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**6.453 Quantum Optical Communication  
Lecture 1**

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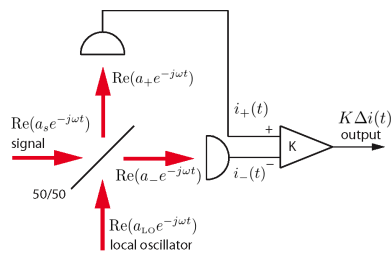
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## **6.453 Quantum Optical Communication — Lecture 1**

- Handouts
  - Syllabus, schedule/policy, probability chapter, lecture notes, slides, problem set 1
  - Sign-up on class list
- Introductory Remarks
  - Subject organization
  - Subject outline
- Technical Overview
  - Optical eavesdropping tap — quadrature-noise squeezing
  - Action at a distance — polarization entanglement
  - Long-distance quantum state transmission — qubit teleportation

### Optical Homodyne Detection — Semiclassical

Balanced Homodyne Receiver



- Signal is weak, LO is strong
- Energy conservation
 
$$a_{\pm} \equiv \frac{a_s \pm a_{LO}}{\sqrt{2}}$$
- Detectors are noisy square laws
 
$$i_{\pm}(t) \text{ Poisson distributed}$$

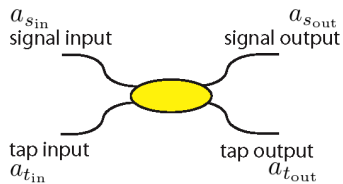
$$\text{mean} = |a_{\pm}|^2$$
- Output mean and variance
 
$$\langle K\Delta i(t) \rangle = 2K\text{Re}(a_s a_{LO}^*)$$

$$\text{var}(K\Delta i(t)) = K^2 |a_{LO}|^2$$



### Optical Waveguide Tap — Semiclassical

Fused Fiber Coupler



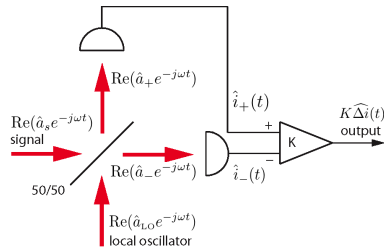
- Coupler is a beam splitter
 
$$a_{s_{out}} = \sqrt{T} a_{s_{in}} + \sqrt{1-T} a_{t_{in}}$$

$$a_{t_{out}} = \sqrt{1-T} a_{s_{in}} - \sqrt{T} a_{t_{in}}$$
- Tap input is zero
- Homodyne SNR at signal input
 
$$\text{SNR}_{in} = 4|a_{s_{in}}|^2$$
- Homodyne SNR at signal output
 
$$\text{SNR}_{out} = 4T|a_{s_{in}}|^2$$
- Homodyne SNR at tap output
 
$$\text{SNR}_{tap} = 4(1-T)|a_{s_{in}}|^2$$



**Quantum Homodyne Detection and Waveguide Tap**

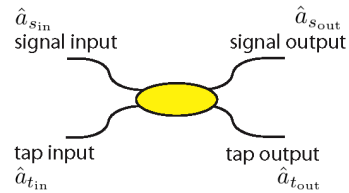
Balanced Homodyne Receiver



Homodyne SNR at signal output

$$\text{SNR}_{\text{out}} \approx 4|a_{s_{\text{in}}}|^2$$

Fused Fiber Coupler



Homodyne SNR at tap output

$$\text{SNR}_{\text{tap}} \approx 4|a_{s_{\text{in}}}|^2$$



**Billiard-Ball Photons and the Poincaré Sphere**

- Polarization of +z-going photon:

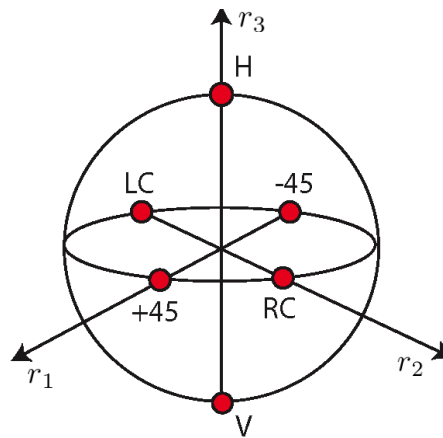
$$\mathbf{i} \equiv \begin{bmatrix} \alpha_x \\ \alpha_y \end{bmatrix}, \quad \mathbf{i}^\dagger \mathbf{i} = 1$$

- Poincaré sphere representation

$$\mathbf{r} \equiv \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 2\text{Re}(\alpha_x^* \alpha_y) \\ 2\text{Im}(\alpha_x^* \alpha_y) \\ |\alpha_x|^2 - |\alpha_y|^2 \end{bmatrix}$$

- $\pm \mathbf{r}_m$  polarization measurement

$$\text{Pr}(\text{polarized } \pm \mathbf{r}_m) = \frac{1 \pm \mathbf{r}_m^T \mathbf{r}}{2}$$



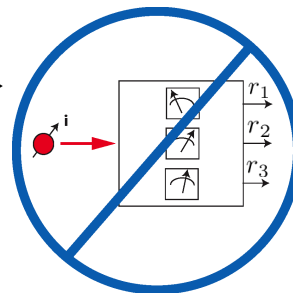
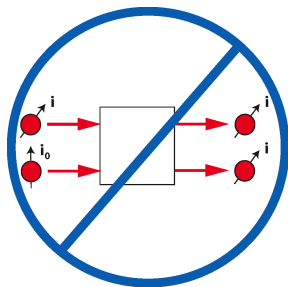
**Classical Correlation vs. Quantum Entanglement**

- Classically-Correlated, Randomly-Polarized Photons
  - Source produces  $\pm \mathbf{r}$  photon pair with  $\mathbf{r}$  completely random  
 $\Pr(\text{photon 1} = \pm \mathbf{r}_m) = \Pr(\text{photon 2} = \mp \mathbf{r}_m) = 1/2$
  - Conditional probability given photon 1 is  $\mathbf{r}_m$  instead of  $-\mathbf{r}_m$   
 $\Pr(\text{photon 2} = -\mathbf{r}_m \mid \text{photon 1} = \mathbf{r}_m) = 2/3$
- Maximally-Entangled Photons
  - Source produces  $\pm \mathbf{r}$  photon pair with  $\mathbf{r}$  completely random  
 $\Pr(\text{photon 1} = \pm \mathbf{r}_m) = \Pr(\text{photon 2} = \mp \mathbf{r}_m) = 1/2$
  - Conditional probability given photon 1 is  $\mathbf{r}_m$  instead of  $-\mathbf{r}_m$   
 $\Pr(\text{photon 2} = -\mathbf{r}_m \mid \text{photon 1} = \mathbf{r}_m) = 1$



**Properties of Single-Photon Polarization States**

- Polarization cannot be perfectly measured  $\rightarrow$



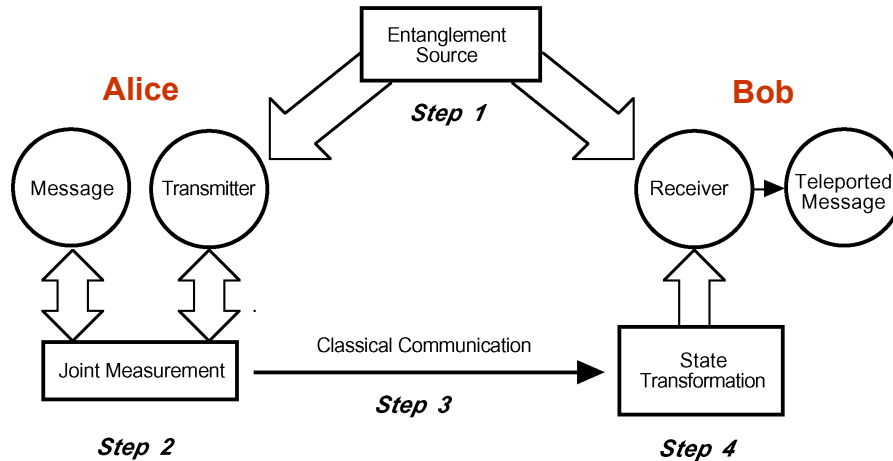
- $\leftarrow$  Polarization cannot be perfectly cloned

- Photons can be lost in propagation:

$$\Pr(\text{photon loss in 50 km of low-loss fiber}) = 0.9$$



## Photon Polarization States Can Be Teleported



## The Road Ahead: Problem Set 1, Lectures 2 and 3

- Problem Set 1
  - Reviews of essential probability theory and linear algebra
- Lectures 2 and 3:
  - Fundamentals of Dirac-Notation Quantum Mechanics
    - Quantum systems
    - States as ket vectors
    - State evolution via Schrödinger's equation
    - Quantum measurements — observables
    - Schrödinger picture versus Heisenberg picture

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