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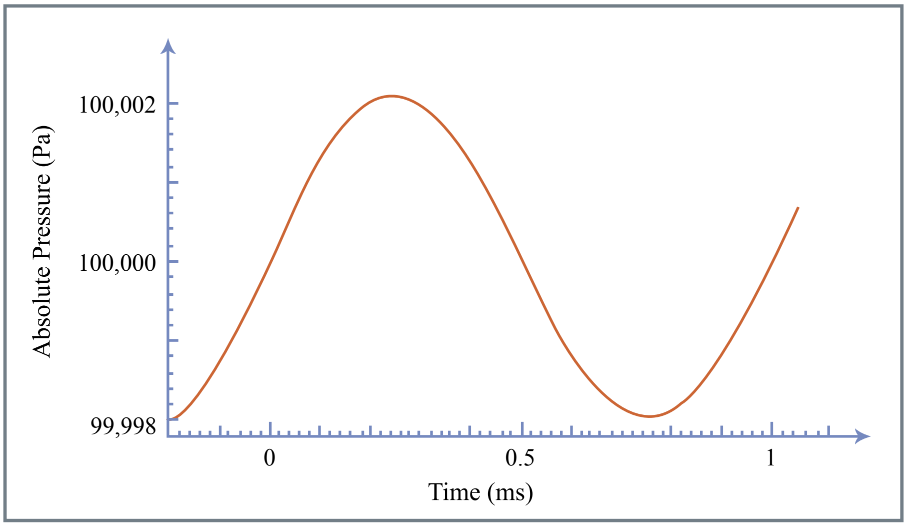
Department of Electrical Engineering and Computer Science
Harvard-MIT Division of Health Science and Technology
6.551J/HST.714J Acoustics of Speech and Hearing

Distributed Thursday 9 Sept 2004
Due Thursday 16 Sept 2004

How long did it take to complete this problem set? _______

Problem1. Some tonal exercises.

Figure 1.1
The *total pressure* as a function of time $p_T(t)$ at a point Q in space in the presence of a propagating plane wave produced by a sound source playing a continuous tone. The sound pressure is zero at time $t =$ ... 0, 0.5 ms, 1 ms ...



a). In the presence of a continuous tonal drive from a distant loudspeaker the *absolute pressure* at a point Q in space is plotted as a function of time $p(t)$ for a short segment around $-0.2 < t < 1.1$ *ms* in Figure 1.1.

- a.i. What is the frequency f in Hz of the continuous tone?
- a.ii. What is the radian frequency ω?
- a.iii. The total pressure is the sum of a constant steady-state pressure P_0 and a time varying sound pressure $p(t)$: $p_T(t)=P_0+ p(t)$.
  What is the *peak-peak* amplitude of the *sound pressure* of the sinusoidal variation?
  What is the *peak* amplitude of the *sound pressure*?
  What is the *rms* amplitude of the *sound pressure?*
  What is the *total pressure* at time t=0?
  What is the *average total pressure* for $0 < t < 1$ ms*?*
  What is the *average sound pressure* for $0 < t < 1$ ms?
- a.iv. Describe the illustrated *sound pressure p(t)* by a sine function in t.
- a.v. Describe the *sound pressure p(t)* by a cosine function.
- a.vi. What is the cosine phase of the *sound pressure p(t)* at time t= 0?
- a.vii. Describe the illustrated *sound pressure* in terms of an exponential function in t with a magnitude, frequency and angle.
- a.viii. We often describe the amplitude of an oscillating sound pressure in terms of decibels or dB. The decibel is a measure of energy relative to some standard, where energy is proportional to (sound pressure)^2:

$$\text{The dB value of a pressure } P = 10\log_{10}\left(\frac{|P|}{P_{Ref}}\right)^2 = 20\log_{10}\left(\frac{|P|}{P_{Ref}}\right).$$

  What is the sound pressure amplitude in Figure 1.1 in dB relative to P_{Ref}=1 Pa rms?

The international sound pressure reference for dB SPL is $2\mathrm{x}10^{-5}$ Pa rms. What is the sound pressure amplitude in Figure 1.1 in dB SPL?

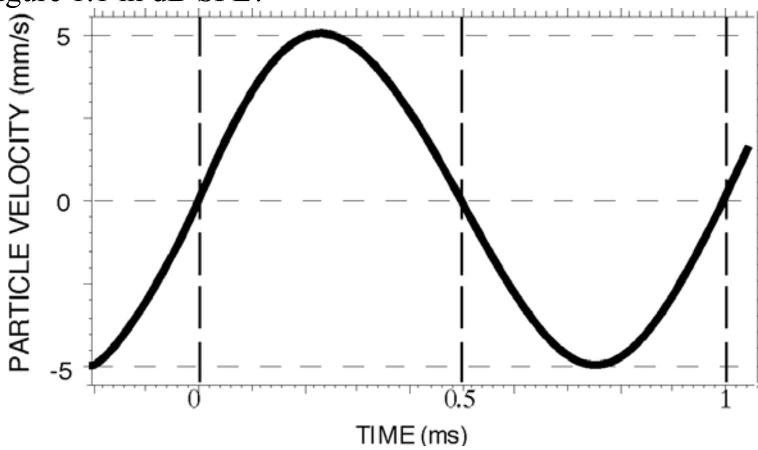


Figure 1.2
The *x* component of sound induced particle velocity $v_x(t)$ (as it propagates in the *x* direction) associated with the pressure variations of Figure 1.1

b). The particle velocity *along the direction of propagation* of the wave $v_x(t)$ at the same point in space (point *Q*) produced by the same tone is plotted as a function of time for a short segment around $-0.2 < t < 1.1$ *ms* in Figure 1.2.

b.i. Define an acoustic air particle.

b.ii. Define particle velocity.

b.iii. In quantifying the plotted particle velocity we were careful to define a direction (‘*along the direction of propagation*’). Why is that? Why didn’t we define a direction for the pressure?

b.iv. Describe $v_x(t)$ at point *Q* in terms of a cosine function.

b.v. Describe $v_x(t)$ via an exponential. How do the magnitude and angle of the exponential notation relate to the magnitude and angle of a cosine function?

b.vi. The specific acoustic impedance at a point in a propagating plane wave is determined by the characteristic impedance of the media $z_0=\rho_0 c$, where ρ_0 is the static density of air at the experimental pressure and temperature, and *c* is the propagation velocity under the same conditions. What is z_0 of air at standard temperature and pressure?

b.vii. The characteristic impedance of a medium, which relates pressure and velocity, has units of rayls, named after Lord Rayleigh. Describe 1 rayl in terms of pascals and meters. Do the same for newtons and meters. Do the same for kilograms and meters.

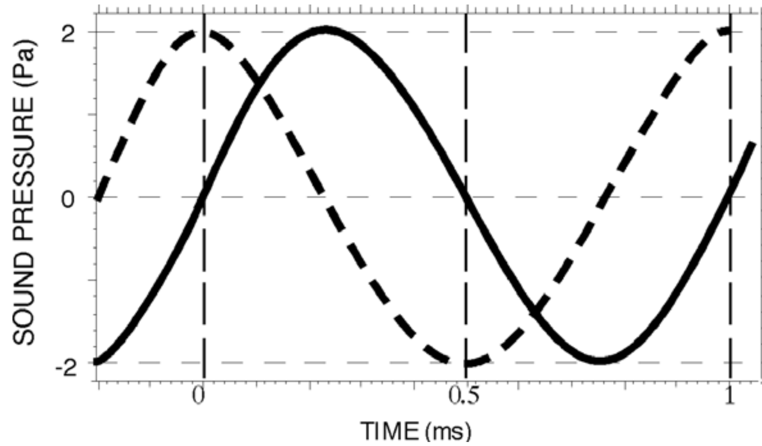
b.viii. The characteristic impedance of air is a real number, i.e. angle(z_0) = 0. Are the pressure and velocity time traces at point *Q* consistent with the being related by z_0? Specificall y are the relative phase angles of the sound pressure and particle velocity equal, as they should be if *p(t)* and $v_x(t)$ are proportional? The characteristic impedance also relates the amplitude of *p(t)* and $v_x(t)$ are the amplitudes in Figures 1.1 and 1.2 consistent with z_0?

b.ix. The characteristic impedance of the media can also be defined in terms of the density ρ_0 and Bulk modulus *B* (a measures of the stiffness of a medium) such that: $z_0 = \sqrt{\rho_0 B}$.

Define the Bulk modulus in terms of ρ_0 and *c.*

Give another definition of the Bulk modulus of air in terms of volume and pressure changes.

Figure 1.3
The sound pressure as a function of time *p(t)* at two points (*S-the dashed line-* & *Q-the solid line*) in space in the presence of the sinusoidal propagating plane wave.



c). Figure 1.3 is a comparison of the sound pressures *p(t)* measured at two points *S* & *Q* in the sound field produced by the propagating plane wave. The line connecting the two points in space is parallel with the direction of sound propagation. Furthermore, the environment is anechoic, so we need not be concerned with reflected sounds, i.e. the only relevant sound wave is the one that is propagating from the distant speaker.

c.i. What is the propagation velocity of sound in air at standard temperature and pressure?

c.ii. What is the smallest propagation time necessary to explain the phase difference between the sound pressures at *S* & *Q* ?

c.iii. Based on this smallest propagation time, what is the shortest distance between *S* & *Q*?

c.iv. Define a few other measurement-point separations that can fit the data of Figure 1.3. Hint: The amplitude of a propagating plane wave is constant, i.e. independent of the distance from the source.

c.v. Describe the sound pressure as a function of time at any point on the plane that contains *S* and is orthogonal to the direction of propagation.

d). Sound Intensity is defined by the Real part of the product of sound pressure and the complex conjugate of particle velocity:

$$I = \frac{1}{2}\mathrm{Real}\left\{\underline{P}\underline{V}^*\right\} = \frac{1}{2}\mathrm{Real}\left\{\underline{P}\left(\frac{\underline{P}}{z_0}\right)^*\right\} = \frac{1}{2z_0}\left|\underline{P}\right|^2 = \frac{1}{2}\mathrm{Real}\left\{\left(\underline{V}z_0\right)\underline{V}^*\right\} = \frac{1}{2}z_0\left|\underline{V}\right|^2 .$$

d.i. What are the units of sound intensity?

d.ii. How does intensity compare to power (units of Pa-m^3/s)? Explain why we equate sound intensity with power density.

d.iii We can also use decibels to quantify the relative intensity of a sound. Since intensity is proportional to energy,

$$\text{The dB value of an intensity } I = 10\log_{10}\left(\frac{|I|}{I_{Ref}}\right).$$

What is the reference intensity level I_{Ref} for dB SPL?

Problem 2. The magnitude of sound pressures.

Sound pressures with an amplitude of 1 pascal are rather loud, e.g. the sound level that results from a very loud voice. This question asks you to compute how much of a change in elevation is required to produce a change in atmospheric pressure of 1 pascal. (In answering this question, there is no need to maintain an accuracy greater than 2 decimal places.)

The table on the right lists the pressure produced by the atmosphere as a function of elevation above sea level

Elevation in meters	Atmospheric pressure in pascals
Sea level (0 meters)	101,300 pascals
+ 500	95,400
+1000	89,800

a. Assuming a linear relationship between elevation and atmospheric pressure near sea level, what is the change in elevation required to produce an atmospheric pressure of 1 Pa above the sea level value?

b. The lowest audible sound pressure has an amplitude of about $2\text{x}10^{-5}$ Pa. What is the change in elevation required to produce an atmospheric pressure of $2\text{x}10^{-5}$ Pa above the sea level value?

c. If you are like me. You head bobs up and down by 2 to 3 cm when you walk. What kind of sound pressure variation would be produced by such a motion? Why is that variation not audible?

Problem 3. Time and Frequency (To be answered from your reading and with a little computation.)

Fourier's Theorem relates the temporal variations in a periodic signal to the summation of sinusoidal waves of different frequencies. Specifically for a waveform *p(t),* we can define a Fourier series:

$$p(t) = P_0 + \sum_{n=1}^{\infty} |P_n| \cos(n\omega t + \angle P_n)$$

where P_0 is a static pressure term, n is the component number, ω is the base radian frequency of the periodic signal and $|\underline{P}_n|$ and $\angle \underline{P}_n$ are the magnitudes and angles of the nth complex Fourier component.

(a) What is the difference between a *periodic* and an *aperiodic* waveform? Can both be described in terms of a Fourier series?

(b) Determine the Fourier series that describes the waveform in Figure 1.1. What is (are) the magnitudes and angles of the Fourier component(s) in the series?

(c) The periodic glottal pulse that is the fundamental source of 'voicing' is often described as a sawtooth function, e.g. Figure 2.

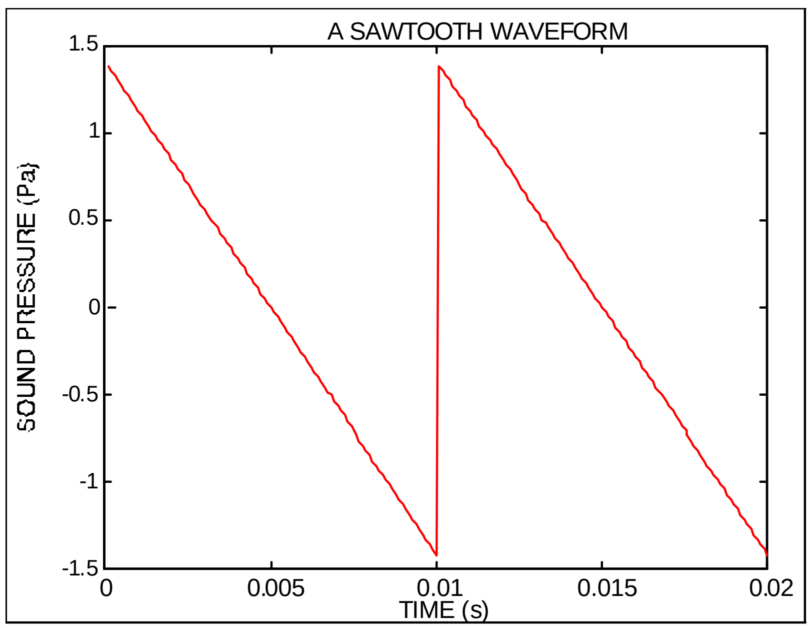


Figure 2: A saw-toothed description of the temporal variations in sound pressure associated with the glottal pulse.

Demonstrate (here's where the computation is necessary) that the waveform of Figure 2 can be approximated by the first four terms of the following infinite Fourier series, where A=1 and ω=2π100:

$$x(t) = A\cos(\omega t - \frac{\pi}{2}) + \frac{A}{2}\cos(2\omega t - \frac{\pi}{2}) + \frac{A}{3}\cos(3\omega t - \frac{\pi}{2}) + \ldots.$$

(d) Calculate the sum of the first four "odd" terms of the same infinite series, i.e.

$$y(t) = A\cos(\omega t - \frac{\pi}{2}) + \frac{A}{3}\cos(3\omega t - \frac{\pi}{2}) + \frac{A}{5}\cos(5\omega t - \frac{\pi}{2}) + \frac{A}{7}\cos(7\omega t - \frac{\pi}{2}) .$$

What's the common name for the waveform that is approximated by the above series?

(e) The reading for this week points out that difference in the phase relationship between different frequency components can lead to variations in time wave forms.

i) What are the phases of the Fourier components you used in part b & c of this problem?

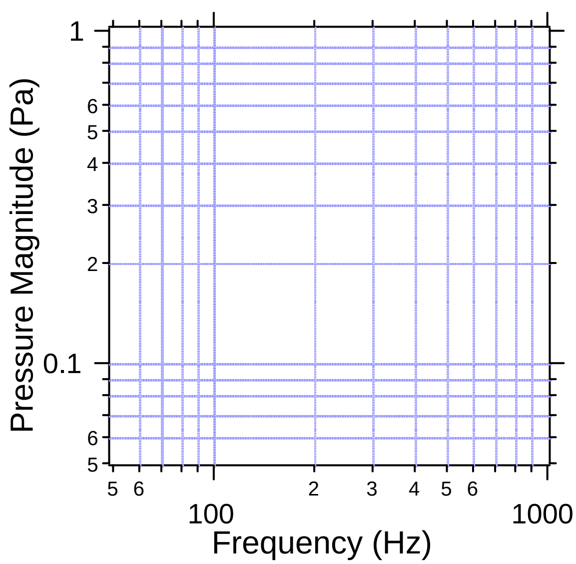
ii) Would you expect the same time waveform to result if the phase of the 2ω component in the series of 2c was set to 0?

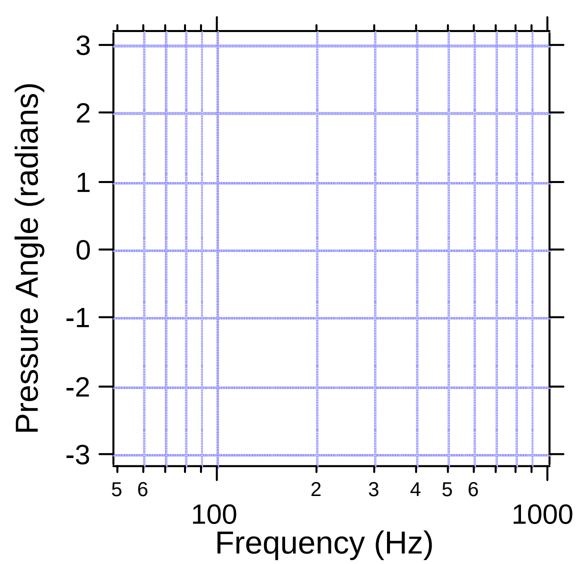
iii) Would you expect the same time waveform to result if the phases of all of the components in the series of 2c were set to 0?

(f) What is the rms sound pressure of the saw-toothed wave?

(g) What is the average intensity (in dB SPL) of the sound ?

(f) A common format for displaying the magnitude and angle of complex components that vary with frequency is the Bode plot, which is the combination of a log-log plot of component magnitude vs. frequency and a linear-log plot of the angle of the component vs. frequency. Prepare a Bode plot (using the scales below) of the first 6 components of the infinite series described in 2c. Assume
A = 1 Pa and $\omega = 2\pi 100$ Hz





Problem 4. One-dimensional wave propagation in a long duct: The acoustic transmission line

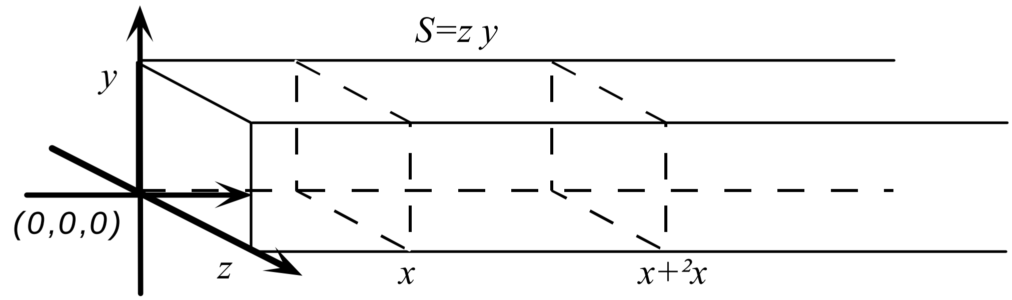


Figure 4.1: A rigid-walled rectangular duct.

It has been demonstrated that the 'Wave Equation' describing the variation of pressure within an infinite duct as a function of space and time can be simplified to one-dimension, e.g.

$$\frac{\partial^2 p(x,t)}{\partial x^2} = \frac{\rho_0}{B}\frac{\partial^2 p(x,t)}{\partial t^2},$$

as long as the dimensions orthogonal to the long axis of the duct (y and z) are much less than a wave length, z & $y << \lambda = c/f$.

In the case of a sinusoidal stimulus which has been on for some time ('the *Sinusoidal Steady State*), we can define the sound pressure at the entrance to the duct by:

$$p(0,t) = P\cos(\omega t)$$

where P is a real number and $\omega = 2\pi f$.

Assume a forward moving sound wave down the tube:

a.) How much time is required for the 'wave front', defined by the maximum in pressure that occurs at $x = 0$ at $t=0$, to propagate 1 meter down the tube?

b.) Assume the sinusoidal frequency f is 85 Hz and the propagation velocity of sound is 340 Hz. Sketch the relationship between pressure and time for $0 \leq t \leq 1/85$ seconds at $x=0$. On a separate set of axes, sketch the pressure observed at $x= 1$ meter over the same time period.

c.) How does the wave front defined by the region of maximum pressure travel along the duct? Does the pressure measured at $x=0$ foretell the future of what will occur at $x=1$ meter or recap the past? Explain.

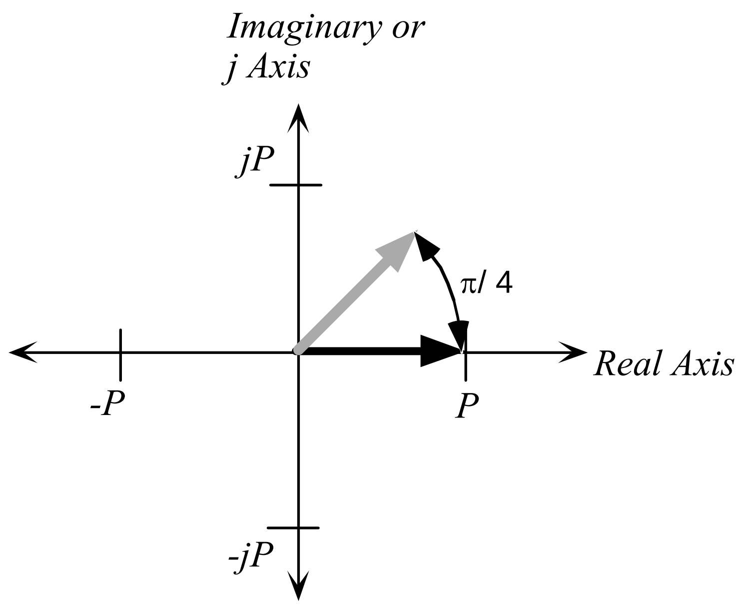


Figure 4.2. Phaser representation on the complex plane. The thick black arrow represents a cosine function $P\cos(0)$, while the gray arrow is the phaser representation of the function $P\cos(\pi/4)$.

d.) Lets look at the trading between time and distance in a different light. The dark thick arrow in Figure 4.2 is a 'phaser' display in the complex plane of the cosine function $P\cos(\omega t-kx)$ when $(\omega t-kx)=0$. The thick gray arrow in Figure 4.2 represents the 'phaser' of $P\cos(\omega t-kx)$ when $(\omega t-kx)=\pi/4$. Any change that increases the argument to the cosine function makes the phaser rotate in a counter-clockwise direction. Any change that decreases the argument to the cosine function makes the phaser rotate in a clockwise direction.

d.i) What is the Real component (projection on the Real axis) of the phaser when t=0? How about when $\omega t = \pi/4$?

d.ii) Sketch a plot of the real projection of the function $P\cos(\omega t)$ vs. ωt, as ωt varies between 0 and 2π.

d.iii) Sketch a phaser plot that describes the magnitude and phase angle of the pressure in the duct at time t=0 at three positions, a x=0, $\lambda/4$ and $\lambda/2$.

d.iv) At any one position in the duct, does advancing time cause a clockwise or counterclockwise rotation of the phaser? At any one time does moving forward in the duct cause a clockwise or counterclockwise rotation of the phaser?

d.v) Discuss in a paragraph, the notion that the motion of a wave front in space and time can be described in terms of a phaser display (like Figure 4.2) where the rotation of the phaser due to advancing time is offset by the rotation of the phaser due to increasing distance from the sound source.