MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 4 | 6.630 Electromagnetics | Issued: Week 4
Fall Term 2006 | Due: Week 5

Reading assignment: Section 1.5-1.6; J. A. Kong, "*Electromagnetic Wave Theory*," EMW, 2005.
Additional materials: Hertian dipole MATLAB scripts in Tools section.

Problem P4.1

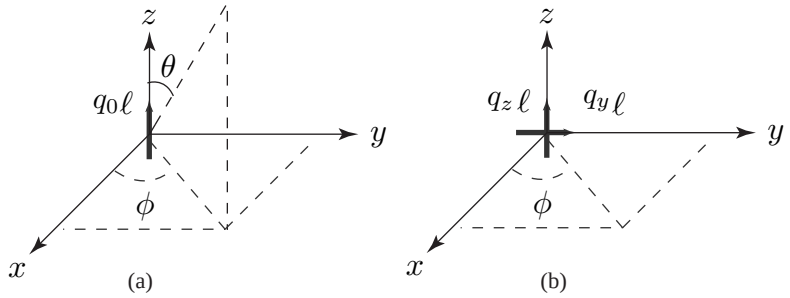

Fig.1

For a z-oriented dipole (Fig.1 a) the far field expression of $\overline{E}$ and $\overline{H}$ are:

$$\overline{E} = -\hat{\theta} k^2 q_0 \ell \frac{\cos(kr - \omega t)}{4\pi r \epsilon_0} \sin\theta \qquad \overline{H} = -\hat{\phi} \omega k q_0 \ell \frac{\cos(kr - \omega t)}{4\pi r} \sin\theta$$

Consider two identical dipoles oriented as shown in Fig.1 (b).
(a) What is the far field expression for E-field on the $x-y$ plane in the first quadrant?
(b) What is the time average Poynting power at $\phi = 0$ on the $x-y$ plane?

Problem P4.2

Two Hertzian dipole antennas are located at $(0,0,0)$ and $(0,d,0)$ with dipole moments $p_1 = q_1 l$ and $p_2 = q_2 l$ as shown in Figure 2. The two in phase dipoles are oriented in z and x direction respectively.

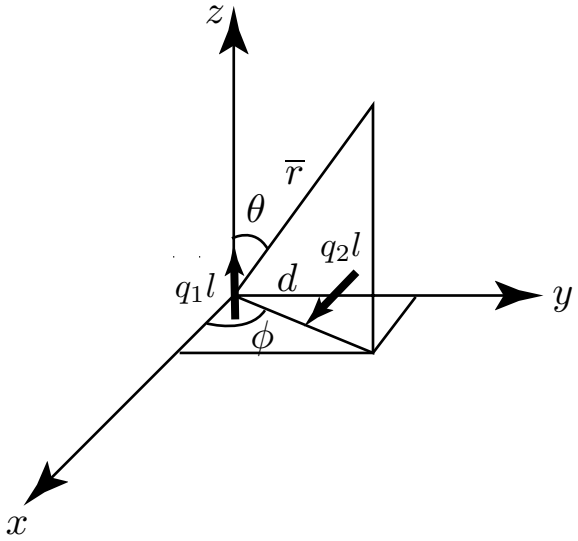

Fig.2 Two-diploe problem

(a) For the x-oriented dipole, the far field ($r \gg 1$) expression of $\overline{E}$ on the yz-plane is:

$$\overline{E}_2 = \hat{x}\frac{k^2 q_2 \ell}{4\pi r \epsilon_0}\cos(k\sqrt{x^2+(y-d)^2+z^2} - \omega t)$$

Show that as $d \ll \sqrt{x^2+y^2+z^2} = r$

$$\overline{E}_2 = \hat{x}\frac{k^2 q_2 \ell}{4\pi r \epsilon_0}\cos(kr - kd\sin\theta - \omega t)$$

(b) Find the total far field $\overline{E}$ on the yz-plane.

(c) Let q_1 and q_2 be real and positive. On the yz-plane, if the far field $\overline{E}$ for $\theta = 45°$ is circularly polarized,

(i) Find the minimum d in terms of λ.

(ii) What is the ratio of q_1/q_2 ?

(iii) Specify the handness of the circularly polarized field.

Problem P4.3

The gas laser depicted in Fig. 3 uses "Brewster angle" quartz windows on the gas discharge tube in order to minimize reflection losses. Determine the angle θ if the index of refraction for quartz at the wavelength of interest is $n = 1.46$. Because of these windows, the laser output is almost completely linearly polarized. What is the direction of polarization, i.e., is $\overline{E}$ parallel or perpendicular to the paper? Why?

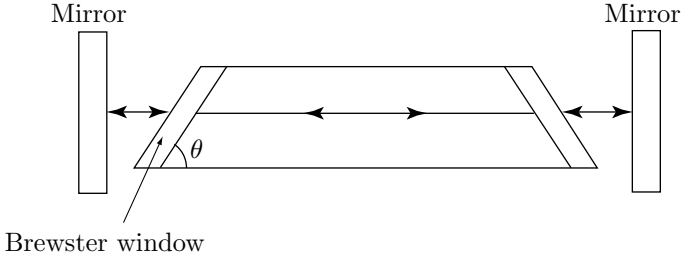

Fig.3 A gas laser with Brewster windows.

Problem P4.4

Sun light glares caused by reflections from plane surfaces are partially linearly polarized.

(a) Determine the Brewster angle for $\epsilon_t = 9\epsilon_o$. The Brewster angle, θ_B, is also called the polarization angle because at θ_B the reflected wave is entirely TE polarized.

(b) Your polaroid glasses absorb one linear component of incident light. To minimize sun glare, what component, TE or TM, reaches your eyes after passing through the glasses? Explain why.