes out is not a sinusoid, but a modulated sinusoid. It's something spread out over a region of frequencies.

So one of your favorite tools for dealing with linear time invariant systems is no longer adequate. Doesn't work. Just make a mark of that, because every time you see a problem like this, everytime I see it, the first thing I try to do is go through that most familiar and most favorite form of linear system analysis, which is the Fourier transform of convolution, is the same as multiplication in the frequency domain. You cannot do that anymore. However, convolution still work, so that's the next thing we want to look at.

So the thing that we have is the output of the system is now going to be this integral over frequency -- x-hat of f times the frequency response function for the linear but time varying system, times e to the 2 pi ift.

This is what we derived on the last page, on the last slide. It is the thing which just automatically happens here. If I let h of tau and t be the inverse Fourier transform of h-hat of ft -- and here what I'm doing is I'm regarding t as a parameter. So this is a function of tau now and this is a function of tau for a given t, I can take the Fourier transform of this. Take the Fourier transform of any old thing at all, so long as it's L2. I will sort of half-pretend it's L2. We'll worry about that later. So this is the Fourier transform of this.

So then, the thing that happens is that y of t is going to be equal to this quantity here, except in place of the system function, h and f of t, for a particular value of t, I'm going to put in this inverse Fourier transform. So it'll be h of tau and t, e to the minus 2 pi i of tau d tau. This integral here is just h-hat of f and t.

If I take this quantity here and I move this term inside and I interchange orders of integration -- incidentally, when we're dealing with wireless, we're going to forget about all of the nice things that we know about L2 functions. There's just too much new stuff that's going on here to worry about that. So what you want to do is just take Fourier transforms like while, interchange orders of integration, interchange everything you want to and simply forgot about all the mathematical problems that might arise. After you understand this, at that point, go back and straighten out the mathematical issues.

This in fact is the way we deal with any problem, or the way you should deal with any problem. You don't bring the mathematics in unless it's going to help you solve the problem. You don't bring it in to frustrate yourselves.

So the thing we're going to do now is to interchange these orders of integration. We're going to integrate over tau on the outside and f on the inside. So we're going to bring the function of tau outside. This is an integral in tau. The function of f is going to go on the inside. we have e to the 2 pi ft -- that's this quantity here. We have this term there. When we look at this, we see something very nice because this is in fact just the Fourier transform of x of t minus tau. When we take that out, what we get is the integral of x of t minus tau times h of tau and t be tau. In other words, this is time varying convolution. Nice, simple equation, makes a lot of sense. It says you have this impulse response here. h of tau and t now can be interpreted as the response at time t to an impulse tau seconds earlier. If you have a system which is changing very, very very, slowly, then this is essentially just a function of tau and it's the usual impulse response that you're familiar with -- namely, this convolution equation gives you the response at time t to an impulse tau seconds early.

Now we just have something which says this is a linear time varying filter and in all the cases, we're interested in this linear time varying filter, changes its impulse response very slowly as time changes. Relatively fast change with tau, relatively slow change with t. So this is very similar to linear time invariant convolution. Channel behaves like a slowly time varying filter and that's the bottom line of this.

For these ray tracing models we were looking at, the system function is a sum of terms at the sum of attenuations times phase change terms. If we take the inverse

Fourier transform of this -- remember, we're taking the inverse Fourier transform on f and putting in a tau. So we're taking this inverse Fourier transform for a particular t. We have a function of f and of t. We take the inverse Fourier transform with respect to the f and get a tau here. This then becomes this quantity here. How do I interpret that? If I look at a single term here, what is it? A single term here is just an attentuation factor, a constant times a sinusoid. At a particular value of t, this is just the constant here also. So for a particular t, all I have is a sinusoid. What's the inverse Fourier transform of a sinusoid? I told you all along that it doesn't exist, but for the time being we will assume that it's what you learned early, that the inverse Foruier transform of the sinusoid is an impulse. So here we are with our impulse there. This says that the response at time t to an impulse at tau is going to be zero unless tau is equal to one of these propogation delay terms.

In other words, I have a system where I'm putting in an input and this input comes in. The response to the input is a number of different path delays and at each path delay, what I'm going to get out of the system is just a delayed and attenuated version of what I put in. Namely, the system isn't a function of frequency at all. That's what I get through using ray tracing. I mean, it's one of the consequences of using ray tracing. So what I wind up with is a system function which is a string of impulses and the output then, the convolution of this with that -- you're probably better at working with impulses than I am -- and it's y of t is just the sum of these attenuation terms times x of t at these various delays. So what we're doing is we're putting in an arbitrary input. What's coming out is that attenuated input coming out at various different times, due to these various different paths. I get various paths that are delaying the input by different amounts and out that delayed input comes at various times.

This is a nice sanity check because if you think about it, that's exactly what ought to come out of here. On the other hand, you ought to wonder about this impulsive impulse response, because that clearly doesn't make any sense physically. So what's going on here? The thing that's going on is that when we started, we said, if we're putting in a narrow band input, we don't care about the frequency response because the frequency response on these different paths cannot change very quickly and therefore, we're just going to have a fixed frequency response term. Then we've worked with that thing which is not a function of frequency and then finally we get down here, where in fact, what we're doing is looking at the output. Due to a bunch of delayed input terms, if this input term here -- if x of t is in fact the narrow band term -- I guess the way to see that as to look at -- where do I look at it? I want to look at this expression here. If I have a narrow band or maybe -- I guess this one is better here. Let's look at this expression. If my input is in fact narrow band, it's only going to be non zero over a small range of frequencies. If it's only non zero over a small range of frequencies, I don't care what this is, except over that small range of frequencies. All of this gets filtered out. In other words, this filters out this, opposite of the usual case. So I don't care about what this is at different frequency ranges and therefore, we simply have the consequences of this, which is something that you see in linear system theory all the time. We've sort of ruled out impulses and sine waves because they don't carry information, but in terms of looking at things as intermediate points and going through filters and things like that, they're perfectly fine

So here, for simplicity, we assume that these channel filters really do not have any -- don't respond to frequency changes, whereas in fact they do, and all we're doing is modelling them in certain frequency bands, which, when we get all done, is what really gets rid of all our problems, due to this sort of input because this input is smooth now and therefore, the output is smooth also.

The next thing I want to spend a little bit of time on and we'll come back to it next time is, how do you deal with all of this at baseband? I should warn you here that the notes don't do a terribly good job of this. They have all of the results that you need. They don't seem to put them in a very nice, well organized fashion. I'm not sure there is a nice, well organized fashion to put them in. But anyway there's a lot of stuff going on when you try to move this down from pass band down to baseband. I will try to change the notes a little bit to make it clear, but I'm not sure that I can.

The kind of system that we're looking at now is our usual QAM type system, which can be generalized somewhat. We have a binary input coming in. We have a baseband encoder. That baseband encoder is creating baseband signals, which are being added together to give us a baseband complex input to the channel. This is being frequency modulated up to some function, x of t, which is just the real part of ut times e to the 2 pi if of ct as usual. So we now have a real part of this signal here, modulated up by the carrier frequency. This is going through what we'll now regard as a time varying channel filter. We talked a little bit about channel filters before when we were talking about Nyquist theory, because we said in general, you want to take your input, you want to pass it through a pulse p of t. That goes through another filter, which is some h of t and that goes into another filter, which is at the receiver, which is q of t and you want the product of t and you want the convolution of all of those to satisfy the Nyquist criteria.

So we're back with that in spades because now this is varying with t also. So this goes through this channel filter, up at pass band. The channel filter up at pass band, we've seen that one of the things that it can be viewed as doing is putting Doppler shift into this input. So what comes in at some frequency f is now going to be coming through here at some slightly different frequency. We then add white noise to it. We then get y of t out, which has now been shifted around and smudged in frequency a little bit. We go through a frequency demodulation. We get down frequency to modulation by this carrier frequency. We get down to v of t, which is now a baseband complex function again, which is supposed to be the same as this except for the white Gaussian noise and except for the fact we've gone through this filtering operation here. Then we want to do base band detection at this point.

What we would like to do and what will make life a little easier for ourselves because we all got sick of this business of looking at filters at baseband and also looking at filters at pass band, we would like to be able to take some baseband equivalent of this filter here. So what we're going to do is look at the baseband equivalent. The system function at baseband corresponding to this system at carrier frequency will just be this response moved down by s of t. In other words, when you take what comes out of here, you multiply it, and you shift it down in frequency by s sub c, what happens is that this gets shifted down in frequency by s sub c and this channel filter gets shifted down in frequency by f of c and therefore, what happens is the effect of passing y of t through a base band filter -- h-hat of f plus fc and t or 0 for f, less than or equal to minus fc.

So far, this is all kind of straightforward and not too mysterious. So you wind up then in -- and this is pure analogy to what we did before -- the output is then going to be the integral of this Fourier transform of the input. The system function for the baseband filter times e to the 2 pi ift df. Same equation as we had before, but before we did it at pass band and now we're doing it at baseband . For the ray tracing model that we looked at, this function here down at baseband is going to be the same as it was at pass band, except in place of the 2 pi i tau jft, we now have f plus the carrier frequency times tau jft.

So when we take the inverse Fourier transform of this, we're given parameter t. What we're going to wind up with is this quantity here. This is the same as we had before, with the difference that now we're stuck with this carrier frequency times propogation delay, which is occurring in time t. We still have the same delta functions here and the output v of t is the same time variant convolution as we had before.

I'm doing this much too fast for you to follow this in real time. What I'm saying is, this is exactly the same thing as we did before and the only thing new that happens is now this carrier frequency term is coming in here on this term. If we're using the ray tracing model then, v of t is equal to this quantity here. The reason I write this down is that this shows you quite simply what's going on in terms of the Doppler shift, because these terms here, these delay terms are changing with Doppler shift and now we're going to see exactly what they do to us.

So we're going to represent the propogation delay on each path. That's tau j of t, which is tau j prime, which is the propogation delay of time zero minus the Doppler shift times t divided by f. This is just Doppler shift, which is a frequency term, and it's increasing linearly. The propogation delay is increasing linearly with t. The Doppler shift is a shift in frequency and, as I think we saw before, you need a 1 over f to compensate for this Doppler shift. Namely, this is in terms of hertz. This is in terms of hertz. This is in terms of time. This is in terms of time. So you need this frequency here to be dimensionally right at any rate.

If we're using narrow band communication, then v of t is just going to be this difference here, which is just the last equation with the Doppler shift put into it. So all of this says that when we're now looking at things at base band instead of a pass band, what has happened is that this term has been added. Before, we just had these delay terms here and we had these attenuation terms. Now what's happening is because we are modulating up with carrier frequency s sub c, we then get a Doppler shift that moves us down a little bit. We're then moving down by s sub c. What we wind up with is something which is off by that Doppler shift. If we only had a single term, we wouldn't be demodulating by s sub c. We'd be demodulating by something which would get rid of this term for us. Then all we'd have is just some arbitrary phase term here, because this isn't important. This is just a phase term. This is the term which is important.

The reason for going through all of this -- and I'm going to come back to this next time -- is that when you use frequency recovery at the receiver and you always use frequency recovery at the receiver, what's going to happen is you're not going to shift down by s of c. You're going to shift down s of c, plus some average value of Doppler shift. If they have a bunch of terms, each with different Doppler shifts in them, when you try to measure how much Doppler -- when you try to measure what the received carrier is, you're going to be frustrated by all of these different Doppler shifts. You're going to come up with some average value in your frequency circuit, which means that in place of this quantity, you will get something which replaces each of these D sub j's by not the actual Doppler shift, but by how far this Doppler shift is away from the main Doppler shift.

So these terms here, which are the things that make this system function in here change with time, are in fact changing according to how far away each of these Doppler shifts are from the main Doppler shift, which means that the amount of time this system function in here is going to remain stable depends on how much the Doppler shifts vary from each other on these different paths. In other words, the important thing is the Doppler spread between the biggest Doppler shifts and the smallest Doppler shifts. That Doppler spread is going to determine how long you've got something that looks like a linear time invariant system function which says that every once in awhile, if you're trying to measure what's going on at the channel, at intervals of time approximately one over two times the Doppler spread, you're going to have to change those measurements. So whether you can make cellular telephony work or not depends on whether you can make measurements at a speed which is equal to one over two times the Doppler spread.

I'm going to do more about that next time. I don't expect you to understand it now. Maybe after you read about it and we talk about it more, because in fact that's -- if you look at this question of Doppler spread and what its effect is on how long a system looks like it's time invariant, this is one of the key parameters to understanding how any kind of wireless system is going to work.