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6.453 *Quantum Optical Communication* Lecture 21

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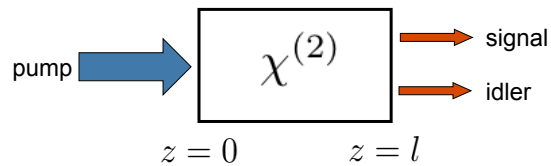
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6.453 *Quantum Optical Communication* - Lecture 21

- Announcements
 - Pick up lecture notes, slides
- Nonlinear Optics of $\chi^{(2)}$ Interactions
 - Coupled-mode equations for parametric downconversion
 - Phase-matching for efficient interactions
 - Classical and quantum solutions
 - Gaussian-state characterization

Second-Order Nonlinear Optics

- Spontaneous Parametric Downconversion



- Strong pump at frequency $\omega_P = \omega_S + \omega_I$
- No input at signal frequency ω_S
- No input at idler frequency ω_I
- Nonlinear mixing in $\chi^{(2)}$ crystal produces signal and idler outputs

Coupled Equations for Plane-Wave Modes

- Monochromatic Pump, Signal, and Idler Electric Fields:

$$\begin{aligned} \vec{E}(z, t) = & (A_S(z)e^{-j(\omega_S t - k_S z)} + cc)\vec{i}_S/2 \\ & + (A_I(z)e^{-j(\omega_I t - k_I z)} + cc)\vec{i}_I/2 \\ & + (A_P e^{-j(\omega_P t - k_P z)} + cc)\vec{i}_P/2 \end{aligned}$$

- Non-depleting pump
- Slowly-varying signal and idler complex amplitudes
- Photon-Units Coupled-Mode Equations:

$$\begin{aligned} \frac{dA_S(z)}{dz} &= j\kappa A_I^*(z)e^{j\Delta k z} \\ \frac{dA_I(z)}{dz} &= j\kappa A_S^*(z)e^{j\Delta k z} \end{aligned}$$

Type-II Phase Matched Operation at Degeneracy

- Phase Matching for Efficient Coupling: $\Delta k = 0$
 - Conservation of photon momentum: $k_P = k_S + k_I$
 - Type-II system: $\vec{i}_S = \vec{i}_x, \vec{i}_I = \vec{i}_y$
- Operation at Frequency Degeneracy: $\omega_S = \omega_I = \omega_P/2$
- Classical Input-Output Relation:

$$A_S(l) = \cosh(|\kappa|l)A_S(0) + j \frac{\kappa}{|\kappa|} \sinh(|\kappa|l)A_I^*(0)$$

$$A_I(l) = \cosh(|\kappa|l)A_I(0) + j \frac{\kappa}{|\kappa|} \sinh(|\kappa|l)A_S^*(0)$$

Quantum Coupled-Mode Equations

- Strong, Monochromatic, Coherent-State Pump
- Positive-Frequency Signal and Idler Field Operators:

$$\hat{E}_S^{(+)}(z, t) = \int \frac{d\omega}{2\pi} \hat{A}_S(z, \omega) e^{-j[(\omega_P/2 + \omega)t - k_S(\omega_P/2 + \omega)z]}$$

$$\hat{E}_I^{(+)}(z, t) = \int \frac{d\omega}{2\pi} \hat{A}_I(z, \omega) e^{-j[(\omega_P/2 - \omega)t - k_I(\omega_P/2 - \omega)z]}$$

- Quantum Coupled-Mode Equations:

$$\frac{\partial \hat{A}_S(z, \omega)}{\partial z} = j\kappa \hat{A}_I^\dagger(z, \omega) e^{j\omega \Delta k' z}$$

$$\frac{\partial \hat{A}_I(z, \omega)}{\partial z} = j\kappa \hat{A}_S^\dagger(z, \omega) e^{j\omega \Delta k' z}$$

Quantum Input-Output Relation

- Two-Mode Bogoliubov Relation

$$\hat{A}_S(l, \omega) = \mu(\omega)\hat{A}_S(0, \omega) + \nu(\omega)\hat{A}_I^\dagger(0, \omega)$$

$$\hat{A}_I(l, \omega) = \mu(\omega)\hat{A}_I(0, \omega) + \nu(\omega)\hat{A}_S^\dagger(0, \omega)$$

where

$$\mu(\omega) = \left(\cosh(pl) - \frac{j\omega\Delta k' l \sinh(pl)}{2pl} \right) e^{j\omega\Delta k' l/2}$$

$$\nu(\omega) = j\kappa l \frac{\sinh(pl)}{pl} e^{j\omega\Delta k' l/2},$$

$$p = \sqrt{|\kappa|^2 - (\omega\Delta k'/2)^2}$$

Gaussian-State Characterization

- Signal and Idler at $z = 0$ are in Vacuum States
- Signal and Idler at $z = l$ are in Zero-Mean Gaussian States
- Baseband Signal and Idler Field Operators:

$$\hat{E}_m^{(+)}(l, t) = \hat{E}_m(t) e^{-j(\omega_P t/2 - k_m(\omega_P/2)l)}, \quad \text{for } m = S, I$$

- Non-Zero Covariance Functions:

$$K_{SS}^{(n)}(\tau) = K_{II}^{(n)}(-\tau) = \int \frac{d\omega}{2\pi} |\nu(\omega)|^2 e^{j\omega\tau}$$

$$K_{SI}^{(p)}(\tau) = \int \frac{d\omega}{2\pi} \mu(-\omega)\nu(-\omega) e^{j\omega(\tau + \Delta k' l)}$$

Operation in the Low-Gain Regime

- Low-Gain Regime: $|\kappa|l \ll 1$

- Approximate Bogoliubov Parameters:

$$\mu(\omega) \approx 1 \quad \text{and} \quad \nu(\omega) \approx j\kappa l \frac{\sin(\omega\Delta k'l/2)}{\omega\Delta k'l/2} e^{j\omega\Delta k'l/2}$$

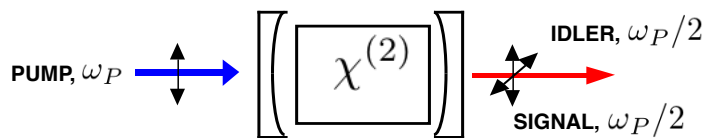
- Normally-Ordered and Phase-Sensitive Spectra:

$$\mathcal{S}_{SS}^{(n)}(\omega) = \mathcal{S}_{II}^{(n)}(\omega) \approx |\kappa|^2 l^2 \left(\frac{\sin(\omega\Delta k'l/2)}{\omega\Delta k'l/2} \right)^2$$

$$\mathcal{S}_{SI}^{(p)}(\omega) \approx j\kappa l \frac{\sin(\omega\Delta k'l/2)}{\omega\Delta k'l/2} e^{j\omega\Delta k'l/2}$$

Type-II Optical Parametric Amplifier

- Doubly-Resonant Operation at Frequency Degeneracy



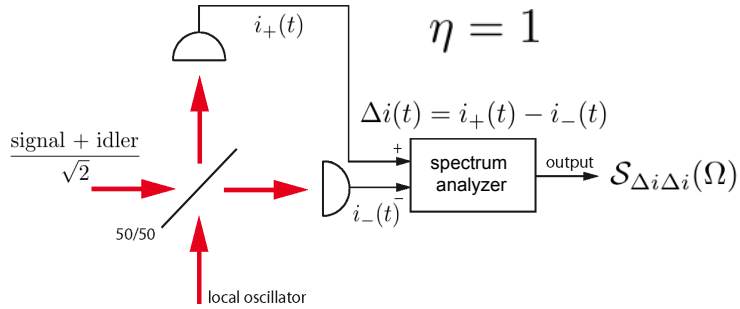
- Normally-Ordered and Phase-Sensitive Covariances:

$$K^{(n)}(\tau) = \frac{G\Gamma}{2} \left[\frac{e^{-(1-G)\Gamma|\tau|}}{1-G} - \frac{e^{-(1+G)\Gamma|\tau|}}{1+G} \right]$$

$$K_{SI}^{(p)}(\tau) = \frac{G\Gamma}{2} \left[\frac{e^{-(1-G)\Gamma|\tau|}}{1-G} + \frac{e^{-(1+G)\Gamma|\tau|}}{1+G} \right]$$

Quadrature Noise Squeezing

- Homodyne Detection of 45° Polarization (Signal + Idler)



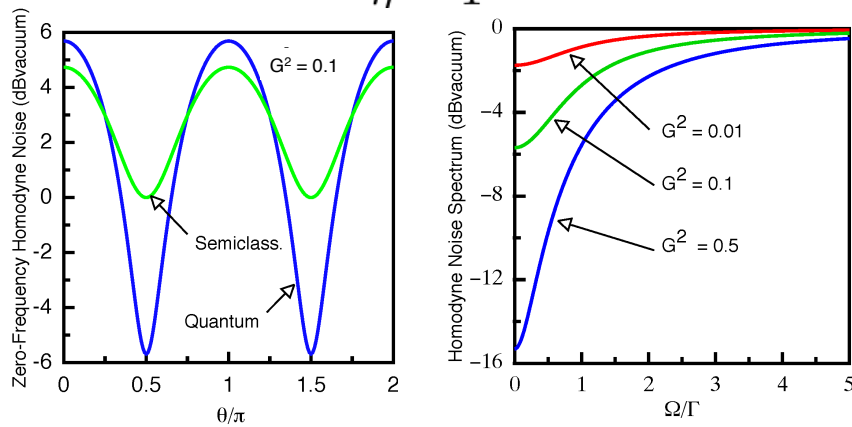
$$\frac{\mathcal{S}_{\Delta i \Delta i}(\Omega)}{\mathcal{S}_{\Delta i \Delta i}(\Omega)|_{\text{CS}}} = |\mu(\Omega) + \nu(\Omega)e^{-2j\theta}|^2$$

$$\mu(\Omega) \equiv \frac{1 + G^2 + \Omega^2/\Gamma^2}{1 - G^2 - \Omega^2/\Gamma^2 - 2j\Omega/\Gamma} \quad \text{and} \quad \nu(\Omega) \equiv \frac{2G}{1 - G^2 - \Omega^2/\Gamma^2 - 2j\Omega/\Gamma}$$

Quadrature Noise Squeezing: Quantum Efficiency 1

- Homodyne Detection of 45° Polarization (Signal + Idler)

$$\eta = 1$$



Coming Attractions: Lecture 22

- Lecture 22:
Quantum Signatures from Parametric Interactions
 - Hong-Ou-Mandel dip produced by parametric downconversion
 - Polarization entanglement produced by parametric downconversion
 - Photon twins from parametric amplifiers

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